Assignment - 1

Discrete Mathematics

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① Determine whether these conditional statements are a tautology are not by using truth tables.

a) $\neg p \to (p \to q)$

b) $[(p \to q) \wedge (q \to r)] \to (p \to r)]$

Ans: a)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

$[\neg p \to (p \to q)]$ is a tautology

b)

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology

② What is the truth value of $\forall x (x^2 > x)$ if the domain consists of all real numbers? What is the truth value of this statement if the domain consists of all integers.

Ans: • Domain is all Real numbers.

Let, $P(x) : \forall x (x^2 > x)$

for $x = 1$

$(1)^2 > 1$ is false

for $x = 0.1$

$(0.1)^2 > 0.1$ is false

$\therefore$ Truth value is False.

• Domain is all Integers

Let, $P(x) : \forall x (x^2 > x)$

for $x = 2$

$(2)^2 > 2$ is true

but, for $x = 1$

$(1)^2 > 1$ is false

∴ Truth value is <u>False</u>

(3) Prove that if n is a positive integer, then n is even if and only if 7n + 4 is even.

<u>Ans</u>:

$P(n)$: n is even

$q(n)$: 7n + 4 is even

<u>To prove</u>:

$p(n) \leftrightarrow q(n)$

we are proving $p(n) \rightarrow q(n)$ and $q(n) \rightarrow p(n)$

i) $p(n) \rightarrow q(n)$

let P(n) is even

There exist an integer K where

$n = 2K$

$7n + 4 = 7(2K) + 4$

$= 14K + 4$

$= 2(7K + 2)$

$= 2K'$ $[K' = 7K + 2]$

It is in the form of 2K

∴ 7n + 4 is even.

∴ $p(n) \rightarrow q(n)$

ii) $q(n) \rightarrow p(n)$

let q(n) be even

$7n + 4 = 2K$

$7n = 2K - 4$

$= 2(K - 2)$

$= 2K'$ $[K' = K - 2]$

$\therefore q(n) \to p(n)$

which implies $p(n) \leftrightarrow q(n)$

(4) Show that these three statements are equivalent when a and b are real numbers.

(i) a is less than b

(ii) The average of a and b is greater than a and

(iii) The average of a and b is less than b

Ans:

Let us consider,

a, b are real numbers

p, q, r be the conditions

$p : a < b$

$q : \frac{a+b}{2} > a$

$r : \frac{a+b}{2} < b$

To prove:

These statement are equivalent;

$p \to q$; $p \to r$; $q \to p$; $r \to p$

(i) To prove:

$p \to q$

$p : a < b$

Adding (a) on both sides

$a + a < a + b$

$2a < a + b$

$a < \frac{a+b}{2}$

where $\frac{a+b}{2} > a$

$\therefore$ $p \to q$ is true

ii) To prove:

$p \to r$

$p : a < b$

Adding (b) on both sides

$a+b < b+b$

$a+b < 2b$

$\frac{a+b}{2} < b$

where $\frac{a+b}{2} < b$

$\therefore$ $p \to r$ is true

iii) To prove:

$q \to p$

$q : \frac{a+b}{2} > a$

$a+b > 2a$

Subtracting (a) on both sides

$a+b-a > 2a-a$

$b > a$

where, $a < b$

$\therefore$ $q \to p$ is true

iv) To prove:

$r \to p$

$\frac{a+b}{2} < b$

$a+b < 2b$

Subtracting (b) on both sides

$a+b-b < 2b-b$

$a < b$

where $a < b$

$\therefore r \to p$ is true

which implies that the three statements are equivalent.

⑤ Determine if these definitions are valid recursive definitions of a function and prove that your formula is valid.

a) $f(0) = 1$; $f(n) = -f(n-1)$ for $n >= 1$

b) $f(0) = 1$; $f(1) = 0$; $f(2) = 2$; $f(n) = 2f(n-3)$ for $n >= 3$

Ans:

a) $f(n) = -f(n-1)$

$f(1) = -f(1-1)$

$f(1) = -f(0)$

$f(1) = -1$ $(f(0) = 1)$

$f(2) = -f(2-1)$

$f(2) = -f(1)$

$f(2) = -(-1)$

$f(2) = 1$

It follows the pattern of $f(n) = (-1)^n$

$\therefore$ It is a recursive function.

b) $f(0) = 1$; $f(1) = 0$; $f(2) = 2$; $f(n) = 2f(n-3)$ for $n >= 3$

$f(n) = 2f(n-3)$

$f(3) = 2f(3-3)$

$f(3) = 2f(0)$

$f(3) = 2$ $(f(0) = 1)$

$f(4) = 2f(4-3)$

$f(4) = 2f(1)$

$f(4) = 0 \quad [f(1) = 0]$

$f(5) = 2f(5-3)$

$f(5) = 2f(2)$

$f(5) = 2.2 \quad [f(2) = 2]$

$f(5) = 4$

we can see that the values depend on the previous values.

$\therefore$ It is a recursive function.