#1. a) $y = \ln x^x = x \ln x$

$y' = \frac{d}{dx}[x] \cdot \ln x + x \cdot \frac{d}{dx}[\ln x] = \ln x + x \cdot \frac{1}{x} = \boxed{\ln x + 1}$

b) $y = \ln(2x+1)^{3x} = 3x \cdot \ln(2x+1) = \frac{d}{dx}[3x] \cdot \ln(2x+1) + 3x \cdot \frac{d}{dx}[\ln(2x+1)]$

$= 3\ln(2x+1) + 3x \cdot \frac{1}{2x+1} \cdot \overset{\text{CH. R}}{2} = \boxed{3\ln(2x+1) + \frac{6x}{2x+1}}$

c) $y = \ln \cos(x^2)$

$y' = \frac{1}{\cos(x^2)} \cdot -\sin(x^2) \cdot \overset{\text{CH. R.}}{2x} = \frac{-2x\sin(x^2)}{\cos(x^2)} = \boxed{-2x\tan(x^2)}$

#2. a) $y = (4x+3)^2(x^2-1)^3 \rightarrow \ln y = \ln(4x+3)^2(x^2-1)^3$

$\ln y = 2\ln(4x+3) + 3\ln(x^2-1)$

$\frac{1}{y} \cdot \frac{dy}{dx} = 2 \cdot \frac{1}{4x+3} \cdot \overset{\text{CH. R}}{4} + 3 \cdot \frac{1}{x^2-1} \cdot \overset{\text{CH. R}}{2x}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{8}{4x+3} + \frac{6x}{x^2-1} \rightarrow \frac{dy}{dx} = \left(\frac{8}{4x+3} + \frac{6x}{x^2-1}\right) \cdot y$

$\frac{dy}{dx} = \left(\frac{8}{4x+3} + \frac{6x}{x^2-1}\right)(4x+3)^2(x^2-1)^3$

b) $y = \frac{\sin^2 x}{x^2\sqrt{1+\tan x}} \rightarrow \ln y = \ln\left[\frac{\sin^2 x}{x^2\sqrt{1+\tan x}}\right]$

$$\ln y = 2\ln \sin x - 2\ln x - \tfrac{1}{2}\ln(1+\tan x)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 2\cdot\frac{1}{\sin x}\cdot\cos x - 2\cdot\frac{1}{x} - \frac{1}{2}\cdot\frac{1}{1+\tan x}\cdot\sec^2 x$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 2\cot x - \frac{2}{x} - \frac{\sec^2 x}{2(1+\tan x)}$$

$$\frac{dy}{dx} = \left(2\cot x - \frac{2}{x} - \frac{\sec^2 x}{2(1+\tan x)}\right)\cdot y$$

$$\boxed{\frac{dy}{dx} = \left(2\cot x - \frac{2}{x} - \frac{\sec^2 x}{2(1+\tan x)}\right)\cdot\frac{\sin^2 x}{x^2\sqrt{1+\tan x}}}$$

#3. a) $\int \frac{x^2}{x^3-1}\,dx$ $u = x^3-1$, $du = 3x^2\,dx$, $\frac{du}{3x^2} = dx$

$$\rightarrow \int \frac{\cancel{x^2}}{u}\cdot\frac{du}{3\cancel{x^2}} = \frac{1}{3}\int\frac{1}{u}\,du$$

$$= \frac{1}{3}\ln|u| = \boxed{\frac{1}{3}\ln|x^3-1| + c}$$

b) $\int \frac{\sec^2(2x)}{5-\tan(2x)}\,dx$ $u = 5-\tan(2x)$, $du = -2\sec^2(2x)\,dx$, $\frac{du}{-2\sec^2(2x)} = dx$

$$= \int \frac{\sec^2(2x)}{u}\cdot\frac{du}{-2\sec^2(2x)} = -\frac{1}{2}\int\frac{1}{u}\,du = -\frac{1}{2}\ln|u|$$

$$= \boxed{-\frac{1}{2}\ln|5-\tan(2x)| + c}$$

#4. $f(x)=\frac{x+1}{2x-1}$; $(1,2)$ $f(1)=\frac{1+1}{2\cdot1-1}=\frac{2}{1}=2$ ✓

$g'(2)=\frac{1}{f'(g(2))}$ $f'(x)=\frac{(2x-1)\cdot1-(x+1)\cdot2}{(2x-1)^2}$

$=\frac{1}{-3}=\boxed{\frac{-1}{3}}$ $=\frac{2x-1-2x-2}{(2x-1)^2}=\frac{-3}{(2x-1)^2}$

$f'(g(2))=f'(1)=-3$

SINCE $g=f^{-1}$, $f(1)=2\rightarrow f^{-1}(2)=1\rightarrow g(2)=1$

#5. $\lim_{x\to\infty}\left(\frac{5x^2-1}{x^2+7}\right)e^{-0.3T}=\left(\frac{5}{1}\right)\cdot e^{-\infty}=5\cdot0=\boxed{0}$

#6. a) $y=(e^{2x}-e^{-3x})^{1/2}\rightarrow y'=\frac{1}{2}(e^{2x}-e^{-3x})^{-1/2}\cdot\frac{d}{dx}[e^{2x}-e^{-3x}]$

$y'=\frac{1}{2}(e^{2x}-e^{-3x})^{-1/2}\cdot(2e^{2x}+3e^{-3x})=\boxed{\frac{2e^{2x}+3e^{-3x}}{2\sqrt{e^{2x}-e^{-3x}}}}$

CH.R.

b) $y=(\ln(x^2)+e^{2x})^3\rightarrow y'=3(\ln(x^2)+e^{2x})^2\cdot\frac{d}{dx}[\ln(x^2)+e^{2x}]$

$y'=3(\ln(x^2)+e^{2x})^2\cdot\left(\frac{1}{x^2}\cdot2x+2e^{2x}\right)=\boxed{3(\ln(x^2)+e^{2x})^2\left(\frac{2}{x}+2e^{2x}\right)}$

CH.R

c) $y=e^{x^2}\cos(3x)\rightarrow y'=\frac{d}{dx}[e^{x^2}]\cdot\cos(3x)+e^{x^2}\cdot\frac{d}{dx}[\cos(3x)]$

CH.R

$=2xe^{x^2}\cos(3x)+e^{x^2}(-\sin(3x)\cdot3)=2xe^{x^2}\cos(3x)-3e^{x^2}\sin(3x)$

$=\boxed{e^{x^2}(2x\cos(3x)-3\sin(3x))}$

#7. a) $\int (x^2-1)e^{x^3-3x}\,dx$

$u = x^3-3x$

$du = 3x^2-3\,dx$

$\frac{du}{3(x^2-1)} = dx$

$\rightarrow \int \cancel{(x^2-1)}e^{u}\frac{du}{3\cancel{(x^2-1)}}$

$= \frac{1}{3}\int e^u du = \frac{1}{3}e^u = \boxed{\frac{1}{3}e^{x^3-3x}+C}$

b) $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$

$u = e^x-e^{-x}$

$du = (e^x+e^{-x})dx$

$\frac{du}{e^x+e^{-x}} = dx$

$\rightarrow \int \frac{\cancel{e^x+e^{-x}}}{u}\cdot\frac{du}{\cancel{e^x+e^{-x}}}$

$= \int \frac{1}{u}\,du = \ln|u| = \boxed{\ln|e^x-e^{-x}|+C}$

#8. a) $y = 3^{\tan(x^2)} \rightarrow y' = (\ln 3)\cdot 3^{\tan(x^2)}\cdot\frac{d}{dx}[\tan(x^2)]$

$= (\ln 3)3^{\tan(x^2)}\cdot\sec^2(x^2)\cdot 2x$ CH. R $= \boxed{2x(\ln 3)\sec^2(x^2)\,3^{\tan(x^2)}}$

b) $y = x^2 5^{x^3} \rightarrow y' = \frac{d}{dx}[x^2]\cdot 5^{x^3} + x^2\cdot\frac{d}{dx}[5^{x^3}]$

$= 2x5^{x^3} + x^2(\ln 5)5^{x^3}\cdot\frac{d}{dx}[x^3] = 2x5^{x^3} + x^2(\ln 5)5^{x^3}\cdot 3x^2$

$= 2x5^{x^3} + 3x^4(\ln 5)5^{x^3} = \boxed{x5^{x^3}(2+3x^3\ln 5)}$

c) $y = \log(3x^2+1)^{1/2} \rightarrow y' = \frac{1}{(\ln 10)\sqrt{3x^2+1}}\cdot\frac{d}{dx}\left[(3x^2+1)^{1/2}\right]$

$= \frac{1}{(\ln 10)\sqrt{3x^2+1}}\cdot\frac{1}{\cancel{2}}(3x^2+1)^{-1/2}\cdot\cancel{6x}_{3}$ CHR. $= \frac{3x}{(\ln 10)\sqrt{3x^2+1}\cdot\sqrt{3x^2+1}}$

$= \boxed{\frac{3x}{(\ln 10)(3x^2+1)}}$

OR $y = \frac{1}{2}\log(3x^2+1) = \dfrac{\ln(3x^2+1)}{2(\ln 10)} = \dfrac{1}{2(\ln 10)} \cdot \ln(3x^2+1)$

$$y' = \frac{1}{2(\ln 10)} \cdot \frac{1}{(3x^2+1)} \cdot 6x = \boxed{\frac{3x}{(\ln 10)(3x^2+1)}}$$

#9. a) $y = \sin x^{\tan x} \rightarrow \ln y = \ln \sin x^{\tan x}$

$$\ln y = \tan x \ln \sin x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}[\tan x] \ln(\sin x) + \tan x \frac{d}{dx}[\ln(\sin x)]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sec^2 x \ln(\sin x) + \tan x \cdot \frac{1}{\sin x} \cdot \cos x = 1$$

$$\frac{dy}{dx} = (\sec^2 x \ln(\sin x) + 1) \cdot y = \boxed{(\sec^2 x \ln(\sin x) + 1) \sin x^{\tan x}}$$

b) $y = (x^2+2x)^{x^2} \rightarrow \ln y = \ln(x^2+2x)^{x^2}$

$$\ln y = x^2 \ln(x^2+2x) \rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}[x^2] \ln(x^2+2x) + x^2 \cdot \frac{d}{dx}[\ln(x^2+2x)]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x\ln(x^2+2x) + x^2 \cdot \frac{1}{x^2+2x} \cdot \underbrace{(2x+2)}_{\text{CH. R}}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2x\ln(x^2+2x) + \frac{2x^3+2x^2}{x^2+2x}$$

$$\frac{dy}{dx} = \left(2x\ln(x^2+2x) + \frac{2x^3+2x^2}{x^2+2x}\right) \cdot y = \boxed{\left(2x\ln(x^2+2x) + \frac{2x^3+2x^2}{x^2+2x}\right)(x^2+2x)^{x^2}}$$

#10. $\int 3^x \sec^2(3^x)\,dx$ $\quad u = 3^x$, $du = (\ln 3)3^x\,dx$, $\dfrac{du}{(\ln 3)3^x} = dx$ $\quad \rightarrow \int \cancel{3^x} \sec^2 u \cdot \dfrac{du}{(\ln 3)\cancel{3^x}}$

$$= \frac{1}{\ln 3}\int \sec^2 u\,du = \frac{1}{\ln 3}\tan u = \boxed{\frac{\tan(3^x)}{\ln 3} + C}$$

#11. a) $y = 4\sec^{-1}(x^2) \rightarrow y' = \dfrac{4}{|x^2|\sqrt{(x^2)^2-1}} \cdot \dfrac{d}{dx}[x^2]$

$$y' = \frac{8x}{\underbrace{|x^2|}_{\text{ALWAYS }(+)}\sqrt{x^4-1}} = \frac{8\cancel{x}}{x^{\cancel{2}}\sqrt{x^4-1}} = \boxed{\frac{8}{x\sqrt{x^4-1}}}$$

b) $y = e^{2x}\sin^{-1}(5x) \rightarrow y' = \dfrac{d}{dx}[e^{2x}]\sin^{-1}(5x) + e^{2x}\dfrac{d}{dx}[\sin^{-1}(5x)]$

$$y' = \underset{\text{CH.R}}{2}e^{2x}\sin^{-1}(5x) + e^{2x} \cdot \frac{1}{\sqrt{1-(5x)^2}} \cdot \overset{\text{CH.R}}{5}$$

$$y' = \boxed{2e^{2x}\sin^{-1}(5x) + \frac{5e^{2x}}{\sqrt{1-25x^2}}}$$

#12. a) $\int \dfrac{1}{\sqrt{1-16x^2}}\,dx \rightarrow \int \dfrac{1}{\sqrt{1-(4x)^2}}\,dx$ $\quad u = 4x$, $du = 4\,dx$, $\dfrac{du}{4} = dx$ $\quad \rightarrow \int \dfrac{1}{\sqrt{1-u^2}} \cdot \dfrac{du}{4}$

$$= \tfrac{1}{4}\sin^{-1} u = \boxed{\tfrac{1}{4}\sin^{-1}(4x) + C}$$

$$x = \frac{u}{3}$$

b) $\displaystyle\int \frac{1}{2x\sqrt{9x^2-1}}\,dx = \frac{1}{2}\int \frac{1}{x\sqrt{(3x)^2-1}}\,dx$ $\qquad u = 3x$, $du = 3dx$, $\frac{du}{3} = dx$

$$= \frac{1}{2}\int \frac{1}{\frac{u}{\cancel{3}}\sqrt{u^2-1}}\cdot\frac{du}{\cancel{3}} = \frac{1}{2}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$= \tfrac{1}{2}\sec^{-1}|u| = \boxed{\tfrac{1}{2}\sec^{-1}|3x| + C}$$

c) $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{e^x}{\sqrt{1-(e^x)^2}}\,dx$ $\qquad u = e^x$, $du = e^x dx$, $\frac{du}{e^x} = dx$

$$= \int \frac{\cancel{e^x}}{\sqrt{1-u^2}}\,\frac{du}{\cancel{e^x}} = \int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \sin^{-1}u = \boxed{\sin^{-1}(e^x) + C}$$

#13. a) $y = \cosh\left(\sqrt[3]{x^2-1}\right) = \cosh\left[(x^2-1)^{1/3}\right]$

$$y' = \sinh\left(\sqrt[3]{x^2-1}\right)\cdot\frac{d}{dx}\left[(x^2-1)^{1/3}\right]$$

CH.R

$$= \sinh\left(\sqrt[3]{x^2-1}\right)\,\frac{1}{3}(x^2-1)^{-2/3}\cdot 2x$$

$$y' = \boxed{\frac{2x\sinh\left(\sqrt[3]{x^2-1}\right)}{3(x^2-1)^{2/3}}}$$

b) $y = e^{-x}\operatorname{sech}(x^2) \to y' = \frac{d}{dx}\left[e^{-x}\right]\operatorname{sech}(x^2) + e^{-x}\cdot\frac{d}{dx}\left[\operatorname{sech}(x^2)\right]$

CH.R $\qquad$ CH.R.

$$= -e^{-x}\operatorname{sech}(x^2) + e^{-x}\left(-\operatorname{sech}(x^2)\tanh(x^2)\cdot 2x\right)$$

$$= -e^{-x}\operatorname{sech}(x^2) - 2xe^{-x}\operatorname{sech}(x^2)\tanh(x^2) = \boxed{-e^{-x}\left[\operatorname{sech}(x^2) + 2x\operatorname{sech}(x^2)\tanh(x^2)\right]}$$

c) $y = \sinh^{-1}(x^2) \to y' = \frac{1}{\sqrt{(x^2)^2+1}} \cdot \overset{\text{CH.R}}{2x} = \boxed{\frac{2x}{\sqrt{x^4+1}}}$

#14. $\int \frac{\sinh x}{1+\cosh x}\,dx \qquad u = 1+\cosh x$

$du = \sinh x\,dx \to \int \frac{\cancel{\sinh x}}{u} \cdot \frac{du}{\cancel{\sinh x}}$

$\frac{du}{\sinh x} = dx$

ALWAYS (+)

$= \int \frac{1}{u}\,du = \ln|u| = \ln|1+\cosh x| + c$

$= \boxed{\ln(1+\cosh x) + c}$

#15. a) $\lim_{x\to\infty} \frac{2x+\cos x}{4x+1} \overset{\div x}{=} \lim_{x\to\infty} \frac{\frac{2x}{x}+\frac{\cos x}{x}}{\frac{4x}{x}+\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{2+\frac{\cos x}{x}}{4+\frac{1}{x}} = \frac{2+0}{4+0} = \boxed{\frac{1}{2}}$

b) $\lim_{x\to 0} \frac{e^{x^2}-1}{1-\cos x} \xrightarrow{L'} \lim_{x\to 0} \frac{2xe^{x^2}}{\sin x} \xrightarrow{L'} \lim_{x\to 0} \frac{\overbrace{2e^{x^2}+4x^2e^{x^2}}^{\text{AFTER PROD. RULE}}}{\cos x}$

$= \frac{2\cdot e^0 + 4\cdot 0\cdot e^0}{\cos(0)} = \frac{2\cdot 1+0}{1} = \boxed{2}$

c) $\lim_{x\to 0^+} x^{\sin x} \to e^{\lim_{x\to 0^+} \sin x \ln x} = e^{\lim_{x\to 0^+}\left[\frac{\ln x}{\csc x}\right]}$

$\xrightarrow{L'} e^{\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x}} = e^{\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{\sin x}\cdot\frac{\cos x}{\sin x}}} = e^{\lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}}}$

$= e^{\lim_{x\to 0^+} \frac{-\sin^2 x}{x\cos x}} = e^{\lim_{x\to 0^+} -\underbrace{\frac{\sin x}{x}}_{=1}\cdot\underbrace{\frac{\sin x}{\cos x}}_{\tan x}} = e^{-1\cdot 0} = e^0 = \boxed{1}$