谓词逻辑

谓词逻辑定义

引入　Predicate 谓词 Constant 常量　Variable 变量　Quantifier 量词
相关概念
Truth set

辨析与命题逻辑的区别

Predicates

- The statement $P(x)$ is not a proposition since there are more objects that it can be applied to.

- But the difference is:

  ◇ predicate logic allows us to explicitly manipulate and substitute for the objects

  ◇ predicate logic permits quantified sentences where variables are substituted for statements about the group of objects

P（x）带有变量，会因为x的选值不同真值发生改变，因此不是命题
然而：加上全称 和 特称量词 之后 就会成为命题

◇ Is $P(x)$ a proposition?　　No. Many possible substitutions.
◇ Is $\forall x\ P(x)$ a proposition?
Yes. True if for all x from the unvierse $P(x)$ is ture.

量词

1.取值不仅依赖于量词，还依赖于**Universe**

- Sentence: All SUSTech students are smart.

  - ◇ universe: SUSTech students

    translation: $\forall x\ Smart(x)$

  - ◇ universe: all students

    translation: $\forall x\ (At(x, SUSTech) \rightarrow Smart(x))$

  - ◇ universe: people

    translation: $\forall x\ (Student(x) \wedge At(x, SUSTech) \rightarrow Smart(x))$

2.全称量词和 特称量词作用相当于　整体合取　和　整体析取

- Suppose that the elements in the universe can be enumerated as $x_1, x_2, \ldots, x_n$ then:

  - ◇ $\forall x\ P(x)$ is true whenever $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$ is true
  - ◇ $\exists x\ P(x)$ is true whenever $P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$ is true.

3. 量词的否定：量词的德摩根定律

Conclusion: $\neg \forall x\ P(x)$ is equivalent to $\exists x\ \neg P(x)$

注意易错点

translation: $\forall x\ (At(x, SUSTech) \rightarrow Smart(x))$

translation: $\exists x\ (At(x, SUSTech) \wedge Smart(x))$

4. 量词 嵌套

相同量词可以交换顺序，**不同量词不能交换顺序**