IF YOU FIND A MISTAKE, PLEASE E-MAIL ME.

TEST #2 REVIEW:

#1. a) $\int x^2 e^x dx \qquad u = x^2 \qquad v = \int e^x dx$

$du = 2x\,dx \qquad v = e^x$

$\rightarrow x^2 e^x - \int e^x \cdot 2x\,dx = x^2 e^x - 2\int x e^x dx$ * $\qquad$ * $u = x \qquad v = \int e^x dx$

$du = dx \qquad v = e^x$

$\rightarrow x e^x - \int e^x dx = x e^x - e^x$

$\Rightarrow x^2 e^x - 2\left[x e^x - e^x\right] = x^2 e^x - 2x e^x + 2e^x = \boxed{e^x(x^2 - 2x + 2) + C}$

b) $\int x^5 \ln x\,dx \qquad u = \ln x \qquad v = \int x^5 dx = \frac{1}{6}x^6$

$du = \frac{1}{x}dx$

$\rightarrow \frac{1}{6}x^6 \ln x - \int \frac{1}{6}x^6 \cdot \frac{1}{x}dx = \frac{1}{6}x^6 \ln x - \frac{1}{6}\int x^5 dx$

$= \frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 = \boxed{\frac{1}{6}x^6\left(\ln x - \frac{1}{6}\right) + C}$

c) $\int e^x \sin 2x\,dx \qquad u = e^x \qquad v = \int \sin 2x\,dx = -\frac{1}{2}\cos 2x$

$du = e^x dx$

$\rightarrow -\frac{1}{2}e^x \cos 2x - \int -\frac{1}{2}\cos 2x \cdot e^x dx$

$= -\frac{1}{2}e^x \cos 2x + \frac{1}{2}\int e^x \cos 2x\,dx$ * $\qquad u = e^x \qquad v = \int \cos 2x\,dx$

$du = e^x dx \qquad = \frac{1}{2}\sin 2x$

$* = \frac{1}{2}e^x \sin 2x - \int \frac{1}{2}\sin 2x\, e^x dx = \frac{1}{2}e^x \sin 2x - \frac{1}{2}\int e^x \sin 2x\,dx$

$\Rightarrow \int e^x \sin 2x\,dx = -\frac{1}{2}e^x \cos 2x + \frac{1}{2}\left[\frac{1}{2}e^x \sin 2x - \frac{1}{2}\int e^x \sin 2x\,dx\right]$

$$\int e^x \sin 2x\,dx = -\tfrac{1}{2}e^x\cos 2x + \tfrac{1}{4}e^x \sin 2x - \tfrac{1}{4}\int e^x \sin 2x\,dx$$

$$+\tfrac{1}{4}\int \qquad\qquad +\tfrac{1}{4}\int$$

$$\tfrac{5}{4}\int e^x \sin 2x\,dx = -\tfrac{1}{2}e^x\cos 2x + \tfrac{1}{4}e^x \sin 2x$$

$$\int e^x \sin 2x\,dx = -\tfrac{2}{5}e^x\cos 2x + \tfrac{1}{5}e^x\sin 2x + c$$

$$= \boxed{\tfrac{1}{5}e^x(\sin(2x) - 2\cos(2x)) + c}$$

#2. a) $\int \sin^3 x \cos^2 x\,dx = \int \sin^2 x \cos^2 x \sin x\,dx$

$$= \int (1-\cos^2 x)\cos^2 x \underline{\sin x\,dx} \qquad u = \cos x \qquad du = -\sin x\,dx$$

$$-du = \underline{\sin x\,dx}$$

$$= \int (1-u^2)u^2 \cdot -du = -\int u^2 - u^4\,du$$

$$= -(\tfrac{1}{3}u^3 - \tfrac{1}{5}u^5) = \boxed{-\tfrac{1}{3}\cos^3 x + \tfrac{1}{5}\cos^5 x + c}$$

b) $\int \sin^4 x \cos^2 x\,dx = \int [\sin^2 x]^2 \cos^2 x\,dx$

$$= \int \left[\frac{1}{2}(1-\cos 2x)\right]^2 \cdot \frac{1}{2}(1+\cos 2x)\,dx$$

$$= \int \frac{1}{4}(1-\cos 2x)^2 \cdot \frac{1}{2}(1+\cos 2x)\,dx$$

$$= \tfrac{1}{8}\int (1 - 2\cos 2x + \cos^2 2x)(1+\cos 2x)\,dx$$

$$= \tfrac{1}{8}\int 1 + \cos 2x - 2\cos 2x - 2\cos^2 2x + \cos^2 2x + \cos^3 2x\,dx$$

$$= \tfrac{1}{8}\int 1 - \cos 2x - \cos^2 2x + \cos^3 2x\,dx$$

$$= \frac{1}{8}\int 1 - \cos 2x - \left[\frac{1}{2}(1+\cos 4x)\right] + \cos^2(2x)\cos 2x\,dx$$

$$= \frac{1}{8}\int 1 - \cos 2x - \frac{1}{2} - \frac{1}{2}\cos(4x) + (1-\sin^2 2x)\cos 2x\,dx$$

$$= \frac{1}{8}\int \frac{1}{2} - \cos 2x - \frac{1}{2}\cos(4x)\,dx + \frac{1}{8}\int(1-\sin^2 2x)\underline{\cos 2x\,dx}$$

$u = \sin 2x$

$du = 2\cos 2x\,dx$

$\frac{1}{2}du = \underline{\cos 2x\,dx}$

$$= \frac{1}{8}\left(\frac{1}{2}x - \frac{1}{2}\sin 2x - \frac{1}{8}\sin 4x\right) + \frac{1}{16}\int 1 - u^2\,du$$

$$= \frac{1}{16}x - \frac{1}{16}\sin 2x - \frac{1}{64}\sin 4x + \frac{1}{16}\left(u - \frac{1}{3}u^3\right)$$

$$= \frac{1}{16}x - \cancel{\frac{1}{16}\sin 2x} - \frac{1}{64}\sin 4x + \cancel{\frac{1}{16}\sin 2x} - \frac{1}{48}\sin^3 2x$$

$$= \frac{1}{16}x - \frac{1}{64}\sin(4x) - \frac{1}{48}\sin^3(2x) + C$$

c) $\int \tan^3 x \sec^4 x\,dx$; $\int \tan^2 x \sec^3 x \cdot \sec x \tan x\,dx$

$$= \int(\sec^2 x - 1)\sec^3 x \cdot \underline{\sec x \tan x\,dx}$$

$u = \sec x$

$du = \underline{\sec x \tan x\,dx}$

$$= \int(u^2-1)u^3\,du = \int u^5 - u^3\,du = \frac{1}{6}u^6 - \frac{1}{4}u^4$$

$$= \boxed{\frac{1}{6}\sec^6 x - \frac{1}{4}\sec^4 x + C}$$

d) $\int \tan^2 x \sec^4 x\,dx = \int \tan^2 x \cdot \sec^2 x \cdot \sec^2 x\,dx$

$$= \int \tan^2 x(\tan^2 x + 1)\cdot \underline{\sec^2 x\,dx}$$ $u = \tan x$ $du = \underline{\sec^2 x\,dx}$

$$= \int u^2(u^2+1)\,du = \int u^4 + u^2\,du = \frac{1}{5}u^5 + \frac{1}{3}u^3$$

$= \boxed{\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C}$

e) $\int \csc^4 x \cot^4 x \, dx = \int \csc^2 x \cot^4 x \cdot \csc^2 x \, dx$

$= \int (\cot^2 x + 1)\cot^4 x \cdot \underline{\csc^2 x \, dx}$ $\quad u = \cot x \quad du = -\csc^2 x \, dx$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad -du = \underline{\csc^2 x \, dx}$

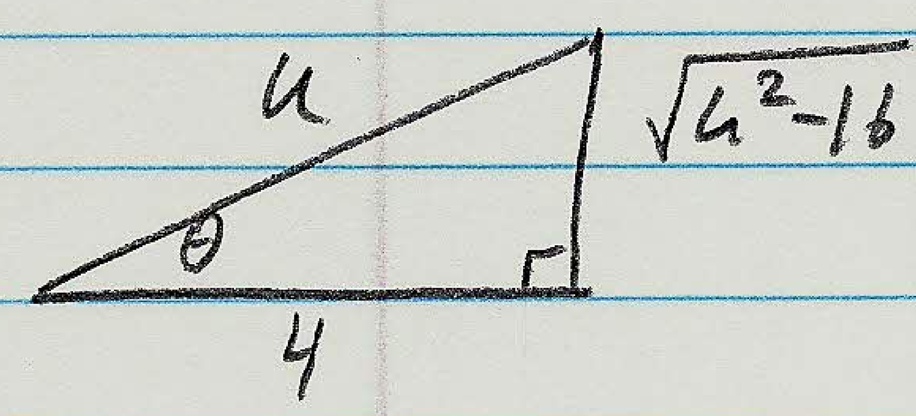
$= \int (u^2+1)\, u^4 \cdot -du$

$= -\int u^6 + u^4 \, du = -\left(\frac{1}{7}u^7 + \frac{1}{5}u^5\right) = \boxed{-\frac{1}{7}\cot^7 x - \frac{1}{5}\cot^5 x + C}$

#3. a) $\int \frac{1}{\sqrt{49x^2-16}} \, dx = \int \frac{1}{\sqrt{(7x)^2-16}} \, dx$ $\quad u = 7x \to du = 7dx$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{1}{7} du = dx$

$= \int \frac{1}{\sqrt{u^2-16}} \cdot \frac{1}{7} \, du = \frac{1}{7}\int \frac{1}{\sqrt{u^2-16}} \, du \to$



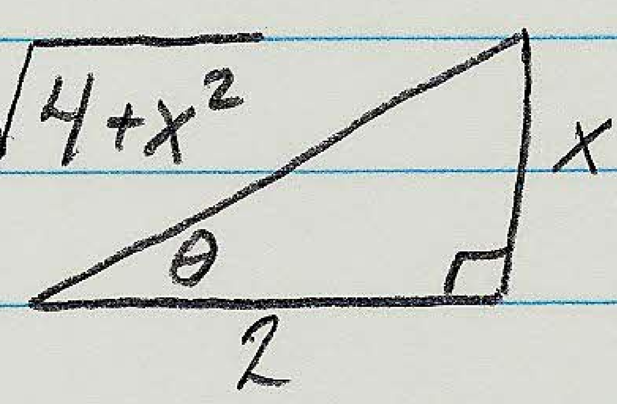
$\sec\theta = u/4 \to 4\sec\theta = u \to 4\sec\theta\tan\theta \, d\theta = du$

$\to \frac{1}{7}\int \frac{4\sec\theta\tan\theta}{\sqrt{(4\sec\theta)^2-16}} \, d\theta = \frac{1}{7}\int \frac{\cancel{4}\sec\theta\tan\theta}{\cancel{4}\sqrt{\sec^2\theta-1}} \, d\theta$

$= \frac{1}{7}\int \frac{\sec\theta\,\cancel{\tan\theta}}{\cancel{\sqrt{\tan^2\theta}}} \, d\theta = \frac{1}{7}\int \sec\theta \, d\theta = \frac{1}{7}\ln\left|\sec\theta + \tan\theta\right|$

$= \frac{1}{7}\ln\left|\frac{u}{4} + \frac{\sqrt{u^2-16}}{4}\right| = \boxed{\frac{1}{7}\ln\left|\frac{7x}{4} + \frac{\sqrt{49x^2-16}}{4}\right| + C}$

b) $\int \frac{1}{\left(\sqrt{4+x^2}\right)^3} \, dx \to$



$\tan\theta = x/2 \quad 2\tan\theta = x$

$2\sec^2\theta \, d\theta = dx$

$= \int \frac{2\sec^2\theta \, d\theta}{\left[\sqrt{4+(2\tan\theta)^2}\right]^3} = \int \frac{2\sec^2\theta}{\left[2\sqrt{1+\tan^2\theta}\right]^3} \, d\theta = \int \frac{2\sec^2\theta \, d\theta}{8\left[\sqrt{\sec^2\theta}\right]^3}$

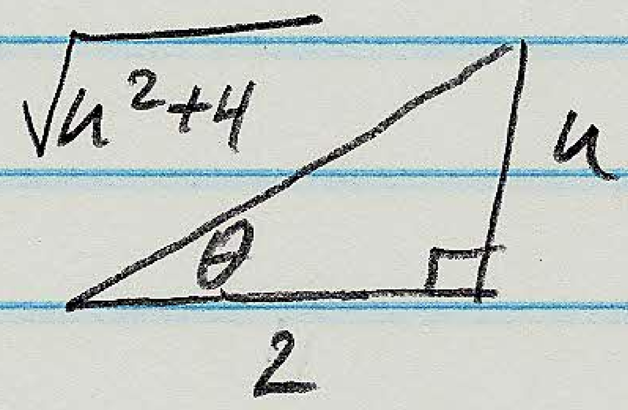
$$\frac{1}{4}\int \frac{\sec^2\theta}{\sec^3\theta}\,d\theta = \frac{1}{4}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{4}\int \cos\theta\,d\theta = \frac{1}{4}\sin\theta$$

$$= \frac{1}{4}\,\frac{x}{\sqrt{4+x^2}} = \frac{x}{4\sqrt{4+x^2}} + c$$

c) $\displaystyle\int \frac{\sqrt{(3x)^2+4}}{x^4}\,dx$ $\qquad u = 3x \rightarrow x = u/3$

$\qquad du = 3dx \rightarrow 1/3\,du = dx$

$$= \int \frac{\sqrt{u^2+4}}{(u/3)^4}\cdot\frac{1}{3}\,du = \int \frac{\sqrt{u^2+4}}{u^4/81}\,\frac{du}{3} = 27\int \frac{\sqrt{u^2+4}}{u^4}\,du$$



$\tan\theta = \frac{u}{2} \rightarrow 2\tan\theta = u \rightarrow 2\sec^2\theta\,d\theta = du$

$$= 27\int \frac{\sqrt{(2\tan\theta)^2+4}}{(2\tan\theta)^4}\cdot 2\sec^2\theta\,d\theta$$

$$= 27\int \frac{2\sqrt{\tan^2\theta+1}}{16\tan^4\theta}\cdot 2\sec^2\theta\,d\theta = \frac{27}{4}\int \frac{\sqrt{\sec^2\theta}\cdot\sec^2\theta\,d\theta}{\tan^4\theta}$$

$$= \frac{27}{4}\int \frac{\sec^3\theta}{\tan^4\theta}\,d\theta \qquad \frac{\frac{1}{\cos^3\theta}}{\frac{\sin^4\theta}{\cos^4\theta}} = \frac{\cos\theta}{\sin^4\theta}$$

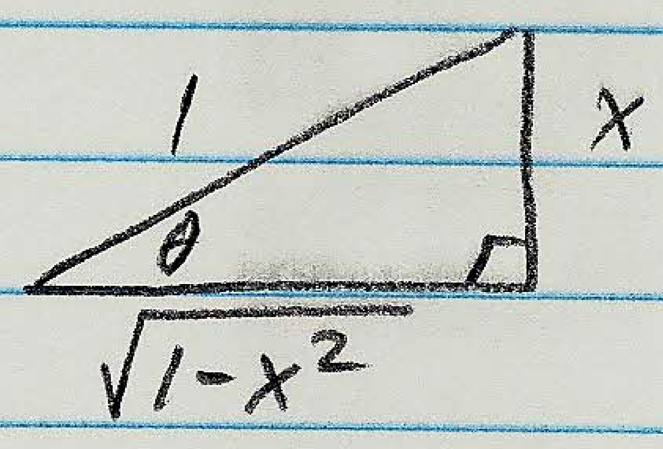
$$= \frac{27}{4}\int \frac{\cos\theta}{\sin^4\theta}\,d\theta \qquad \begin{aligned} u &= \sin\theta \\ du &= \underline{\cos\theta\,d\theta} \end{aligned} \qquad = \frac{27}{4}\int \frac{1}{u^4}\,du$$

$$= \frac{27}{4}\int u^{-4}\,du = \frac{27}{4}\cdot\frac{u^{-3}}{-3} = -\frac{9}{4}\cdot\frac{1}{u^3} = \frac{-9}{4\sin^3\theta}$$

$$= -9/4\csc^3\theta = \frac{-9}{4}\left[\frac{\sqrt{u^2+4}}{u}\right]^3 = \frac{-9\left(\sqrt{9x^2+4}\right)^3}{4(3x)^3} = \boxed{\frac{-(9x^2+4)^{3/2}}{12x^3} + c}$$

$4\cdot 27x^3$

d) $\displaystyle\int \frac{1}{x\sqrt{1-x^2}}\,dx$



$\sin\theta = x \qquad \cos\theta\, d\theta = dx$

$$= \int \frac{1}{\sin\theta\sqrt{1-\sin^2\theta}}\cdot\cos\theta\, d\theta = \int \frac{\cancel{\cos\theta}}{\sin\theta\,\cancel{\sqrt{\cos^2\theta}}}\,d\theta = \int \csc\theta\, d\theta$$

$$= \ln\left|\csc\theta - \cot\theta\right| = \ln\left|\frac{1}{x} - \frac{\sqrt{1-x^2}}{x}\right| = \boxed{\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| + C}$$

#4. a) $\displaystyle\int \frac{x^3-2x+7}{(x+2)(x-1)}\,dx$

$$\begin{array}{r l} & x - 1 \\ x^2+x-2 \; \big) & x^3+0x^2-2x+7 \\ & -(x^3+x^2-2x) \\ & -x^2+0x+7 \\ & (-x^2-x+2) \\ & x+5 \end{array}$$

$= x-1 + \dfrac{x+5}{x^2+x-2}$ ★

★ $\dfrac{x+5}{x^2+x-2} = \dfrac{x+5}{(x+2)(x-1)} = \dfrac{A}{x+2} + \dfrac{B}{x-1}$

$x+5 = A(x-1) + B(x+2)$, $x=1;\ 6 = 3B \rightarrow B=2$

$x=-2;\ 3 = -3A \rightarrow A=-1$

$$\int x-1 + \frac{-1}{x+2} + \frac{2}{x-1}\,dx = \boxed{\begin{array}{c} \tfrac{1}{2}x^2 - x - \ln|x+2| + 2\ln|x-1| + C \\ \text{OR} \\ \tfrac{1}{2}x^2 - x + \ln\left|\dfrac{(x-1)^2}{x+2}\right| + C \end{array}}$$

b) $\displaystyle\int \frac{x^2-x-21}{(2x-1)(x^2+4)}\,dx$, $\quad \dfrac{x^2-x-21}{(2x-1)(x^2+4)} = \dfrac{A}{2x-1} + \dfrac{Bx+C}{x^2+4}$

$x^2-x-21 = A(x^2+4) + (Bx+C)(2x-1)$, $x=\tfrac{1}{2};\ -\tfrac{85}{4} = \tfrac{17}{4}A \rightarrow A=-5$

$x^2-x-21 = Ax^2+4A+2Bx^2-Bx+2Cx-C$

$x^2-x-21 = (A+2B)x^2 + (-B+2C)x + (4A-C)$

$A+2B=1 \rightarrow B=3$

$4A-C=-21 \rightarrow C=1$

$$\int \frac{-5}{2x+1} + \frac{3x+1}{x^2+4}\,dx = \int \frac{-5}{2x+1} + \frac{3x}{x^2+4} + \frac{1}{x^2+4}\,dx$$

$u=2x+1$ $\quad u=x^2+4$

$du=2dx$ $\quad du=2x\,dx$

$$= -\frac{5}{2}\ln|2x+1| + \frac{3}{2}\ln|x^2+4| + \int \frac{1}{4\left(\left(\frac{x}{2}\right)^2+1\right)}\,dx \quad \frac{1}{4}\int \frac{1}{u^2+1}\cdot 2du$$

$u=x/2 \rightarrow du=\frac{1}{2}dx \rightarrow 2du=dx$

$\rightarrow \frac{1}{2}\tan^{-1}u = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$

$$= \boxed{-5/2\ln|2x+1| + 3/2\ln(x^2+4) + 1/2\tan^{-1}\left(\frac{x}{2}\right) + C}$$

c) $\displaystyle\int \frac{x^3-x+2}{x^3+2x^2+x}\,dx$

$$\begin{array}{r} 1 \\ x^3+2x^2+x\;\overline{)\,x^3+0x^2-x+2} \\ -(x^3+2x^2+x)\downarrow \\ \hline -2x^2-2x+2 \end{array}$$

$$1+\frac{-2x^2-2x+2}{x^3+2x^2+x} = 1-\frac{2x^2+2x-2}{x^3+2x^2+x}$$

$$\frac{2x^2+2x-2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$2x^2+2x-2 = A(x+1)^2 + Bx(x+1) + Cx \quad x=0;\ -2=A$

$x=-1;\ -2=-C \rightarrow C=2,\ x=1;\ 2=4A+2B+C,\ B=4$

$$\rightarrow \int 1-\left[\frac{-2}{x}+\frac{4}{x+1}+\frac{2}{(x+1)^2}\right]dx = \int 1+\frac{2}{x}-\frac{4}{x+1}-\frac{2}{(x+1)^2}\,dx$$

$u=x+1 \quad du=dx$

$$= \boxed{x+2\ln|x|-4\ln|x+1|+\frac{2}{x+1} \overset{\text{OR}}{=} x+\ln\left(\frac{x^2}{(x+1)^4}\right)+\frac{2}{x+1}+C}$$

#5. a) $\int_0^e \ln x\,dx = \lim_{a\to 0^+} \int_a^e \ln x\,dx = \lim_{a\to 0^+} \left[x\ln x - x\right]_a^e$

$= \lim_{a\to 0^+} \left[(e\ln e - e) - (\overset{0}{\cancel{a\ln a}} - a)\right] \to \lim_{a\to 0^+} \frac{\ln a}{\frac{1}{a}} \overset{L'}{\to} \lim_{a\to 0^+} \frac{1/a}{-1/a^2}$

$= e\cdot 1 - e - 0 = \boxed{0}$ $\qquad = \lim_{a\to 0^+} -a = 0$

b) $\int_{-\infty}^0 \frac{1}{x^2+4}\,dx + \int_0^\infty \frac{1}{x^2+4}\,dx$

$= \lim_{a\to -\infty} \int_a^0 \frac{1}{x^2+4}\,dx + \lim_{b\to \infty} \int_0^b \frac{1}{x^2+4}\,dx$

$^*\int \frac{1}{x^2+4}\,dx = \int \frac{1}{4\left[\left(\frac{x}{2}\right)^2+1\right]}\,dx \qquad u = \frac{x}{2} \to du = \frac{1}{2}dx,\ 2du = dx$

$= \int \frac{1}{4(u^2+1)}\cdot 2du = \frac{1}{2}\int \frac{1}{u^2+1}\,du = \frac{1}{2}\tan^{-1} u = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$

$\to \lim_{a\to -\infty} \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_a^0 + \lim_{b\to \infty} \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^b$

$= \lim_{a\to -\infty} \frac{1}{2}\left[\tan^{-1} 0 - \tan^{-1} \underbrace{\frac{a}{2}}_{-\infty}\right] + \lim_{b\to \infty} \frac{1}{2}\left[\tan^{-1} \underbrace{\frac{b}{2}}_{\infty} - \tan^{-1} 0\right]$

$= \frac{1}{2}\left(0 - \left(-\frac{\pi}{2}\right)\right) + \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4} + \frac{\pi}{4} = \boxed{\frac{\pi}{2}}$

c) $\int_{-\infty}^{-1} x^{-4/3}\,dx + \int_{-1}^{0} x^{-4/3}\,dx + \int_0^1 x^{-4/3}\,dx + \int_1^\infty x^{-4/3}\,dx$

$\to \lim_{b\to 0^-} \left[-3x^{-1/3}\right]_{-1}^b = \lim_{b\to 0^+} \left[\overset{-\infty}{\cancel{-\frac{3}{\sqrt[3]{b}}}} + \frac{3}{\sqrt[3]{-1}}\right] = -\infty$

SO, THIS INTEGRAL <u>DIVERGES</u>

#6. a) $\frac{dy}{dx} = e^{2x} y \rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = e^{2x}$

$\int \left[\frac{1}{y} \cdot \frac{dy}{\cancel{dx}}\right] \cancel{dx} = \int e^{2x} dx \rightarrow \ln|y| = \frac{1}{2} e^{2x} + c$

$y(0) = 1 \rightarrow \ln(1) = \frac{1}{2} e^{2 \cdot 0} + c$

$0 = \frac{1}{2} + c \rightarrow c = -\frac{1}{2}$

$\ln|y| = \frac{1}{2} e^{2x} - \frac{1}{2}$

$\rightarrow 2\ln|y| = e^{2x} - 1 \rightarrow \boxed{\ln(y^2) = e^{2x} - 1}$

b) $\frac{dy}{dx} = \frac{y^2}{x-2} \rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{1}{x-2}$

$\int \left[\frac{1}{y^2} \frac{dy}{\cancel{dx}}\right] \cancel{dx} = \int \frac{1}{x-2} dx \rightarrow -\frac{1}{y} = \ln|x-2| + c$

$y(3) = 1 \rightarrow -\frac{1}{1} = \ln|3-2| + c \rightarrow -1 = \ln(1) + c \rightarrow c = -1$

$-\frac{1}{y} = \ln|x-2| - 1 \rightarrow \frac{1}{y} = 1 - \ln|x-2|$

$$\boxed{y = \frac{1}{1 - \ln|x-2|}}$$