Coursework (5) for *Introductory Lectures on Optimization*

Your name
Your ID

Nov. 29, 2024

Excercise 1. Prove the following theorem:
Let $\|\cdot\|$ be a vector norm in $\mathbb{R}^n$, then

$$\partial \|\cdot\| = \left\{ V(\boldsymbol{x}) \triangleq \left\{ \boldsymbol{v} \in \mathbb{R}^n \middle| \langle \boldsymbol{v},\ \boldsymbol{x} \rangle = \|\boldsymbol{x}\|, \|\boldsymbol{v}\|_* \leq 1 \right\} \right\},$$

where $\|\boldsymbol{v}\|_*$ is the dual norm of $\|\cdot\|$, defined as

$$\|\boldsymbol{v}\|_* \triangleq \sup_{\|\boldsymbol{u}\| \leq 1} \langle \boldsymbol{v},\ \boldsymbol{u} \rangle.$$

Proof of Excercise 1: bla.bla... bla bla.. bla. □

Excercise 2. Write down the subdifferentials of following functions.

1. $f(\boldsymbol{x}) = |\boldsymbol{x}|, \boldsymbol{x} \in \mathbb{R}^1$.
2. $f(\boldsymbol{x}) = \sum_{i=1}^{m} |\langle \boldsymbol{a}_i,\ \boldsymbol{x} \rangle - \boldsymbol{b}_i|$.
3. $f(\boldsymbol{x}) = \max_{1 \leq i \leq n} \boldsymbol{x}^{(i)}$.
4. $f(\boldsymbol{x}) = \|\boldsymbol{x}\|$.
5. $f(\boldsymbol{x}) = \|\boldsymbol{x}\|_1 = \sum_{i=1}^{n} |\boldsymbol{x}^{(i)}|$.

Solution of Excercise 2: bla.bla... bla bla.. bla. □

Excercise 3. Please write down three sequences and prove that they satisfy the following conditions:

$$h_k > 0, h_k \to 0, \sum_{k=0}^{\infty} h_k = \infty.$$

Solution of Excercise 3: bla.bla... bla bla.. bla. □