归纳法的一般形式

2019年4月19日   3:20

弱准则:

- $\forall n \geq 2,\ 2^{n+1} \geq n^2 + 3$

Let $P(n)$ – $2^{n+1} \geq n^2 + 3$ Base Step

(i) Note that for $n = 2,\ 2^{2+1} = 8 \geq 7 = 2^2 + 3$ – $P(2)$

(ii) Suppose that $n > 2$ and that $2^n \geq (n-1)^2 + 3$ $(*)$

$$
\begin{aligned}
2^{n+1} &\geq 2(n-1)^2 + 6 \quad \text{Inductive Hypothesis} \\
&= n^2 + 3 + n^2 - 4n + 4 + 1 \\
&= n^2 + 3 + (n-2)^2 + 1 \\
&> n^2 + 3
\end{aligned}
$$

Inductive Step

Hence, we've just prove that for $n > 2,\ P(n-1) \rightarrow P(n)$.

By mathematical induction, $\forall n > 2,\ 2^{n+1} \geq n^2 + 3$.

Inductive Conclusion

强准则:

- Prove that every positive integer is a power of a prime or the product of powers of primes.

  ◇ Base Step: 1 is a power of a prime number, $1 = 2^0$

  ◇ Inductive Hypothesis: Suppose that every number less than n is a power of a prime or a product of powers of primes.

  ◇ Then, if n is not a prime power, it is a product of two smaller numbers, each of which is, by the inductive hypothesis, a power a prime power or a product of powers of primes.

  ◇ Thus, by the strong principle of mathematical induction, every positive integer is a power of a prime or a product of powers of primes.