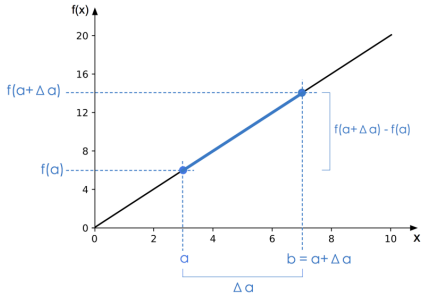
Homework #1
Quiz: August 29, 2018

CS3750: Applied Neural Networks
R. M. Parry

Part I: Candidate Quiz Questions

1. Solve for x and simplify for each of the following:
   (a) $y^{-n} = \frac{1}{x}$
   (b) $y^m \cdot y^n = y^x$
   (c) $\frac{y^m}{y^n} = y^x$
   (d) $(y^m)^n = y^x$
   (e) $y^n \cdot z^n = x^n$
   (f) $\sqrt{y} \cdot \sqrt{z} = \sqrt{x}$
   (g) $\log_b 1 = x$
   (h) $\log_b b = x$
   (i) $\log_b b^a = x$
   (j) $b^{\log_b a} = x$
   (k) $\log_b m + \log_b n = \log_b x$
   (l) $\log_b m - \log_b n = \log_b x$
   (m) $\log_b (m^p) = x \log_b m$
   (n) $(x + 2)(y - 3) = 10$
   (o) $x^2 + 2x + 1 = 25$
   (p) $y_1 + y_2 + \cdots + y_n = \sum_{i=1}^{n} y_x$
   (q) $y_1 \cdot y_2 \cdot \cdots \cdot y_n = \prod_{i=1}^{n} y_x$
2. What is the derivative dy/dx for the following functions:
   (a)



   (b) $y = ax$
   (c) $y = x^a$
   (d) $y = 1/x$
   (e) $y = \log(x)$
   (f) $y = (x^2 - 1)^2$

3. Write a basic Python function (no imports) that creates a list of values, given n as an input argument. As n gets very large, what does the sum of this series approach?

$$\left[1, \frac{1}{4}, \frac{1}{9}, \ldots, \frac{1}{n^2}\right]$$

```
def series(n):
    pass

print(series(3))
print(series(5))
```

Output:

```
[1.0, 0.25, 0.1111111111111111, 0.0625]
[1.0, 0.25, 0.1111111111111111, 0.0625, 0.04, 0.02777777777777776]
```

4. What is the result of each of the following expressions? If an expression cannot be evaluated, explain why.

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   (a) $\mathbf{u} + \mathbf{v}$
   (b) $\mathbf{u} \bullet \mathbf{v}$
   (c) $\mathbf{u} \odot \mathbf{v}$
   (d) $\mathbf{u}\mathbf{v}^T$
   (e) $\mathbf{I}\mathbf{v}$
   (f) $\mathbf{A}^T$
   (g) $\mathbf{A}\mathbf{u}$
   (h) $\mathbf{A} \odot \mathbf{A}$
   (i) $\mathbf{A}\mathbf{A}$
   (j) $\mathbf{A}^{-1}\mathbf{A}$

   Note: $\bullet$ is the dot-product and $\odot$ is the Hadamard (element-wise) product.

Part II: Additional Candidate Exam Questions

1. What is the length of the projection of $\mathbf{v}$ on $\mathbf{w}$? Can the "length" be negative? Why, or why not? The definition of the dot product for your convenience:

$$\mathbf{v} \cdot \mathbf{w} = \sum_{j=1}^{m} v_j w_j = |\mathbf{v}||\mathbf{w}| \cos\theta$$

2. In basic Python (no imports), write a function that computes the dot-product between two lists. You can assume that `u` and `v` have the same length. $x = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$.

```
def dot_product(u, v):
    # compute x
    return x

u = [1, 2, 3]
v = [4, 5, 6]
```

```
print(dot_product(u, v))

u = [1, 2, 3, 4, 10]
v = [-1, 3, 5, 4, 6]
print(dot_product(u, v))
```

Output:

```
32
96
```

3. In basic Python (no imports), write a function that computes matrix multiplication from two lists of lists: $c_j^{(i)} = \sum_{k=1}^{p} a_k^{(i)} b_j^{(k)}$.

```
def matrix_multiply(a, b):
    # compute the list of lists, c
    return c

a = [[1, 3], [2, 4]]
b = [[5, 7], [6, 8]]
print(matrix_multiply(a, b))

a = [[-1, 2], [6, 9]]
b = [[0, -4], [-1, 18]]
print(matrix_multiply(a, b))
```

Output:

```
[[23, 31], [34, 46]]
[[-2, 40], [-9, 138]]
```