

# HomeWork\_5

---

Name: 卫焱滨 (Wei Yanbin)

SID: 11710823

## chapter 3

### Exercise 3.27

Reapresent  $-1.5625 \times 10^{-1}$  as binary :

$$-1.5625 \times 10^{-1} = -0.15625 \times 10^0 = -0.00101 \times 2^0$$

Shift 3 bits to normalization :

$$-0.00101 \times 2^0 = 1.01 \times 2^{-3}$$

Exponenet : add the bias  $-3+15 = 12$ . Fraction:  $-0.0100000000$

By the format, we get the answer:

1011000100000000

Comparision of Range:

Because 1.1111.....(Binary) is near to 2, treat them as 2.0 here.

Range of the 16 bit format :

$$-2.0 \times 2^{15} \sim -1.0 \times 2^{-14} \text{ and } 1.0 \times 2^{-14} \sim 2.0 \times 2^{15}, +\infty, -\infty, \text{NaN}$$

Range of single presicion IEEE 754 :

$$-2.0 \times 2^{127} \sim -1.0 \times 2^{-126} \text{ and } 1.0 \times 2^{-126} \sim 2.0 \times 2^{127}, +\infty, -\infty, \text{NaN}$$

Comparision of accuracy(Here discuss Relative precision and ulp):

Relative precision of the 16 bit format :

$$\Delta A/|A| = 2^{-10} \times 2^{\text{exponent}} / |1 \times 2^{\text{exponent}}| = 2^{-10}$$

ulp of the 16 bit format : **one-half ulp**

Relative precision of single precision IEEE 754 :

$$\Delta A/|A| = 2^{-23} \times 2^{\text{exponent}} / |1 \times 2^{\text{exponent}}| = 2^{-23}$$

ulp of the single precision IEEE 754 : **one-half ulp**

## Exercise 3.29

Step 1 —To Binary normalization form :

1	$2.6125 \times 10^1 = 26.125 = 11010.001 = 1.1010001000 \times 2^4$
2	
3	$4.150390625 \times 10^{-1} = 0.4150390625 = 0.011010100111 = 1.1010100111 \times 2^{-2}$

1. Align binary points

Shift number with smaller exponent 6 bit to align

1	$1.1010100111 \times 2^{-2} = 0.0000011010100111 \times 2^4$
---	--

2. Add significands

1	$1.1010001000 \times 2^4 + 0.0000011010100111 \times 2^4 = 1.101010001010100111 \times 2^4$
---	---

3. Normalize result & check for over/underflow

1	$1.1010100010(10100111) \times 2^4$ No overflow.
---	--

4. Round and renormalize if necessary

1	using GRS round to the nearest even :
2	guard = 1, round = 0, sticky = 1
3	Because round to the nearest even, guard=1 and round=0, the last bit of significant bit is 0 :
4	So The nearest even become $1.1010100010 \times 2^4$
5	There is No need to renormalize.

The answer is  $1.1010100010 \times 2^4$

## Exercise 3.30

Express The Result as :

```
1 | -8.0546875 × -1.79931640625 × 10-1
```

Express every operacand as normalized binary form :

```
1 | -8.0546875 = -1.0000000111 × 23
2 | -1.79931640625 × 10-1 = -1.0111000010 × 2-3
```

Do the multiplication by hand:

```
1 | Exponent:  23 × 2-3 = 20
2 |
3 | Fraction:  1.0000000111
4 |           × 1.0111000010
5 |           -----
6 |           0000000000
7 |           1000000011
8 |           0000000000
9 |           0000000000
10 |          0000000000
11 |          0000000000
12 |          1000000011
13 |          1000000011
14 |          1000000011
15 |          0000000000
16 |         1000000011
17 |        1.0111001100001001110
```

Round to the nearest even(by G.R.S.) and renormalization:

```
1 | 1.0111001100(0001001110)
2 |   guard:0 round: 0 sticky:1
3 | By round to the nearest even: need Truncate
4 | So The answer is 1.0111001100, and no need to renormalize.
```

Express as 16-bit form described in 3.27 and also as a decimal number.

```
1 | 1.0111001100 × 20 = 0100000111001100 (1.0111001100 = 1.44921875)
```

Above is the result computed by hand.

Following is the result by using calculator to compute the product is:

```
1 | -8.0546875 × -1.79931640625 × 10-1 = 1.449293137
```

Accuracy and compare to the number by a calculator is as follows:

```
1 Accuracy:
2   one-half ulp.
3 Compare to the result of calculator:
4   Some information was lost because the result did not fit into the available 10-bit field:
   Answer only off by 0.00007438659667.
```