

# 证明总览

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## 一、定理的分类

Axiom postulate	公理
Theorem	定理
Lemma	引理

二、严格证明：使用推理法则从{公理，定理，引理，前提，其他证明的结论}逐步推得结论

## Formal Proofs

- A **proof** provides an argument supporting the validity of the statement, and may use **premises**, **axioms**, **lemmas**, **results of other theorems**, etc.
- In **formal proofs**, steps follow **logically** from the set of premises, axioms, lemmas, and other theorems.

### 推理法则

- **modus tollens** 否定后件式

$$\frac{p \rightarrow q \quad \neg q}{\therefore \neg p} \quad \text{corresponding tautology: } (\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$$

- **hypothetical syllogism** 假言三段论

$$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r} \quad \text{corresponding tautology: } ((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

- **disjunctive syllogism** 选言三段论

$$\frac{p \vee q \quad \neg p}{\therefore q} \quad \text{corresponding tautology: } (\neg p \wedge (p \vee q)) \rightarrow q$$

- **Addition**

$$\frac{p}{\therefore p \vee q} \quad \text{corresponding tautology: } p \rightarrow (p \vee q)$$

- **Simplification**

$$\frac{p \wedge q}{\therefore q} \quad \text{corresponding tautology: } (p \wedge q) \rightarrow q$$

- **Conjunction**

$$\frac{p \quad q}{\therefore p \wedge q} \quad \text{corresponding tautology: } ((p) \wedge (q)) \rightarrow (p \wedge q)$$

- **Universal Instantiation (UI)**

$$\frac{\forall x P(x)}{\therefore P(c)}$$

- **Universal Generalization (UG)**

$$\frac{\text{true}}{\therefore p \wedge q} \quad ((p) \wedge (q)) \rightarrow (p \wedge q)$$

### ■ Resolution

$$\frac{\neg p \vee r \quad p \vee q}{\therefore q \vee r} \quad \begin{array}{l} \text{corresponding tautology:} \\ ((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r) \end{array}$$

$$\frac{}{\therefore P(c)}$$

### ■ Universal Generalization (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

### ■ Existential Instantiation (EI)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

### ■ Existential Generalization (EG)

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$

## 三、非严格证明的五种基本法：

1.直接证明

2.逆否命题

3.反证法

4.分类讨论

原理：  $P1 \vee P2 \vee P3 \vee P4 \vee P5 \longrightarrow Q$  等价于  $P1 \rightarrow Q \wedge P2 \rightarrow Q \wedge P3 \rightarrow Q \wedge P4 \rightarrow Q \wedge P5 \rightarrow Q$

5.等价证明

### 关于量词的证明方法

#### ■ Universally quantified statements

◇ prove the property holds for all examples

– proof by cases to divide the proof into different parts

◇ counterexamples

– disprove universal statements

#### ■ Existence proof

◇ constructive

– find a specific example to show the statement holds

◇ nonconstructive

– proof by contradiction