EE 235, Winter 2018
Homework 1: Math Review SOLUTIONS
Due Saturday January 6, 2018 by 11:59pm via ONLINE SUBMISSION

HW1 Topics: Complex Numbers, Functions, and Integration
HW1 References: OWN Sections 1.2, 1.2.1, HW1 Supplementary Notes
HW1 Problems (Total = 64 pts):

1. *Complex Numbers - Magnitude and Phase Components, Real and Imaginary Parts.*

   (a) (5 pts) Identify the magnitude component $|z|$ and the phase component $\angle z$ for the following complex numbers:

   i. $z = 4e^{-j}$.
   By inspection, $\boxed{|z| = 4}$ and $\boxed{\angle z = -1}$

   ii. $z = e^{j\frac{\pi}{6}}$.
   By inspection, $\boxed{|z| = 1}$ and $\boxed{\angle z = \frac{\pi}{6}}$

   (b) (5 pts) Identify the real part $Re\{z\}$ and the imaginary part $Im\{z\}$ for the following complex numbers:

   i. $z = 2 - j3$.
   By inspection, $\boxed{Re\{z\} = 2}$ and $\boxed{Im\{z\} = -3}$

   ii. $z = j2$.
   We can rewrite z as $z = 0 + j4$.
   Therefore, by inspection, $\boxed{Re\{z\} = 0}$ and $\boxed{Im\{z\} = 2}$

   iii. $z = 3$.
   We can rewrite z as $z = 3 + j0$.
   Therefore, by inspection, $\boxed{Re\{z\} = 3}$ and $\boxed{Im\{z\} = 0}$

2. *Complex Numbers - Polar Form and Rectangular Form.*

   (a) (5 pts) Using the unit circle or formulas for r and θ, convert the following complex numbers in to polar form, $z = re^{j\theta}$. Make sure $r > 0$ and $-\pi < \theta \leq \pi$:

   i. $z = \frac{\sqrt{3}}{2} + j\frac{1}{2}$.
   $r = |z| = \sqrt{x^2 + y^2} = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$
   $\theta = \angle z = arctan(\frac{y}{x}) = arctan(\frac{1/2}{\sqrt{3}/2}) = arctan(\frac{1}{\sqrt{3}}) = \frac{\pi}{6}$
   Therefore, $\boxed{z = e^{j\frac{\pi}{6}}}$

   ii. $z = -2$
   $r = |z| = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (0)^2} = \sqrt{4 + 0} = 2$
   $\theta = \angle z = arctan(\frac{y}{x}) = arctan(\frac{0}{-2}) = \pi + arctan(\frac{0}{2}) = \pi + 0 = \pi$
   Therefore, $\boxed{z = 2e^{j\pi}}$

   (b) (5 pts) Using the complex plane or Euler's formula, convert the following complex numbers in to rectangular form, $z = x + jy$:

   i. $z = 3e^{-j\pi}$
   $z = 3[cos(\pi) - jsin(\pi)] = 3[(-1) - j(0)] = \boxed{-3}$

ii. $z = 2e^{j\frac{\pi}{2}}$

$z = 2[cos(\frac{\pi}{2}) + jsin(\frac{\pi}{2})] = 2[(0) + j(1)] = \boxed{\text{j2}}$

3. *Complex Conjugation*

(a) Using the method of complex conjugation for dividing complex numbers, simplify the expression for each of the following complex numbers so that your answer is in rectangular form, $z = x + y$:

i. (2 pts) $z = \frac{1}{1-j2}$. Show that $z = \frac{1}{5} + j\frac{2}{5}$.

$z = \frac{1}{1-j2} \cdot \frac{1+j2}{1+j2} = \frac{1+j2}{1-j2+j2-4j^2} = \frac{1+j2}{5} = \boxed{\frac{1}{5} + j\frac{2}{5}}$

ii. (5 pts) $z = -\frac{1+j2}{1-j2}$. $z = -\frac{1+j2}{1-j2} \cdot \frac{1+j2}{1+j2} = -\frac{1+j2+j2+4j^2}{1-j2+j2-4j^2} = -\frac{-3+j4}{5} = \boxed{\frac{3}{5} - j\frac{4}{5}}$

(b) Using the method of complex conjugation for finding magnitude, find the magnitude squared component $|z|^2$ for:

i. (2 pts) $z = 1 + j3$.

Show that $|z|^2 = 10$.

$|z|^2 = zz^* = (1 + j3)(1 - j3) = 1 + 9 = \boxed{10}$

ii. (2 pts) $z = 2e^{j3}$.

Show that $|z|^2 = 4$.

$|z|^2 = zz^* = (2e^{j3})(2e^) - j3 = 4e^{j0} = \boxed{4}$

4. *Function Evaluation.*

(a) (5 pts) Let $y(t) = tx(t + 3)$

i. What is the expression for $y(t - 3)$?

For $y(t - 3)$, replace t with t-3: $y(t - 3) = (t - 3)x(t - 3 + 3) = \boxed{y(t - 3) = (t - 3)x(t)}$

ii. What is the expression for $y(2t)$?

For $y(2t)$, replace t with 2t: $\boxed{y(2t) = 2tx(2t + 3)}$

(b) (5 pts) Let $y(t) = \int\limits_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$

i. What is the expression for $y(3)$?

For $y(3)$, simply evaluate $y(t)$ at t = 3: $\boxed{y(3) = \int\limits_{-\infty}^{\infty} x(\tau)h(3 - \tau)d\tau}$

ii. What is the expression for $y(-t)$?

For $y(-t)$, simply evaluate $y(t)$ at -t: $\boxed{y(-t) = \int\limits_{-\infty}^{\infty} x(\tau)h(-t - \tau)d\tau}$

5. *Integration.*

(a) (2 pts) Evaluate the following integral: $\int_3^\infty e^{-6t}dt + \int_{-\infty}^0 e^{6t}dt$.

Show that the answer is $\frac{1}{6}(e^{-18} + 1)$.

$\int_3^\infty e^{-6t}dt = -\frac{1}{6}e^{-6t}|_{t=3}^{t=\infty} = -\frac{1}{6}(e^{-6(\infty)} - e^{-6(3)}) = -\frac{1}{6}(0 - e^{-18}) = \frac{1}{6}e^{-18}$

$\int_{-\infty}^0 e^{6t}dt = \frac{1}{6}e^{6t}|_{t=-\infty}^{t=0} = \frac{1}{6}(e^{6(0)} - e^{6(-\infty)}) = \frac{1}{6}(1 - 0) = \frac{1}{6}$

Therefore, $\int_3^\infty e^{-6t}dt + \int_{-\infty}^0 e^{6t}dt = \frac{1}{6}e^{-18} + \frac{1}{6} = \boxed{\frac{1}{6}(e^{-18} + 1)}$

(b) (2 pts) Evaluate the integral $\int_{t-2}^{5} d\tau$.
Note: τ is the variable of integration and t can be treated as a constant.
Show that the answer is $-t+7$.
$\int_{t-2}^{5} d\tau = \int_{t-2}^{5} 1 d\tau = \tau|_{\tau=t-2}^{\tau=5} = 5-(t-2) = 5-t+2 = \boxed{\text{-t + 7}}$

(c) (2 pts) Suppose $\int_{-\infty}^{\infty} x(t)dt = 3$. Using this known integral and u-substitution, evaluate $\int_{-\infty}^{\infty} x(2t)dt$.
Show that the answer is $\frac{3}{2}$.
First, we can rewrite $\int_{-\infty}^{\infty} x(2t)dt$ using a u-substitution.
Let $u = 2t$, so $t = \frac{u}{2}$ and $dt = \frac{1}{2}du$.
Using $u = 2t$, the bound $t = -\infty$ becomes $u = 2(-\infty) = -\infty$ and the bound $t = \infty$ becomes $u = 2(\infty) = \infty$.
Therefore, $\int_{-\infty}^{\infty} x(2t)dt = \int_{-\infty}^{\infty} x(u)\frac{1}{2}du = \frac{1}{2}\int_{-\infty}^{\infty} x(u)du$
Using the given integral above, $\frac{1}{2}\int_{-\infty}^{\infty} x(u)du = \frac{1}{2}(3) = \boxed{\frac{3}{2}}$

(d) (5 pts) Suppose $\int_{-\infty}^{x} x(t)dt = 2$, where $x(t)$ is a function of t, t is the variable of integration, and x can be treated as a constant. Using u-substitution, evaluate $\int_{-\infty}^{x-1} 2x(t+1)dt$.
First, we can rewrite $\int_{-\infty}^{x-1} 2x(t+1)dt$ using a u-substitution.
Let $u = t+1$, so $t = u-1$ and $dt = du$.
Using $u = t+1$, the bound $t = -\infty$ becomes $u = -\infty + 1 = -\infty$ and the bound $t = x-1$ becomes $u = x-1+1 = x$.
Therefore, $\int_{-\infty}^{x-1} 2x(t+1)dt = \int_{-\infty}^{x} 2x(u)du = 2\int_{-\infty}^{x} x(u)du$
Using the given integral above, $2\int_{-\infty}^{x} x(u)du = 2(2) = \boxed{4}$

(e) (2 pts) Consider $\int_{-\infty}^{t+2} x(\tau - t_o)d\tau$, where τ is the variable of integration and t and t_o can be treated as constants. Using u-substitution, we can rewrite this integral as $\int_{-\infty}^{a} x(u)du$. What is a in terms of t and t_o?
Show that $a = t+2-t_o$.
We can use the substitution $u = \tau - t_o$, so $\tau = u + t_o$ and $d\tau = du$.
Using $u = \tau - t_o$, the bound $\tau = -\infty$ becomes $u = -\infty - t_o = -\infty$ and the bound $\tau = t+2$ becomes $u = t+2-t_o$.
Therefore, $\int_{-\infty}^{t+2} x(\tau - t_o)d\tau = \int_{-\infty}^{t+2-t_o} x(u)du$
Comparing the two final integrals, $\boxed{a = t+2-t_o}$

6. *Homework Self-Reflection*
(10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

`http://bit.ly/2qfmaEQ`