Assignment - 1

Linear Algebra

1) (i) $u = a_0 + a_1 x$

$v = b_0 + b_1 x$

$u + v = (a_0 + a_1 x) + (b_0 + b_1 x)$

$= (a_0 + b_0) + (a_1 + b_1) x \in V$

(ii) $u + v = v + u$

$= (a_0 + a_1 x) + (b_0 + b_1 x)$

$= (a_0 + b_0) + (a_1 + b_1) x$

$= (b_0 + b_1 x) + (a_0 + a_1 x)$

$= v + u \in V$

(iii) $(u+v) + w = u + (v + w)$

$u = (a_0 + a_1 x)$

$v = (b_0 + b_1 x)$

$w = (c_0 + c_1 x)$

$[(a_0 + a_1 x) + (b_0 + b_1 x) + (c_0 + c_1 x)$

$= [(a_0 + b_0) + (a_1 + b_1) x] + (c_0 + c_1 x)$

$= (a_0 + a_1 x) + [(b_0 + b_1 x) + (c_0 + c_1 x)]$

$= (a_0 + a_1 x) + [(b_0 + c_0) + (b_1 + c_1) x]$

$= u + (v + w) \in V$

(iv) $\bar{0} \in V$

$\bar{0} = 0 + 0x$

$\bar{0} + u = (0 + 0x) + (a_0 + a_1 x)$

$= a_0 + a_1 x$

$= u \in V$

v) $-u \in V$

$-u = -a_0 - a_1 x$

$-u + u = -a_0 - a_1 x + a_0 + a_1 x$

$= (a_0 - a_0) + (a_1 x - a_1 x)$

$= \bar{0} \in V$

vi) K is a scalar

$Ku = K(a_0 + a_1 x)$

$= Ka_0 + Ka_1 x$

$= Ku \in V$.

vii) $(u+v)K = K[(a_0 + a_1 x) + (b_0 + b_1 x)]$

$= K[(a_0 + b_0) + (a_1 x + b_1 x)]$

$= [K(a_0 + b_0) + K(a_1 x + b_1 x)]$

$= K(a_0 + a_1 x) + K(b_0 + b_1 x)$

$= Ku + Kv \in V$

viii) $(K+m)u = (K+m)(a_0 + a_1 x)$

$= Ka_0 + ma_0 + Ka_1 x + ma_1 x$

$= K(a_0 + a_1 x) + m(a_0 + a_1 x)$

$= Ku + mu \in V$

ix) $K(mu) = K(m(a_0 + a_1 x))$

$= Km(a_0 + a_1 x)$

$= (Km)u \in V$

x) $1.u = 1.(a_0 + a_1 x)$

$= (a_0 + a_1 x)$

$= u \in V$.

2) $u = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & \cdots & & & 0 \\ \vdots & \vdots & & & & & \vdots \\ a_{n1} & a_{n2} & \cdots & & & & a_{nn} \end{bmatrix}$

$v = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 & \cdots & 0 \\ b_{21} & b_{22} & 0 & \cdots & & & 0 \\ \vdots & & & & & & \\ b_{n1} & b_{n2} & \cdots & & & & b_{nn} \end{bmatrix}$

(i) $u+v = \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

$= \begin{bmatrix} a_{11}+b_{11} & 0 & \cdots & 0 \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & 0 \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nn}+b_{nn} \end{bmatrix}$

$= u+v \in V$

(ii) $u+v = v+u$

$= \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}$

$= \begin{bmatrix} a_{11}+b_{11} & 0 & \cdots & 0 \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & 0 \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nn}+b_{nn} \end{bmatrix}$

$= \begin{bmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

$= v+u \in V$

iv) $\bar{o} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & 0 & \cdots & 0 \\ 0 & & & \end{bmatrix}$

$$\bar{o} + u = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$= u \in V$$

vi) $ku = k \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$

$$= \begin{bmatrix} ka_{11} & 0 & \cdots & 0 \\ ka_{21} & ka_{22} & \cdots & 0 \\ ka_{n1} & ka_{n2} & \cdots & ka_{nn} \end{bmatrix} \in V$$

3) $x-1, (x-1)^2, (x-1)^3$ do not span P_3.

$$(a,b,c,d) = k_1(-1,1,0,0) + k_2(1,-2,1,0) + k_3(-1,3,-3,1)$$

$a = -k_1 + k_2 - k_3$ — ①

$b = k_1 - 2k_2 + 3k_3$ — ②

$c = k_2 - 3k_3$ — ③

$d = k_3$ — ④ ; sub ④ in ③

$c = k_2 - 3d$

$\boxed{k_2 = c + 3d}$

Adding ① & ②

$a + b = -k_2 + 2k_3$

$a + b = -k_2 + 2d$

$a + b - 2d = -k_2$

$\boxed{-a - b + 2d = k_2}$

As we are getting different values for K_2, the system of linear equations are not consistant.

$\therefore$ It does not span in P_3

4) $V_1 = (1,2,3,4)$; $V_2 = (0,1,0,-1)$; $V_3 = (1,3,3,3)$ form L.I in R^4.

$(0,0,0,0) = K_1V_1 + K_2V_2 + K_3V_3$

$(0,0,0,0) = K_1(1,2,3,4) + K_2(0,1,0,-1) + K_3(1,3,3,3)$

$K_1 + K_3 = 0$ - 1

$2K_1 + K_2 + 3K_3 = 0$ - 2

$3K_1 + 3K_3 = 0$ - 3

$4K_1 - K_2 + 3K_3 = 0$ - 4

$\boxed{K_3 = -K_1}$ - 5

sub 5 in 2

$2K_1 + K_2 - 3K_1 = 0$

$K_2 - K_1 = 0$

$\boxed{K_2 = K_1}$

There are non-zero values of K_1, K_2, K_3 which will make $K_1V_1 + K_2V_2 + K_3V_3 = 0$. So, it is linearly dependent.

5) $P_1(x) = 1$; $P_2(x) = 1 - x$; $P_3(x) = 2 - 4x + x^2$;
$P_4(x) = 6 - 18x + 9x^2 - x^3$ form a basis for P_3.

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & -1 & -4 & -18 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

~~1[-1~~

$1[-1(-1-0)]$

$= 1[-1(-1)]$

$= 1(1)$

$= 1$

$\neq 0$

$\therefore$ It is linearly Independent.

6) $B = (u_1, u_2, u_3)$
$B' = (v_1, v_2, v_3)$ for R^3

$u_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$v_1 = \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

$T: R^3 \to R^3$

$u_1 = K_1 v_1 + K_2 v_2 + K_3 v_3$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = K_1 \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix} + K_2 \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} + K_3 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$3K_1 + K_2 - K_3 = 2$ — ①

$K_1 + K_2 = 1$ — ②

$-5K_1 - 3K_2 + 2K_3 = 1$ — ③

$K_1 - K_2 = 5$ — ④

$2K_1 = 6$

$\boxed{K_1 = 3}$

$\boxed{K_2 = -2}$

$3 \times 3 - 2 - K_3 = 2$

$9 - 2 - K_3 = 2$

$\boxed{K_3 = 5}$

$u_2 = K_1 v_1 + K_2 v_2 + K_3 v_3$

$$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = K_1 \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix} + K_2 \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} + K_3 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$3K_1 + K_2 - K_3 = 2$ — ①

$K_1 + K_2 = -1$ — ②

$-5K_1 - 3K_2 + 2K_3 = 1$ — ③

$K_1 - K_2 = 5$ — ④

$2K_1 = +4$

$\boxed{K_1 = 2}$

$\boxed{K_2 = -3}$

$6 - 3 - K_3 = 2$

$\boxed{K_1 = 1}$

$u_3 = K_1 v_1 + K_2 v_2 + K_3 v_3$

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = K_1 \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix} + K_2 \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} + K_3 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

$3K_1 + K_2 - K_3 = 1$ — ①

$K_1 + K_2 = 2$ — ②

$-5K_1 - 3K_2 + 2K_3 = 1$ — ③

$K_1 - K_2 = 3$ — ④

$2K_1 = 5$

$\boxed{K_1 = 5/2}$

$\frac{5}{2} + K_2 = 2$

$\boxed{K_2 = -\frac{1}{2}}$

$3 \times \frac{5}{2} - \frac{1}{2} - K_3 = 1$

$\frac{15}{2} - \frac{1}{2} - K_3 = 1$

$\frac{14}{2} - K_3 = 1$

$\boxed{K_3 = 6}$

$$[T]_{B',B} = \left[[T(u_1)]_{B'} \;\middle|\; [T(u_2)]_{B'} \;\middle|\; [T(u_3)]_{B'} \right]$$

$$[T]_{B',B} = \begin{bmatrix} 3 & 2 & 5/2 \\ -2 & -3 & -1/2 \\ 5 & 1 & 6 \end{bmatrix}$$

7) $V_1 = (1,1,1)$; $V_2 = (2,-1,-1)$; $V_3 = (0,1,-1)$

$\langle V_1, V_2 \rangle = 1.2 + 1.-1 + 1.-1$

$= 2 - 1 - 1$

$= 2 - 2$

$= 0$

$\langle V_2, V_3 \rangle = 2.0 + 1.-1 + -1.-1$

$= 0 - 1 + 1$

$= 0$

$\langle V_1, V_3 \rangle = 1.0 + 1.1 + 1.-1$

$= 0 + 1 - 1$

$= 0$

$\therefore$ They are orthogonal in R^3

8) $\langle u,v\rangle = 4u_1v_1 + u_1v_2 - 3u_2v_2$

$u = (-3,2)$; $v = (1,-2)$

$\|u\| = \sqrt{\langle u,u\rangle}$

$\langle u,u\rangle = 4u_1u_1 + u_1u_2 - 3u_2u_2$

$= 4(-3)(-3) + (-3)(2) - 3(2)(2)$

$= 36 - 6 - 12$

$= 36 - 18$

$\langle u,u\rangle = 18$

$\|u\| = \sqrt{18}$

$\|u\| = 3\sqrt{2}$

$d(u,v) = \sqrt{\langle u-v, u-v\rangle}$

$u - v = (-3,2) - (1,-2)$

$= (-4,4)$

$\langle u-v, u-v\rangle = 4(-4)(-4) + (-4)(4) + 3(4)(4)$

$= 64 - 16 + 48$

$= 64 - 64$

$= 0$

$d(u,v) = \sqrt{0}$

$= 0$

9) $u_1 = (1,1,1)$; $u_2 = (-1,1,0)$; $u_3 = (1,2,1)$

$v_1 = u_1 = (1,1,1)$

$v_2 = u_2 - \dfrac{\langle u_2, v_1\rangle}{\|v_1\|^2} \cdot v_1$

$$V_2 = (-1,1,0) - \frac{1.-1 + 1.1 + 1.0}{\left(\sqrt{1^2+1^2+1^2}\right)^2}.(1,1,1)$$

$$V_2 = (-1,1,0) - \frac{-1+1+0}{(\sqrt{3})^2}.(1,1,1)$$

$$V_2 = (-1,1,0) - 0$$

$$V_2 = (-1,1,0)$$

$$V_3 = u_3 - \frac{\langle u_3, v_1\rangle}{\|v_1\|^2}.v_1 - \frac{\langle u_3, v_2\rangle}{\|v_2\|^2}.v_2$$

$$V_3 = (1,2,1) - \frac{1.1+2.1+1.1}{\left(\sqrt{1^2+1^2+1^2}\right)^2}.(1,1,1) - \frac{1.-1+2.1+1.0}{\left(\sqrt{(-1)^2+(1)^2+0^2}\right)^2}.(-1,1,0)$$

$$V_3 = (1,2,1) - \frac{1+2+1}{(\sqrt{3})^2}.(1,1,1) - \frac{-1+2+0}{(\sqrt{2})^2}.(-1,1,0)$$

$$V_3 = (1,2,1) - \frac{4}{3}(1,1,1) - \frac{1}{2}(-1,1,0)$$

$$V_3 = (1,2,1) - \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right) - +\frac{1}{2}(-1,1,0)$$

$$V_3 = \left(\frac{-1}{3}, \frac{2}{3}, \frac{-1}{3}\right) - \frac{1}{2}(-1,1,0)$$

$$V_3 = \frac{-1}{3}, \frac{2}{3}, \frac{-1}{3} - \left(\frac{-1}{2}, \frac{1}{2}, 0\right)$$

$$V_3 = \left(\frac{1}{6}, \frac{1}{6}, \frac{-1}{3}\right)$$

$$V_1 = (1,1,1)\ ;\ V_2 = (-1,1,0)\ ;\ V_3 = \left(\frac{1}{6}, \frac{1}{6}, \frac{-1}{3}\right)$$

$$q_1 = \frac{v_1}{\|v_1\|}$$

$$q_1 = \frac{(1,1,1)}{\sqrt{3}}$$

$$q_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$q_2 = \frac{v_2}{\|v_2\|}$$

$$q_2 = \frac{(-1,1,0)}{\sqrt{2}}$$

$$q_2 = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$q_3 = \frac{v_3}{\|v_3\|}$$

$$q_3 = \frac{\left(\frac{1}{6}, \frac{1}{6}, \frac{-1}{3}\right)}{\sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{-1}{3}\right)^2}}$$

$$q_3 = \frac{\left(\frac{1}{6}, \frac{1}{6}, -\frac{1}{3}\right)}{\sqrt{\frac{1}{36} + \frac{1}{36} + \frac{1}{9}}}$$

$$q_3 = \frac{\left(\frac{1}{6}, \frac{1}{6}, -\frac{1}{3}\right)}{\sqrt{\frac{1}{6}}}$$

$$q_3 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$$

10) QR decomposition of $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 4 \end{bmatrix}$

$$A = QR$$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 4 \end{bmatrix}$$

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} ; u_2 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$v_1 = u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$v_2 = u_2 - \frac{\langle u_2, v_1 \rangle}{\|v_1\|^2} \cdot v_1$$

$$v_2 = (2,1,4) - \frac{2.1 + 1.0 + 4.1}{\left(\sqrt{1^2+0^2+1^2}\right)^2} \cdot (1,0,1)$$

$$v_2 = (2,1,4) - \frac{2+0+4}{(\sqrt{2})^2} \cdot (1,0,1)$$

$$v_2 = (2,1,4) - \frac{\cancel{6}^{3}}{\cancel{2}} \cdot (1,0,1)$$

$$v_2 = (2,1,4) - (3,0,3)$$

$$v_2 = (-1,1,1)$$

$$q_1 = \frac{v_1}{\|v_1\|}$$

$$q_1 = \frac{(1,0,1)}{\sqrt{1^2+0^2+1^2}} = \frac{(1,0,1)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$q_2 = \frac{v_2}{\|v_2\|}$$

$$q_2 = \frac{(-1,1,1)}{\sqrt{(-1)^2+1^2+1^2}}$$

$$q_2 = \frac{(-1,1,1)}{\sqrt{3}}$$

$$q_2 = \left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$R = \begin{bmatrix} \langle u_1, q_1 \rangle & \langle u_2, q_1 \rangle \\ 0 & \langle u_2, q_2 \rangle \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{2} & 3\sqrt{2} \\ 0 & \sqrt{3} \end{bmatrix}$$

$$QR = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}_{3\times 2} \begin{bmatrix} \sqrt{2} & 3\sqrt{2} \\ 0 & \sqrt{3} \end{bmatrix}_{2\times 2}$$

$$QR = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 4 \end{bmatrix}$$

$$= A \;//.$$

11) Find least square solution and least square error of A

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & -4 & 3 \\ 1 & 10 & -7 \end{bmatrix} ; \quad b = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$$Ax = b$$

$$A^T = \begin{bmatrix} 3 & 1 & 1 \\ 2 & -4 & 10 \\ -1 & 3 & -7 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & -4 & 10 \\ -1 & 3 & -7 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 1 & -4 & 3 \\ 1 & 10 & -7 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 3.3+1.1+1.1 & 3.2+1.-4.1+1.10 & 3.-1+1.3+1.-7 \\ 2.3+1.-4+10.1 & 2.2+-4.-4+10.10 & -1.2+3.-4+10.-7 \\ -1.3+3.1+-7.1 & -1.2+3.-4.-7.10 & -1.-1+3.3+-7.-7 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 11 & 12 & -7 \\ 12 & 120 & -84 \\ -7 & -84 & 59 \end{bmatrix}$$

$$A^T b = \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 2 & -4 & 10 \\ -1 & 3 & -7 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}}_{3\times1}$$

$$A^T b = \begin{bmatrix} 5 \\ 22 \\ -15 \end{bmatrix}$$

$$A^TAx = A^Tb$$

$$\begin{bmatrix} 11 & 12 & -7 \\ 12 & 120 & -84 \\ -7 & -84 & 59 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 22 \\ -15 \end{bmatrix}$$

$11x_1 + 12x_2 - 7x_3 = 5$ - ①

$12x_1 + 120x_2 - 84x_3 = 22$ - ②

$-7x_1 - 84x_2 + 59x_3 = -15$ - ③

① × 12 - ②

$$132x_1 + 144x_2 - \cancel{84x_3} = 60$$
$$12x_1 + 120x_2 - \cancel{84x_3} = 22$$
$$- \quad - \quad + \quad -$$

$120x_1 + 24x_2 = 38$ - ④

① × 59 + ③ × 7

$$649x_1 + 708x_2 - \cancel{413x_3} = 295$$
$$-49x_1 - 588x_2 + \cancel{413x_3} = -105$$

$600x_1 + 120x_2 = 190$ - ⑤

$x_1 = t$

$120t + 24x_2 = 38$

$24x_2 = 38 - 120t$

$$\boxed{x_2 = \frac{38-120t}{24}}$$

~~600~~

$11t + 12\left(\frac{38-120t}{24_2}\right) - 7x_3 = 5$

$11t + 19 - 60t - 7x_3 = 5$

$-49t - 7x_3 = -14$

$-7x_3 = -14 + 49t$

$x_3 = \frac{14-49t}{7}$

$$\boxed{x_3 = 2 - 7t}$$

Least square solutions:

$$\boxed{\begin{aligned} x_1 &= t \\ x_2 &= \frac{38-120t}{24} \\ x_3 &= 2-7t \end{aligned}}$$

$$b - Ax = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 & -1 \\ 1 & -4 & 3 \\ 1 & 10 & -7 \end{bmatrix}_{3\times 3} \begin{bmatrix} t \\ \frac{38-120t}{24} \\ 2-7t \end{bmatrix}_{3\times 1}$$

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} 3t + 38-120t/12 - 2+7t \\ t - 38+120t/6 + 6 - 21t \\ t + 190 - 600t/6 - 14 + 49t \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} 7/6 \\ -1/3 \\ 11/6 \end{bmatrix}$$

$$= \begin{bmatrix} 5/6 \\ -5/3 \\ -5/6 \end{bmatrix}$$

$$\|b - Ax\| = \sqrt{\left(\frac{5}{6}\right)^2 + \left(-\frac{5}{3}\right)^2 + \left(-\frac{5}{6}\right)^2}$$

$$\|b - Ax\| = \sqrt{\frac{25}{36} + \frac{25}{9} + \frac{25}{36}}$$

$$\|b - Ax\| = \sqrt{\frac{50}{36} + \frac{25}{9}}$$

$$\|b - Ax\| = \sqrt{\frac{50 + 100}{36}}$$

$$\|b - Ax\| = \sqrt{\frac{150}{36}}$$

$$\|b - Ax\| = \frac{5}{\sqrt{6}}$$

$$\boxed{\|b - Ax\| = \frac{5\sqrt{6}}{6}}$$

12) $u = (1,0,2)$

$v_1 = (3,1,2)$

$v_2 = (-1,1,1)$

$$\langle u, v_1 \rangle = \cancel{1.-1+0.}$$
$$= 3.1 + 0.1 + 2.2$$
$$= 3 + 0 + 4$$

$$\langle u, v_1 \rangle = 7$$

$$\langle u, v_2 \rangle = 1.-1 + 0.1 + 2.1$$
$$= -1 + 0 + 2$$

$$\langle u, v_2 \rangle = 1$$

$$\|v_1\|^2 = \left(\sqrt{3^2+1^2+2^2}\right)^2$$

$$= \left(\sqrt{9+1+4}\right)^2$$

$$= (\sqrt{14})^2$$

$$\|v_1\|^2 = 14$$

$$\|v_2\|^2 = \left(\sqrt{(-1)^2+1^2+1^2}\right)^2$$

$$= \left(\sqrt{3}\right)^2$$

$$\|v_2\|^2 = 3$$

$$\text{Proj}_w u = \frac{\langle u, v_1\rangle}{\|v_1\|^2} \cdot v_1 + \frac{\langle u, v_2\rangle}{\|v_2\|^2} \cdot v_2$$

$$= \frac{7}{14}(3,1,2) + \frac{1}{3}(-1,1,1)$$

$$= \left(\frac{21}{14}, \frac{7}{14}, \frac{14}{14}\right) + \left(\frac{-1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$= \left(\frac{3}{2}, \frac{1}{2}, 1\right) + \left(\frac{-1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$\text{Proj}_w u = \left(\frac{7}{6}, \frac{5}{6}, \frac{4}{3}\right) \quad //.$$