Coursework (5) for *Introductory Lectures on Optimization*

Your name
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Excercise 1. Prove the following theorem:
for any $\boldsymbol{x}_0 \in \mathrm{dom}\ f$, all vectors $\boldsymbol{g} \in \partial f(\boldsymbol{x}_0)$ are supporting to the level set $\mathcal{L}_f(f(\boldsymbol{x}_0))$:

$$\langle \boldsymbol{g},\ \boldsymbol{x}_0 - \boldsymbol{x} \rangle \geq 0, \quad \forall \boldsymbol{x} \in \mathcal{L}_f(f(\boldsymbol{x}_0)) \equiv \{\boldsymbol{x} \in \mathrm{dom}\ f : f(\boldsymbol{x}) \leq f(\boldsymbol{x}_0)\}.$$

Proof of Excercise 1: bla.bla... bla bla.. bla. □

Excercise 2. Prove the following theorem:
let $Q \subseteq \mathrm{dom}\ f$ be a closed convex set, $\boldsymbol{x}_0 \in Q$ and

$$\boldsymbol{x}^* = \mathrm{argmin}\{f(\boldsymbol{x}) | \boldsymbol{x} \in Q\}.$$

Then for any $g \in \partial f(\boldsymbol{x}_0)$ we have $\langle \boldsymbol{g},\ \boldsymbol{x}_0 - \boldsymbol{x}^* \rangle \geq 0$.

Proof of Excercise 2: bla.bla... bla bla.. bla. □

Excercise 3. Prove the following theorem:
let f be closed and convex. Assume that it is differentiable on its domain. Then $\partial f(\boldsymbol{x}) = \{\nabla f(\boldsymbol{x})\}$ for any $\boldsymbol{x} \in \mathrm{int}(\mathrm{dom}\ f)$.

Proof of Excercise 3: bla.bla... bla bla.. bla. □