(3)

problem on SHAP

$N = \{A, B, c\}$

Subsets

ϕ	0
$\{A\}$	10
$\{B\}$	20
$\{c\}$	30
$\{A, B\}$	50
$\{A, c\}$	60
$\{B, c\}$	70
$\{A, B, c\}$	90

By formula

$$\text{shapely value} \quad \phi(A) = \sum_{S \in N^{i}} \underbrace{\frac{|s|! \, (N - |s| - 1)!}{|N|!}}_{\text{weight}} * \underbrace{(V(S \cup \{A\}) - V(s))}_{\substack{\text{Marginal} \\ \text{Contributib}}}$$

I   To calculate shapely value of A

Coalition (Excluding (Subset) A)	$V(S)$	$V(S \cup \{A\})$		$V(S \cup \{A\} - V(S)$
ϕ	0	10	$\{A\}$	10
$\{B\}$	20	50	$\{A, B\}$	30
$\{c\}$	30	60	$\{A, c\}$	30
$\{B, c\}$	70	90	$\{A, B, c\}$	20

Compute factor weights.

$$\frac{|S|! \; (N-|S|-1)!}{|N|!}$$

$N = 3 \qquad |N|! = 3! = 6$

Subset $s = \phi \; (0)$

$$\frac{|S|! \; (N-|S|-1)!}{|N|!} = \frac{0! \; (3-|0|-1)!}{3!} = \frac{1 * 2!}{6} = \boxed{\frac{1}{3}}$$

Subset $s = \{B\}$

$$\frac{1! \; (3-|1|-1)!}{3!} = \frac{1 \, (1)!}{6} = \boxed{\frac{1}{6}}$$

Subset $s = \{c\}$

$$|s| = 1$$

$$\frac{1! \; (3-|1|-1)!}{3!} = \boxed{1/6}$$

Subset $s = \{B, c\}$

$$|s| = 2$$

$$\frac{2! \; (3-|2|-1)!}{3!} = \frac{2! * 0!}{6} = \frac{2}{6} = \boxed{1/3}$$

Substitute all Values

$$\phi(A) = \frac{1}{3} * 10 + \frac{1}{6} * \overset{5}{\cancel{30}} + \frac{1}{\cancel{6}} * \overset{5}{\cancel{30}} + \frac{1}{3} * 20$$

$$\frac{10}{3} + 5 + 5 + \frac{20}{3} = \frac{60}{3}$$

$$\boxed{\phi(A) = 20}$$

Similarly Calculate shapely value of B ($\phi(B)$)

Coalition Group	V(S)	$V(S \cup \{B\})$	$V(S \cup \{B\} - V(S))$
ϕ	0	20 {B}	20
{A}	10	50 {A,B}	40
{c}	30	70 {B,c}	40
{A,c}	60	90 {A,B,c}	30

Compute weights.

$$S = 0 \{\phi\} = \frac{0! \ (3 - |0| - 1)!}{3!} = \frac{1 * 2!}{6} = 1/3$$

$$\text{Subset } S = \{A\} = \frac{1! \ (3 - |1| - 1)!}{3!} = \frac{1 * 1}{6} = \frac{1}{6}$$
$$(S = 1).$$

$$\text{Subset } S = \{c\} = \frac{1! \ (3 - |1| - 1)!}{.3!} = \frac{1 * 1}{6} = 1/6$$

$$\text{Subset } S = \{A,c\} = \frac{2! \ (3 - |2| - 1)!}{3!} = \frac{2 * 1}{6} = \frac{1}{3}$$

$$\phi(B) = \frac{1}{3} * 20 + \frac{1}{6} * 40 + \frac{1}{6} * 40 + \frac{1}{3} * 30$$

$$\frac{40}{6} + \frac{40}{6} + \frac{40}{6} + \frac{60}{6} = \frac{180}{6}$$

$$\boxed{\phi(B) = 30}$$

To Calculate shapely Value of C

	$V(S)$		$V(S \cup \{c\})$	$V(S \cup \{c\})$
Subset (Excluding c)				
ϕ	0	$\{c\}$ 30	30
$\{A\}$	10	$\{A,c\}$ 60	50
$\{B\}$	20	$\{B,c\}$ 70	50
$\{A,B\}$	50	$\{A,B,c\}$ 90	40

Compute factor weights.

$$S = \phi \implies \frac{0!\,(3-0-1)!}{3!} = 1/3$$

$$S = \{A\} \implies \frac{1!\,(3-|1|-1)!}{3!} = 1/6$$

$$S = \{B\} \implies \frac{1!\,(3-|1|-1)!}{3!} = \frac{1}{6}$$

$$S = \{A,B\}$$

$$\frac{2!\,(3-|2|-1)!}{3!} = 1/3$$

$$\phi(c) = \frac{1}{3}*30 + \frac{1}{6}*50 + \frac{1}{6}*50 + 1/3*40$$

$$\frac{60}{6} + \frac{50}{6} + \frac{50}{6} + \frac{80}{6}$$

$$= \frac{240}{6} = 40$$

$$\boxed{\phi(c) = 40}$$