Methods in Spatial Analysis
PS | LV.Nr. 856.141

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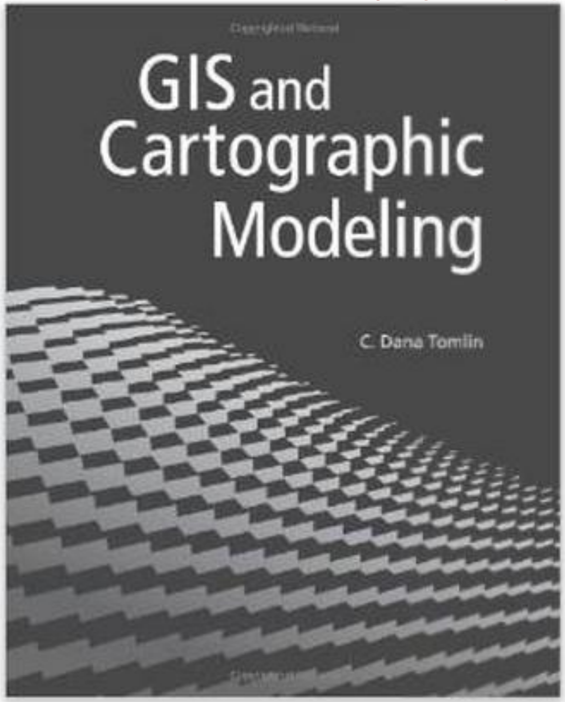
„Spatial Analysis“ || Map Algebra

History – Map Algebra

- **Ian McHarg**:
  - *Design with Nature (1969)*
  - „Layer method“
  - Theoretical basics for GIS developed(!)
- Software GRID, IMGRID (Harvard Labs) 1970ies
- **Dana Tomlin**, Joseph Berry, etc.
  - Map Analysis Package (Mid 1970ies – Early 1980ies)
- PhD Dana Tomlin (1983) >> Map Algebra
- C. D. Tomlin
  - *Geographic Information Systems and Cartographic Modeling* (1990)
  - *GIS and Cartographic Modeling* (2012)



Map Algebra

- **Map Algebra**
  - Not another type of spatial analysis but a „systematical view“ on analytical operations on raster data
  - Set-based algebraic approach to analyze spatial data
  - Defines a number of „operations“ in a GIS, that allow to analyze several raster data layers having similar resolution and extent!

- Properties:
  - The approach is flexible extendable
  - Universal modeling language
  - Concept can be extented to vector data as well

Map Algebra

- Mathematics with map
  - Cartographic Modeling
  - „mapemathics“
  - map modeling

- Attributes in space are represened by numerical values

- Modeling methodologies are based on mathematics

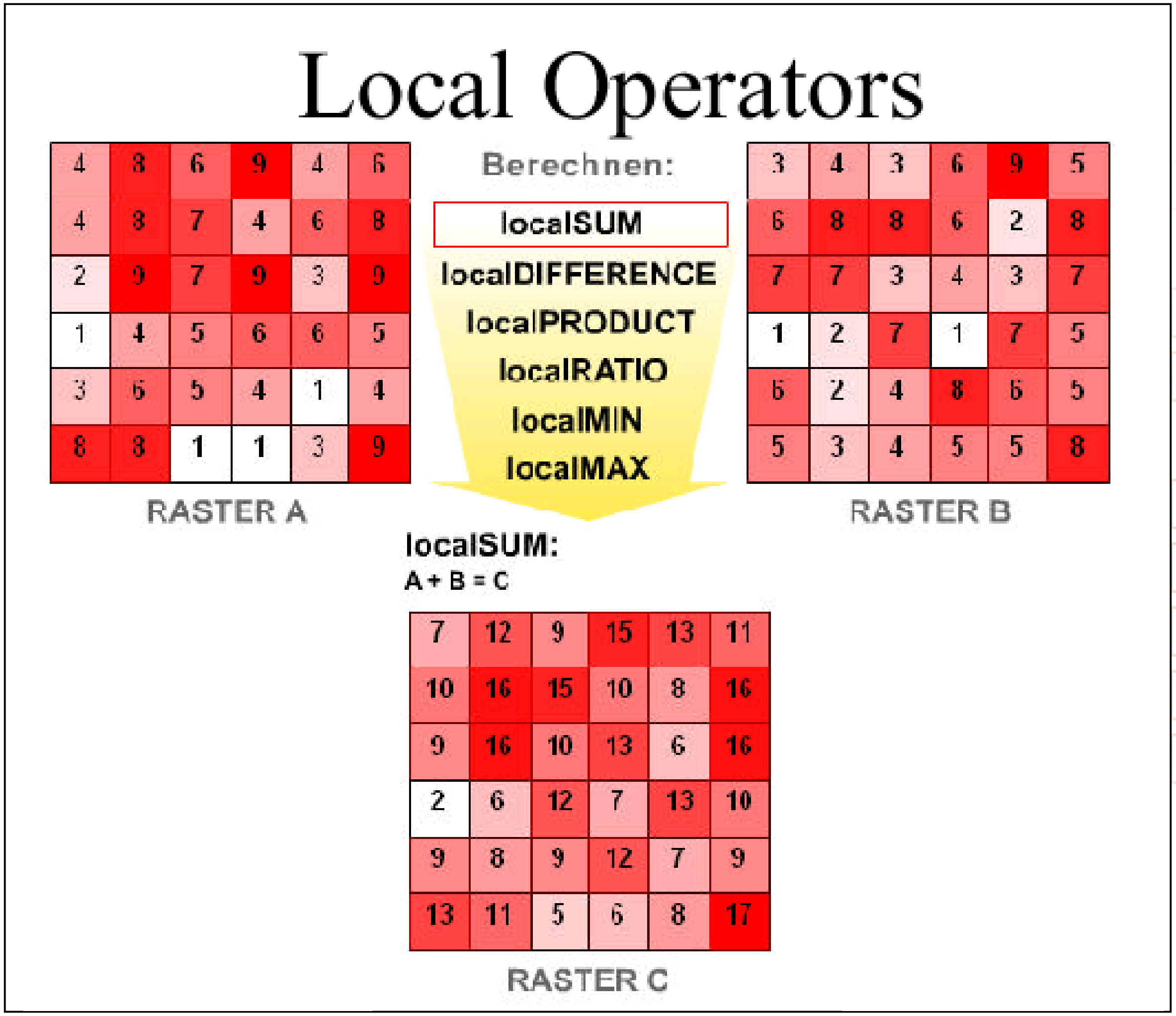
Map Algebra - Operators

- **Local**
  - Non spatial (to be honest!)
  - Process information of exactly on cell location(!)
- **Focal**
  - Information of neighboring cells are processed (nb! neighborhood)
- **Zonal**
  - Processes information of cell zones having the same properties
- **Incremental**
  - Part of global operators
- **Global**
  - Caluclation result at a specific cell is dependent (at least theoretically) on all cells in the raster layer.

Local Operators

- Local operators can combine 1, …, n raster layers
- Only **1 cell of the raster layer** is considered
- Examples:
  - local**Sinus**
  - local**Integer**
  - local**Sum**
  - local**Mean**
  - local**Range**
  - local**Stddev**

Local Operators



Focal Operators

- Neighborhood definition
  - Rook's, Queen's case
  - Circular, donut, wedge
  - Irregular shape

- **Moving Window:** defined by neighborhood (MW moves over the entire raster layer!)
- Examples:
  - focal**Sum**
  - focal**Min**
  - focal**Majority**
  - …

Focal Operators

1	7	4	6	4	1	
3	3	3	9	2	6	6
7	9	3	9	4	3	3
8	8	8	9	9	4	4
2	3	7	6	8	5	
1	8	5	5	8	7	

Eingabe RASTER

- focalSUM
- focalMEAN
- focalMIN
- focalMAX
- focalRANGE

27	33	49	42	39	26
41	40	53	44	44	30
56	52	61	56	55	41
54	55	62	63	57	45
41	50	59	65	61	57
27	40	52	57	59	62

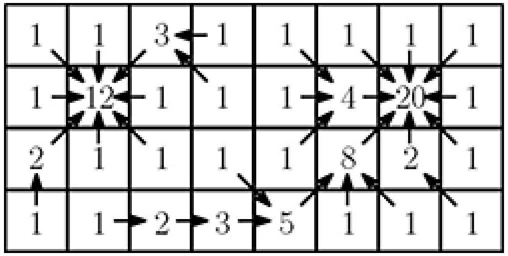
Ausgabe RASTER

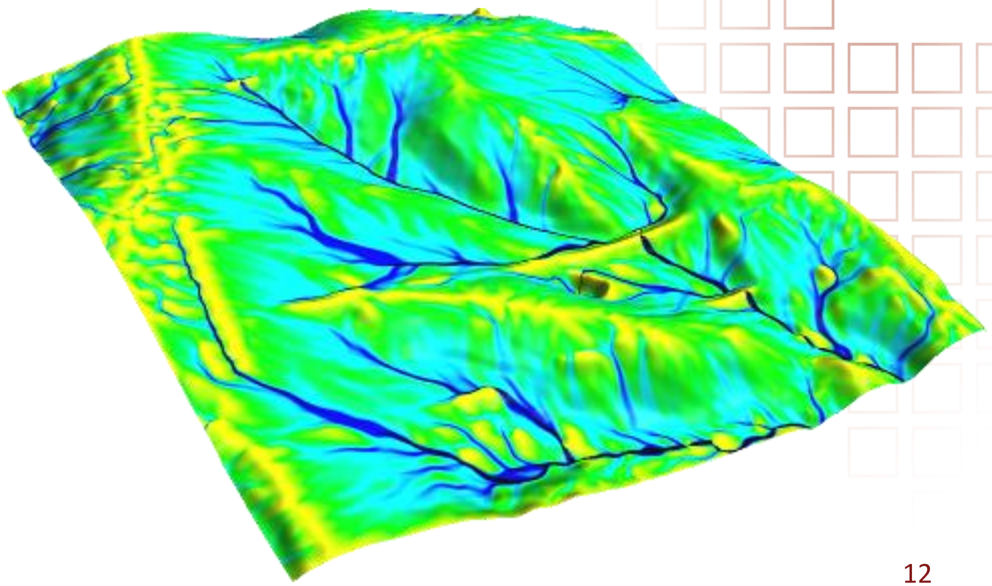
Zonal Operators

- Calculation based on disjoint regions (=zones) having the same properties
- Zones are independent of the neighborhood!

Incremental Operators

- Result of an incremental calculation step are input for the calculation of the neighboring raster cell values.
- Examples:
  - DEM >> slope
  - Hydrological models
  - Distance surfaces



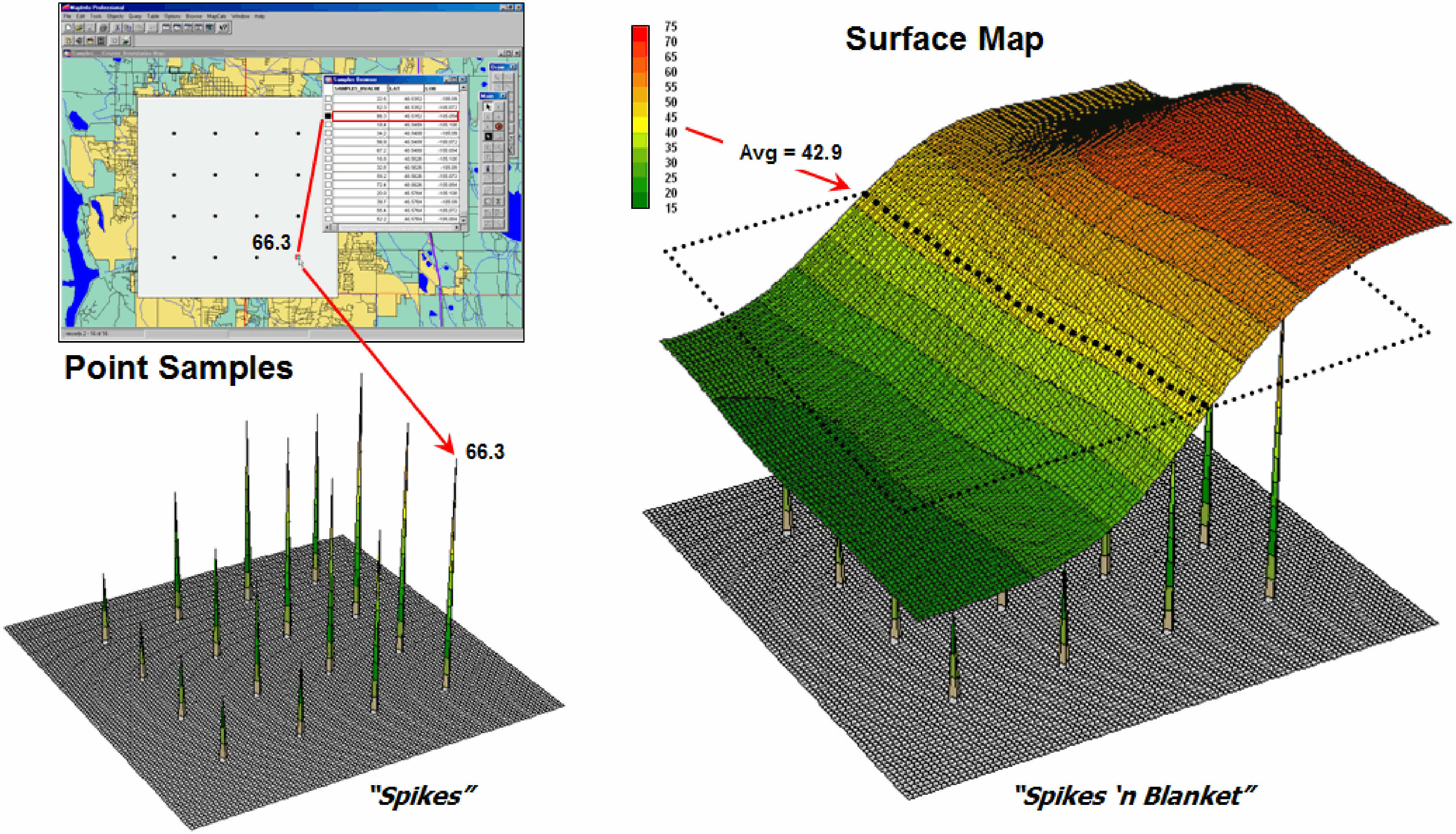


Global Operators

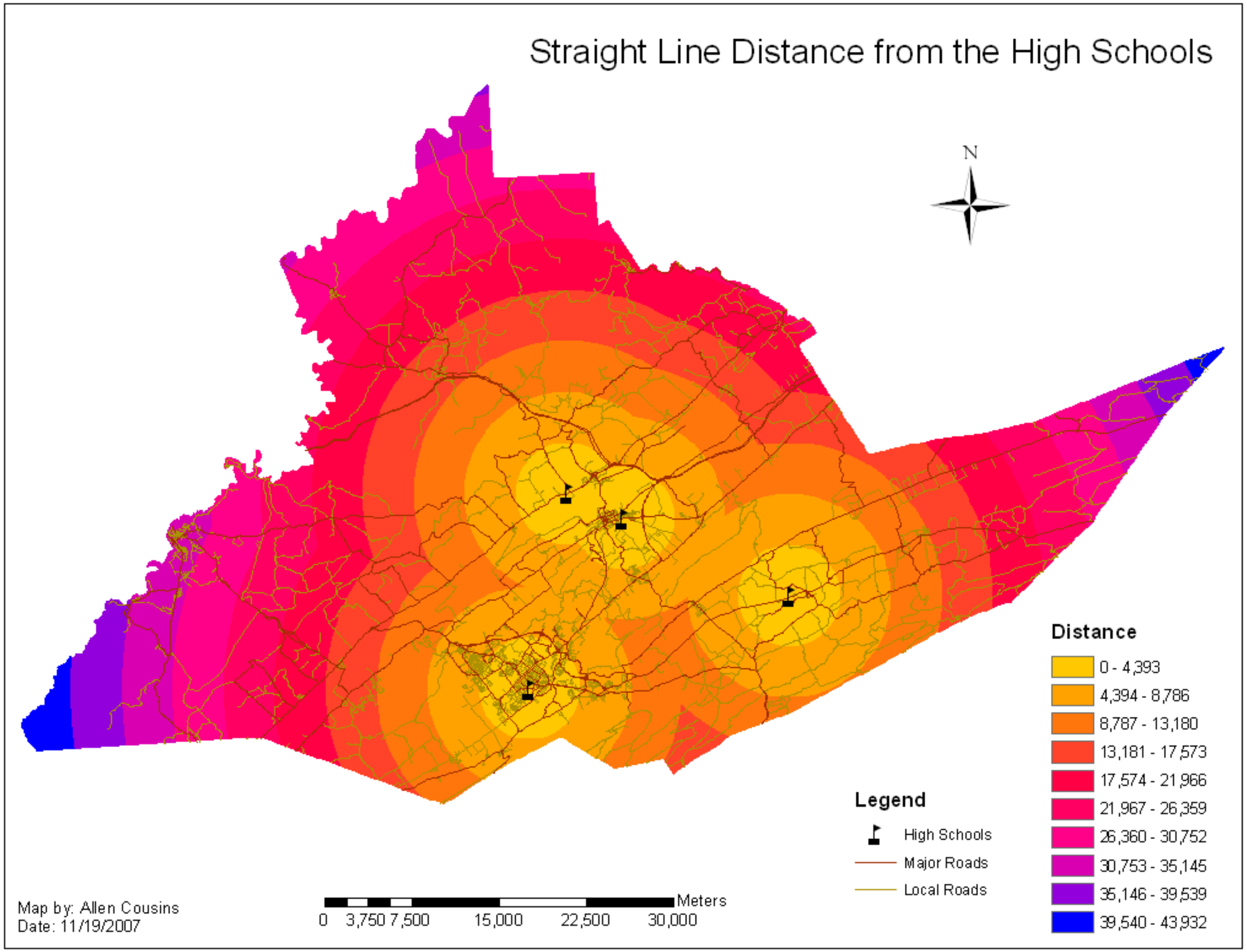
- Caluclation result at a specific cell is dependent (at least theoretically) on all cells present in the raster layer.

- Example:
  - Cost surfaces
  - Interpolation

Global Operators



Global Operators

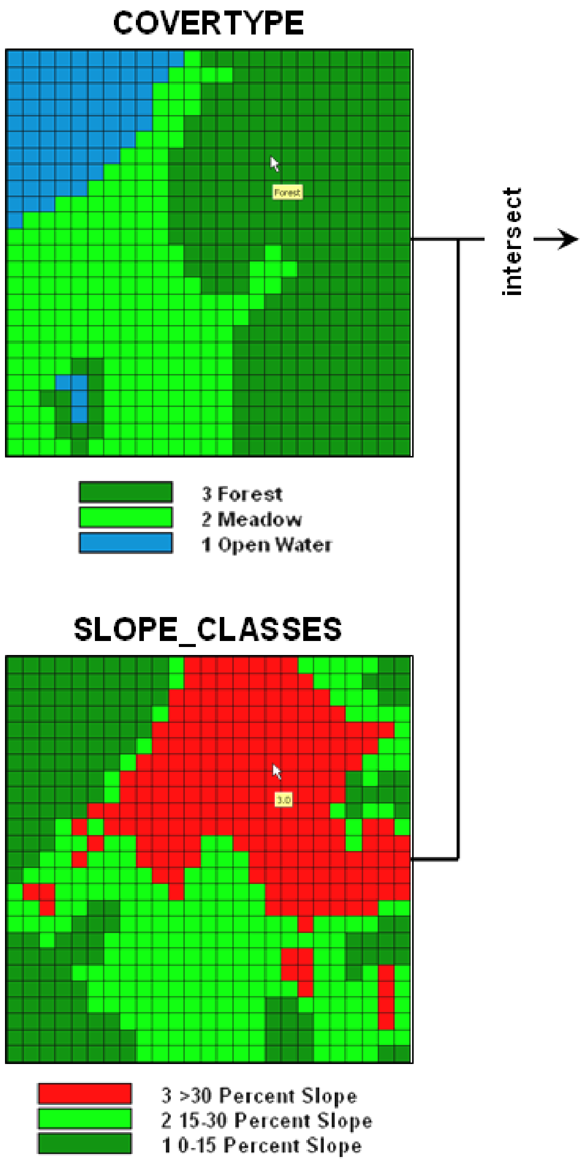


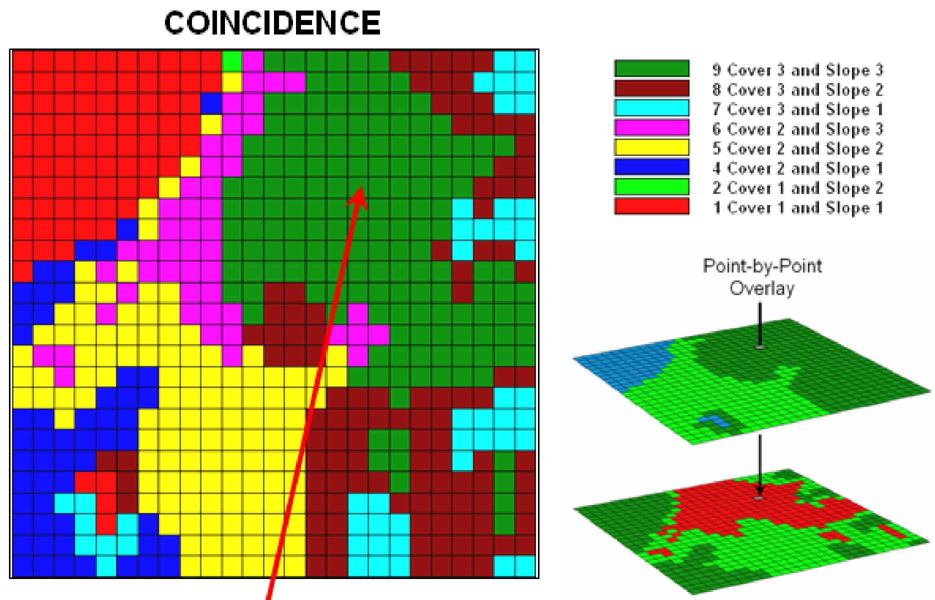
„Spatial Analysis“ || Raster Overlay

Spatial Analysis

- Cross-Tabulation
  - Combines several raster layer in a single raster data set
  - Each possible combination of values is allowed – results in 9 classes when having 2 input layers with 3 classes each.

Spatial Analysis





Cover Type 3 (Forest) coinciding with Slope Class 3 (Steep)

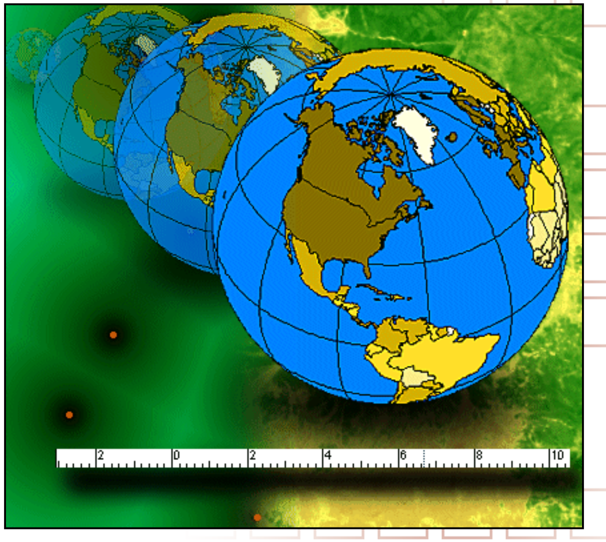
...represents 26.24% of the project area

	Category	Count	acres	% Gridded Area	Color
9.0	Cover 3 and Slope 3	164	405.041	26.24	
8.0	Cover 3 and Slope 2	110	271.674	17.6	
7.0	Cover 3 and Slope 1	48	118.549	7.68	
6.0	Cover 2 and Slope 3	48	118.549	7.68	
5.0	Cover 2 and Slope 2	120	296.372	19.2	
4.0	Cover 2 and Slope 1	53	130.898	8.48	
2.0	Cover 1 and Slope 2	1	2.47	0.16	
1.0	Cover 1 and Slope 1	81	200.051	12.96	

Cost Surfaces

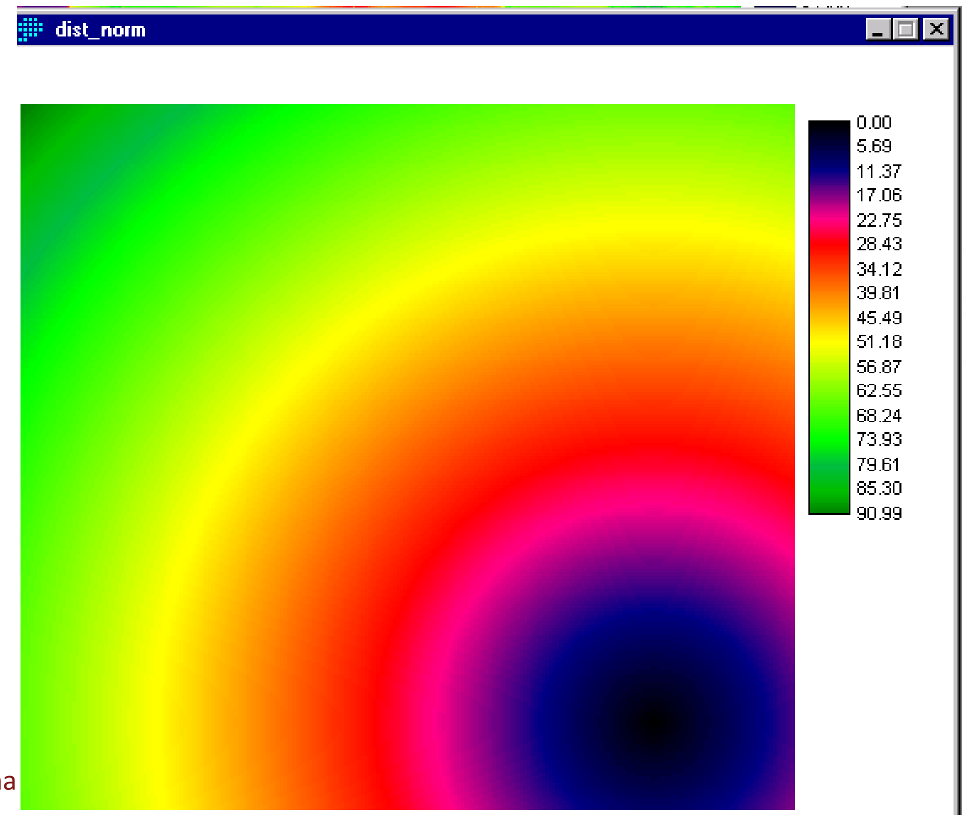
Cost/Distance surfaces

- Distance concepts with raster data
  - Euclidean distances
  - Spherical distances
  - Cost Surfaces



Distance surfaces

- **Euclidean Distance**
  - „as the crow flies …“
  - Calcluation of continuous distance surfaces based on point datasets (e.g. bus stops, schools, …)



Distance & cost surfaces

- Cost surfaces model the „cost“ necessary **to move through space**
  - **Costs** are function of **movement costs** and costs induced by **frictions and forces**

- **Force:** supports movement in space
  - Example:
    - Driving downhill

- **Frictions:** hinder movement in space
  - Example: driving uphill

Distance & cost surfaces

$$\text{distance} = \sqrt{(x^2 + y^2)}$$

$$\text{e.g., } D = \sqrt{(20^2 + 10^2)}$$

$$= 22.4$$

20	10	source cell
22.4	14.1	10
28.3	22.4	20

10 units

cost = distance * fixed cost factor

e.g.,

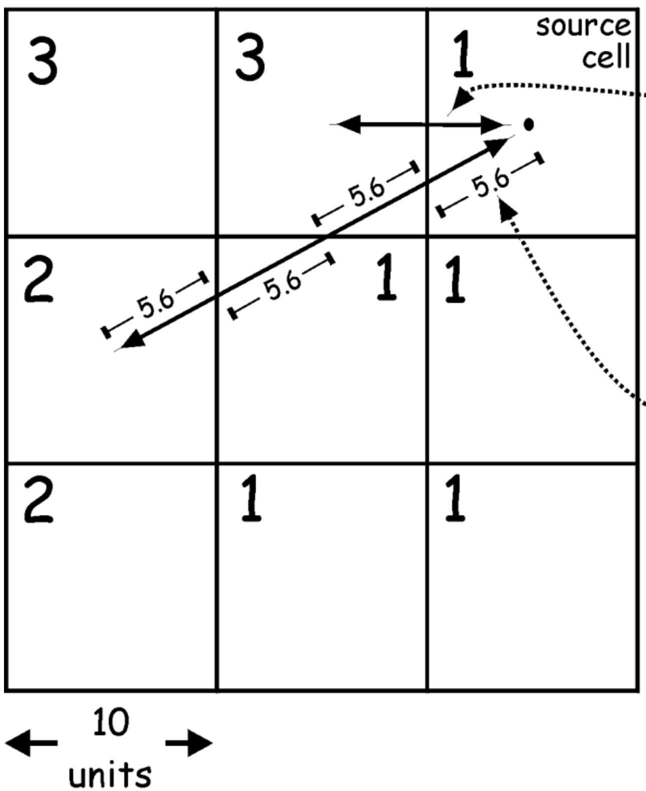
cost = distance * 2

40	20	source cell
44.8	28.2	20
56.6	44.8	40

Distance & cost surfaces

cost = cell distance * friction

friction surface



Cost =
(5 * 1)
\+ (5 * 3)
20

Cost =
(5.6 * 1)
(5.6 * 3)
(5.6 * 1)
\+ (5.6 * 2)
39.1

output cost surface

50	20	source cell
39.1	14.1	10
42.3	22.4	20

10 units

Distance & cost surfaces

cost =
row/column distance * friction

friction surface

3	3	1 source cell
2	1	1
2	1	1

← 10 units →

Cost =

(5 * 1)
+ (5 * 3)
+ (5 * 3)
+ (5 * 1)
+ (5 * 1)
+ (5 * 2)

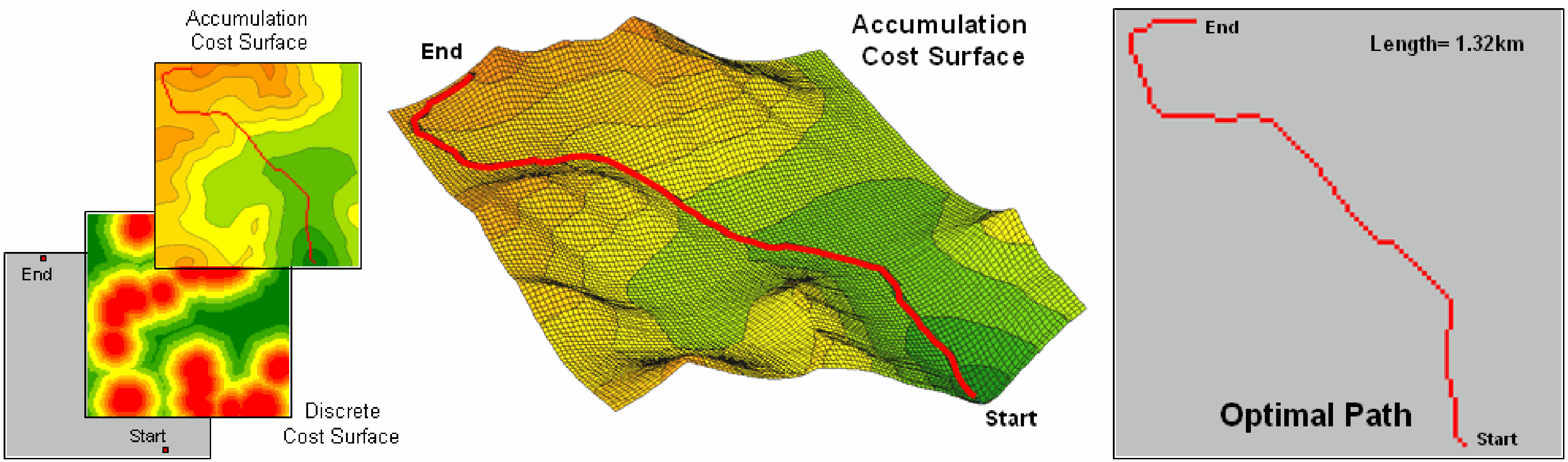
55

output cost surface

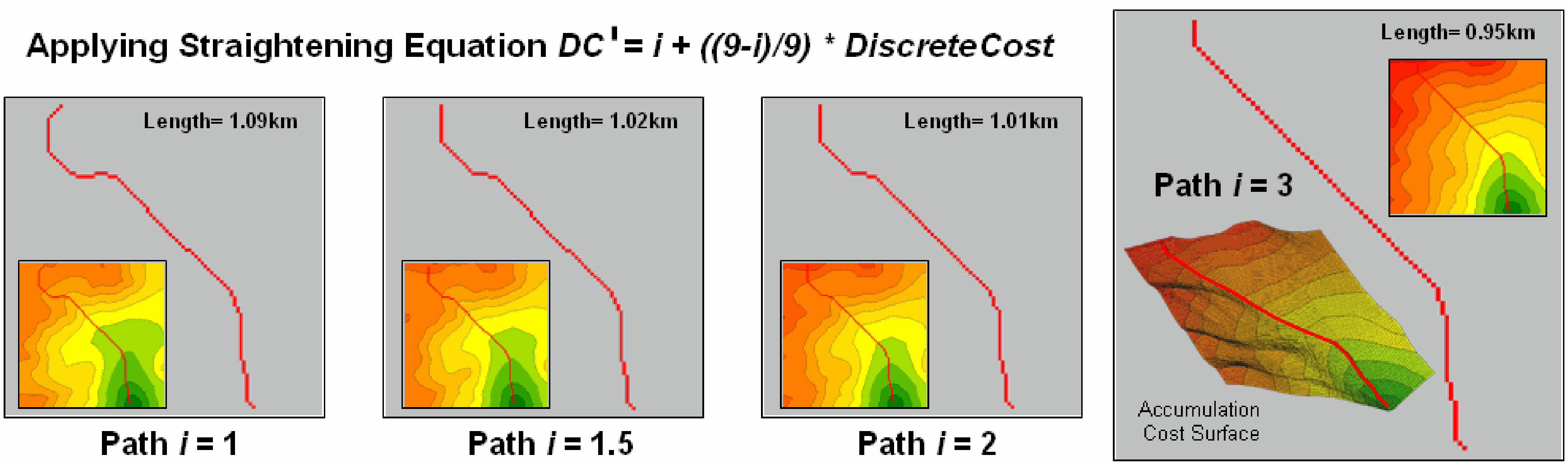
50	20	source cell
55	40	10
45	30	20

← 10 units →

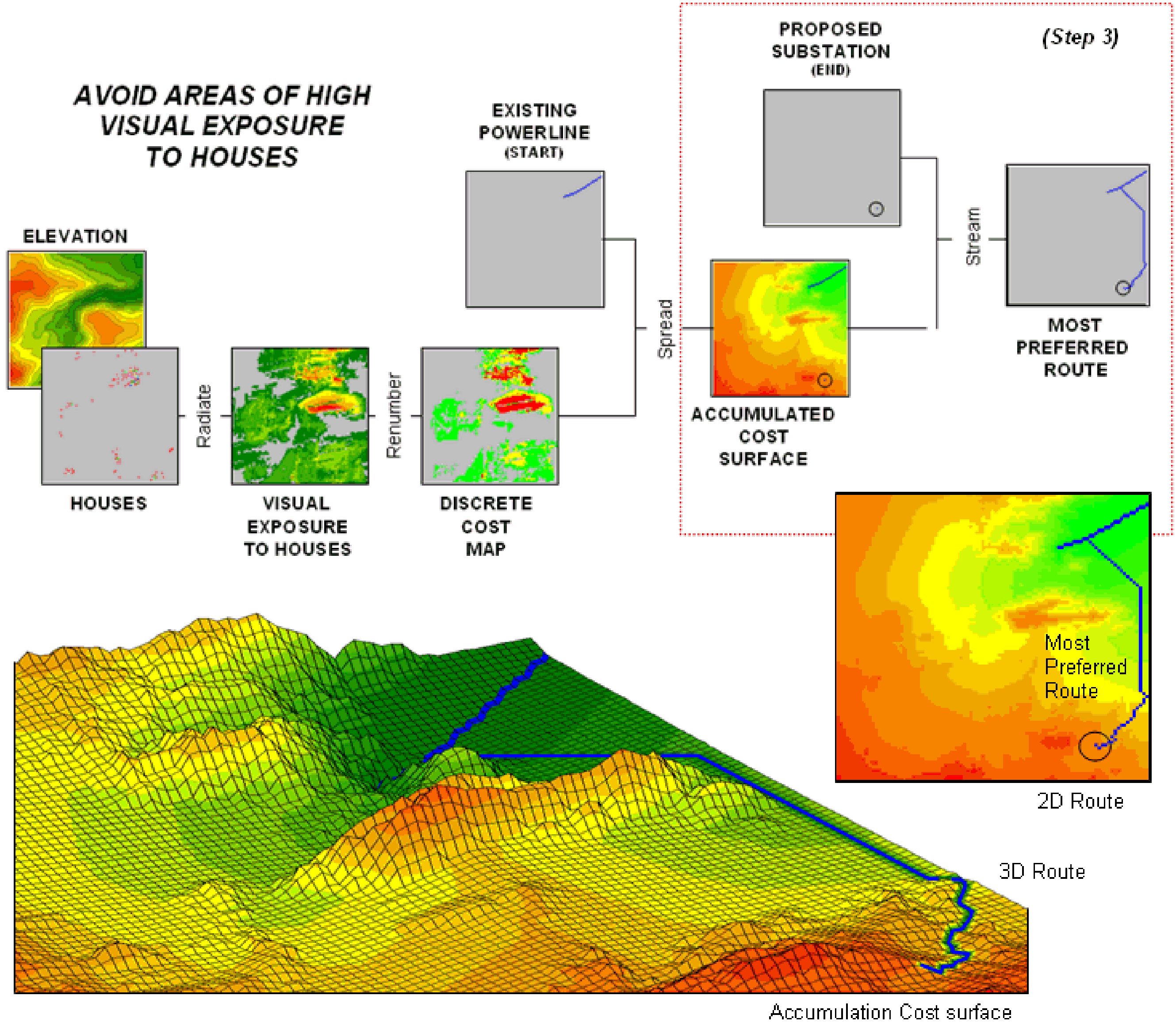
Distance & cost surfaces



Applying Straightening Equation $DC' = i + ((9-i)/9) * DiscreteCost$



Distance & cost surfaces



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