EE 235, Winter 2018, Homework 9: Amplitude Modulation and Sampling
(Due Friday March 2, 2018 via Canvas Submission)
Write down *ALL* steps for full credit

HW9 Topics:

- Sampling Theorem and Aliasing
- Modulation and Demodulation

HW9 Course Learning Goals Satisfied:

- Goal 5: Understand how modulation and sampling affects the frequency components of the input signal.
- Goal 6: Use and understand standard EE terminology associated with filtering and LTI systems (e.g. LPF, HPF, etc.)

HW9 References:

- OWN Sections 7.1 - 7.3, 8.1 - 8.2

HW9 Problems (Total = 134 pts):

1. *Review.* (20 pts)

   (a) *System Properties.* To get you ready for Ch.9 in which we will introduce another way to test system properties, let us review

       i. (4 pts) Suppose a system is described by the following I/O relationship:

   $$y(t) = \int_t^\infty x(\tau - 1)d\tau$$

       Without transforming this system into another system description (e.g. $h(t)$ or $H(j\omega)$), is this system causal? Is this system stable? Justify your answers.

       ii. (4 pts) Suppose an LTI system is described by the following impulse response:

   $$h(t) = (\delta(t) + 2\delta(t-1)) * (\delta(t) - \delta(t-2))$$

       Without transforming this system into another system description (e.g. I/O or $H(j\omega)$), is this system causal? Is this system stable? Justify your answers.

   (b) *LTI System Output.* To get you ready for Ch.9, in which we will introduce another way to find the output of an LTI system, let us review the three different ways we have learned so far to evaluate the output.

       i. (4 pts) $x(t) = e^{-t}u(t)$ and $h(t) = u(t) - 2u(t-1) + u(t-3)$. Using convolution in the time domain, find $y(t)$. Show that $y(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{-t}, & 0 < t < 1 \\ 2e^{-(t-1)} - e^{-t} - 1, & 1 < t < 3 \\ 2e^{-(t-1)} - e^{-t} - e^{-(t-3)}, & t > 3 \end{cases}$

       ii. (4 pts) $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - k\frac{\pi}{2})$ and $h(t) = 4sinc(8t)e^{j(6t)}$. Using Fourier Series, find $y(t)$.
   *Hint: You will need to find $H(j\omega)$ to solve this problem.*
       Show that $y(t) = 1 + e^{j4t} + e^{j8t} + e^{j12t}$.

       iii. (4 pts) $x(t) = e^{-3t}u(t) - 2te^{-3t}u(t)$ and $h(t) = 4e^{-t}u(t) - 2e^{-2t}u(t)$. Using Fourier Transforms, find $y(t)$.

2. *Modulation.* (16 pts) The purpose of these problems is to get you comfortable walking through a block diagram consisting of modulation (by cosine) systems.

(a) (4 pts) Consider a signal $x(t)$ with Fourier Transform $X(j\omega)$ that is input to a system consisting of LTI and modulation systems as shown below:
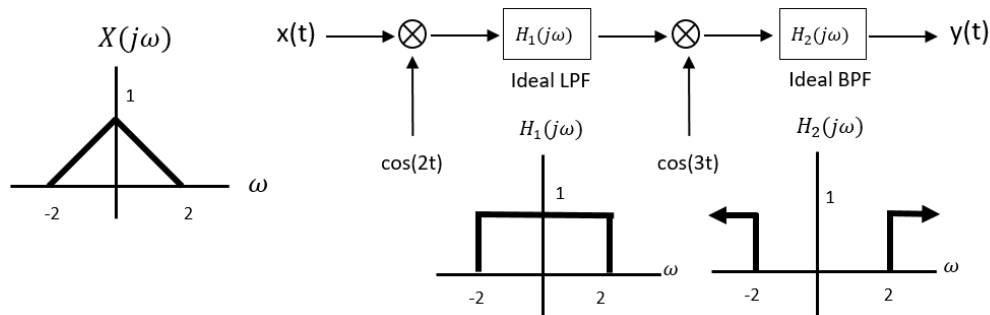
Sketch the output Fourier Transform $Y(j\omega)$.

Hint: The output will consist of two pieces with an amplitude of $\frac{1}{4}$.

(b) (4 pts) Consider a signal $x(t)$ with Fourier Transform $X(j\omega)$ that is input to a system consisting of LTI and modulation systems as shown below:
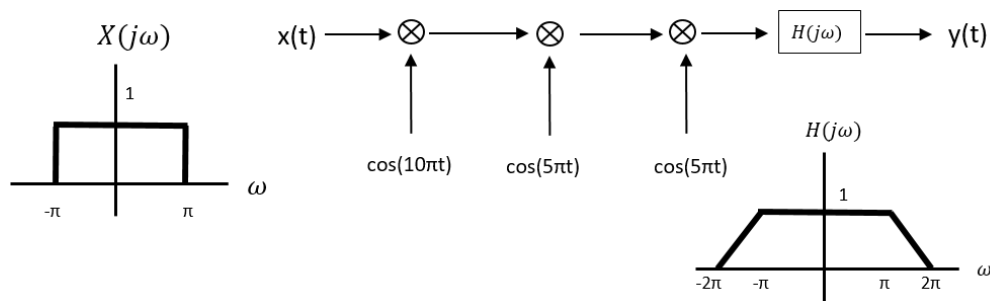
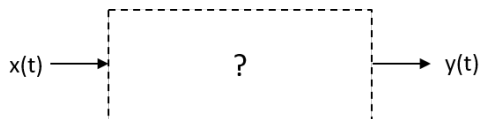Sketch the output Fourier Transform $Y(j\omega)$.

(c) Let us now tackle the problem of deducing an unknown system:

Using only LTI systems and/or modulation systems, draw the full system diagram that produces the output Fourier Transform $Y(j\omega)$ given a known input Fourier Transform $X(j\omega)$ below. If you use LTI systems, please make sure to sketch the corresponding frequency response $H(j\omega)$ that is required.
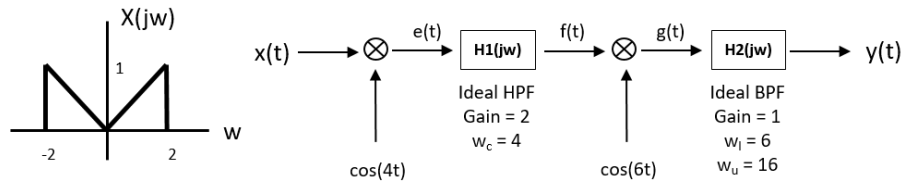
i. (4 pts) Unknown System #1:

2
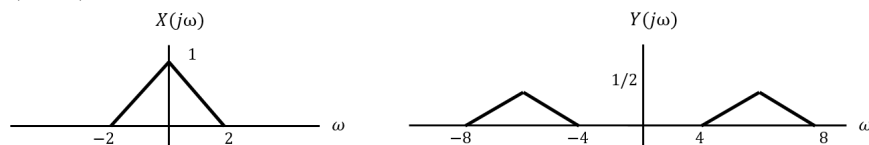
ii. (4 pts) Unknown System #2:

$X(j\omega)$

$Y(j\omega)$

3. *Demodulation.* (26 pts) The purpose of this problem is to apply what you have learned about modulation by cosine to design a demodulation system to recover an input or obtain some other desired result.
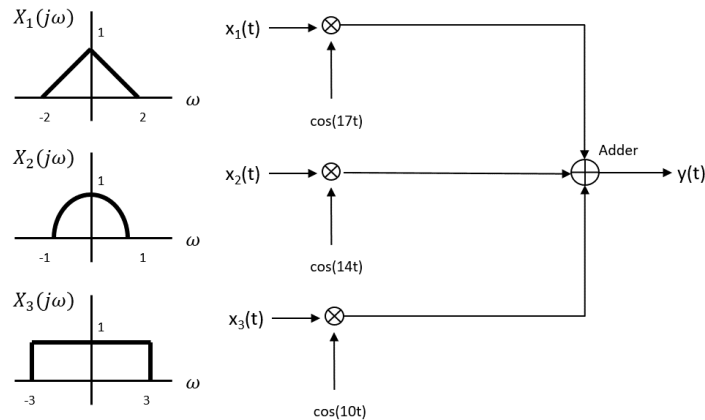
(a) Consider the following block diagram:

i. (4 pts) Sketch the output spectrum $Y(j\omega)$. Precisely label all points and axes.

ii. $X(j\omega)$ can be recovered from $Y(j\omega)$ using a single modulator followed by an ideal LPF with frequency response $H_r(j\omega)$. Answer the following questions:

A. (4 pts) Draw the block diagram for this system. Clearly specify the modulation signal, as well as the filter gain A and cutoff frequency w_c for the ideal LPF.

B. (2 pts) Note that several cutoff frequencies w_c are valid for (A). What <u>range</u> of cutoff frequencies w_c can be used to fully recover $x(t)$ from $y(t)$?

(b) (4 pts) Suppose the output of a modulation system is given by $Y(j\omega)$ below. Draw the demodulation system that will recover the desired input again, shown below as $X_r(j\omega)$:
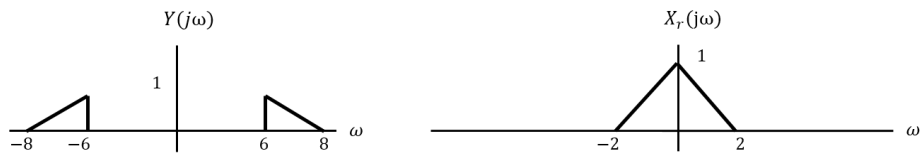

(c) We briefly learned in class that modulation is useful for different radio stations to send radio signals out simultaneously. This is a process called frequency division multiplexing, which we will practice with below.

i. (4 pts) Consider the following frequency division multiplexing system that can be used to transmit multiple signals at the same time:

$X_1(j\omega)$

$x_1(t) \longrightarrow \otimes$

cos(17t)

$X_2(j\omega)$

$x_2(t) \longrightarrow \otimes$

cos(14t)

$X_3(j\omega)$
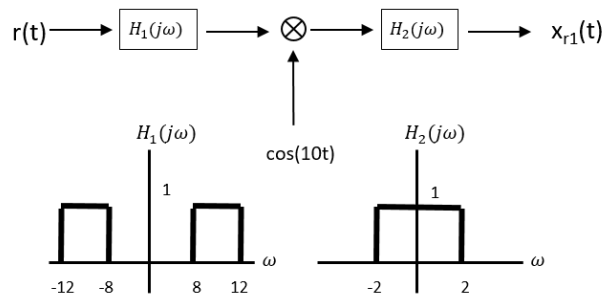
$x_3(t) \longrightarrow \otimes$

cos(10t)

Adder

y(t)

Draw the overall output Fourier Transform $Y(j\omega)$ produced by this system.

ii. (4 pts) Suppose a receiver picks up a signal $r(t)$ whose Fourier Transform is given below:

$R(j\omega)$

1/2

-12   -8   -2   2   8   12 ω
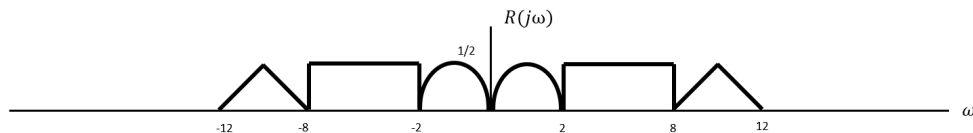
We can get back the received triangle signal $x_1(t)$ using the system below. The idea is to first filter out the desired signal and then shift it back to the center around frequency $\omega = 0$:

r(t) $\longrightarrow$ $H_1(j\omega)$ $\longrightarrow \otimes \longrightarrow$ $H_2(j\omega)$ $\longrightarrow$ $x_{r1}(t)$

$H_1(j\omega)$

cos(10t)

$H_2(j\omega)$

1

1

-12   -8   8   12 ω

-2   2 ω

Sketch the output Fourier Transform $X_{r1}(j\omega)$ of this system.

iii. (4 pts) You will notice the system has an error in the output amplitude. What change can be made to the receiver system to get the desired amplitude back up to 1? Note: there are several answers to this question.
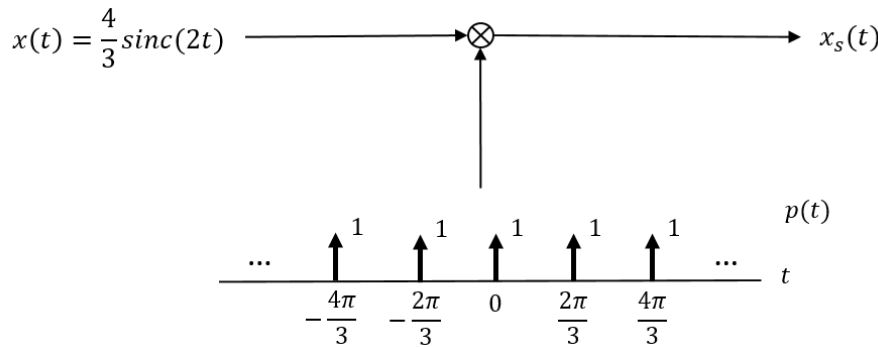
4. *Impulse Train Sampling System.* (34 pts) The purpose of this problem is to get you comfortable walking through a block diagram consisting of an impulse train sampling system and to visually understand the concept of aliasing.

(a) $x(t)$ undergoes impulse train sampling through the following system below:

$x(t)$ ———————⊗———————→ $x_s(t)$

$X(j\omega)$ $\frac{\pi}{15}$

ω

-2   2

$$p(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{\pi}{5}n\right)$$
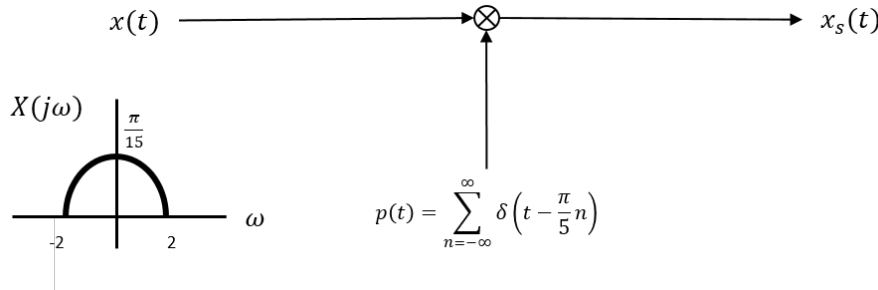
Answer the following questions:

  i. (2 pts) What is the sampling frequency ω_s used by this system? What is the equation for the output Fourier Transform $X_s(j\omega)$ in terms of $X(j\omega)$?

  Show that $\omega_s = 10$ and $X_s(j\omega) = \frac{5}{\pi} \sum_{k=-\infty}^{\infty} X(j(\omega - 10k))$

  ii. (4 pts) Using your equation from (i), sketch the output spectrum $X_s(j\omega)$ vs. ω. Make sure to label all critical points.

  iii. (2 pts) Using your sketch from (ii) and your understanding of the concept of aliasing, explain why this is an example of sampling with no aliasing.

(b) Now consider the following system diagram with a different impulse train and input $x(t)$:

$x(t) = \frac{4}{3}sinc(2t)$ ———————⊗———————→ $x_s(t)$

$p(t)$

1   1   1   1   1

$-\frac{4\pi}{3}$ $-\frac{2\pi}{3}$ 0 $\frac{2\pi}{3}$ $\frac{4\pi}{3}$

t

Answer the following questions:

  i. (4 pts) What is the sampling frequency f_s (not ω_s!) used by this system? What is the equation for the output Fourier Transform $X_s(j\omega)$ in terms of $X(j\omega)$?

  ii. (4 pts) Using your equation from (i), sketch the output spectrum $X_s(j\omega)$ vs. ω. Make sure to label all critical points.
  Note: you need to first find and sketch $X(j\omega)$.

  iii. (2 pts) Using your sketch from (ii), does aliasing of $x(t)$ occur? Justify your answer.

(c) Now consider the following system consisting of both modulation and impulse train sampling systems:

$x_1(t)$

$x(t)$ ———→ ⊗ —————————→ ⊗ ———→ $y(t)$

$X(j\omega)$

2

$\cos(4t)$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{\pi}{2}n\right)$$

ω

-1   1
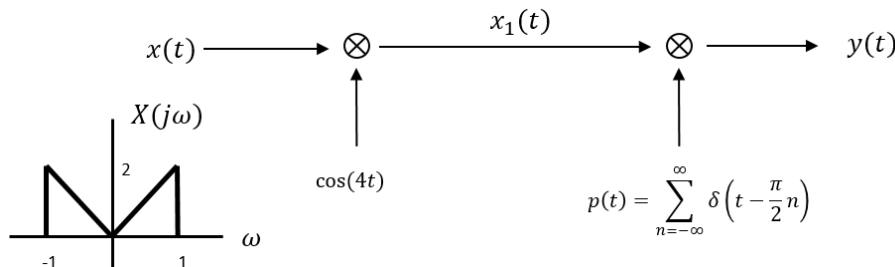
i. (4 pts) What is the sampling period T_s used by the impulse train sampling system? What is the equation for $X_1(j\omega)$ in terms of $X(j\omega)$? What is the equation for $Y(j\omega)$ in terms of $X_1(j\omega)$?

ii. (4 pts) Sketch the final output spectrum $Y(j\omega)$ vs. ω. Make sure to label all critical points.

iii. (4 pts) Using your sketch, is it possible to recover the original input $x(t)$ from $y(t)$? If so, draw or describe a system diagram that will do this. Make sure to clearly specify all system blocks being used either in words or with a graph.

(d) (4 pts) Using your knowledge of the behavior of impulse train sampling systems, draw the appropriate system diagram that will produce the following input-output pair. Justify your design choices.

x(t) $\longrightarrow$ [?] $\longrightarrow$ y(t)

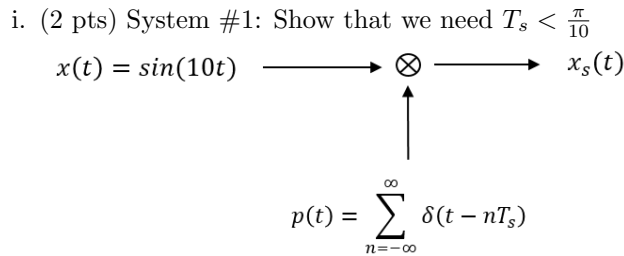$X(j\omega)$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $Y(j\omega)$
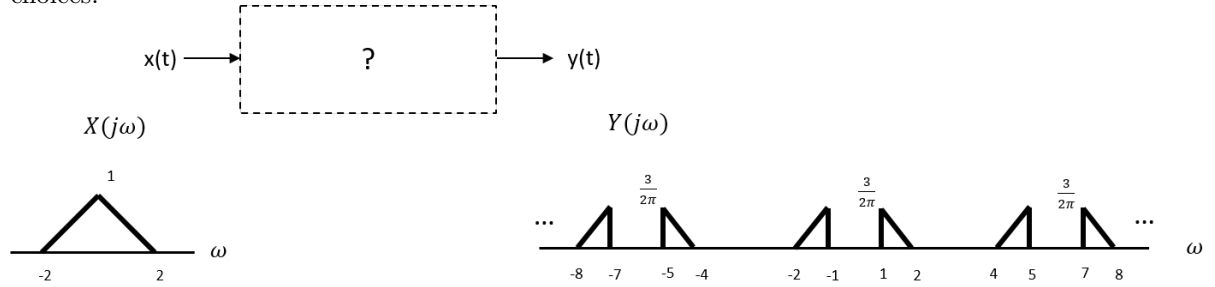
Make sure to clearly specify your systems with an equation, description, or graph.
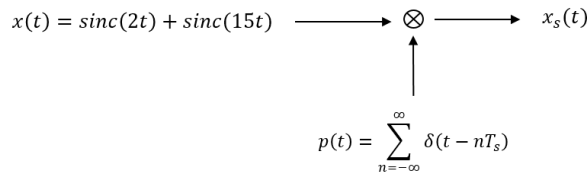
5. *Sampling Theorem and Aliasing.* (28 pts) The purpose of this problem is to apply the sampling theorem to determine how much to sample a continuous-time signal by to avoid aliasing.

(a) Every signal $x(t)$ has a so-called Nyquist rate ω_N used to determine the sampling rate ω_s needed to avoid aliasing. What is the Nyquist rate ω_N for each of the following signals below?

i. (2 pts) $x(t) = sinc(5000t) * \cos(5\pi t)$
Show that $\omega_N = 10\pi$

ii. (4 pts) $x(t) = \frac{\sin(10\pi t)\cos(10\pi t)}{10\pi t}$

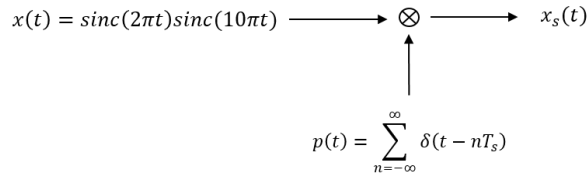iii. (2 pts) $x(t) = sinc(5000t) + \cos(5\pi t)$

(b) Signal $x(t)$ undergoes impulse train sampling. Using the sampling theorem, what should the sampling period T_s be for each sampling system below to avoid aliasing? Justify your answer.

i. (2 pts) System #1: Show that we need $T_s < \frac{\pi}{10}$

$x(t) = \sin(10t) \longrightarrow \otimes \longrightarrow x_s(t)$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

ii. (2 pts) System #2:

$x(t) = sinc(2t) + sinc(15t) \longrightarrow \otimes \longrightarrow x_s(t)$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

iii. (2 pts) System #3:

$$x(t) = sinc(2\pi t)sinc(10\pi t) \longrightarrow \otimes \longrightarrow x_s(t)$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

(c) Consider an impulse train sampling with a fixed sampling period as shown below:

$$x(t) \longrightarrow \otimes \longrightarrow x_s(t)$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{\pi}{100}n\right)$$
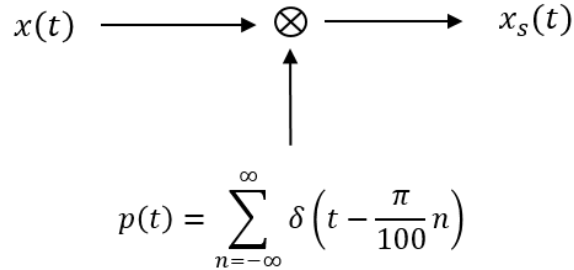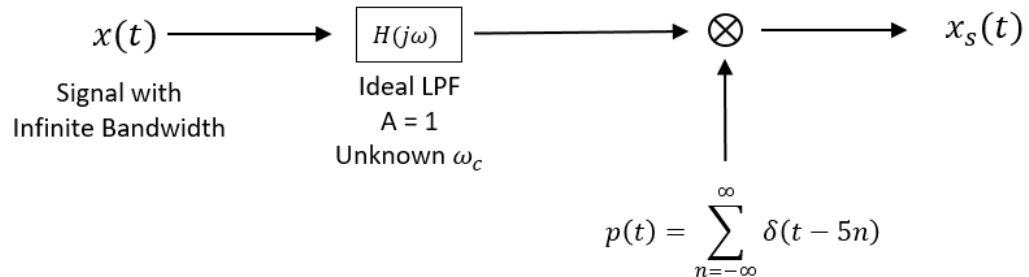
For each input signal $x(t)$ below, what range of values should the unknown parameter ω_x satisfy so that $x(t)$ can be recovered uniquely from its samples in $x_s(t)$? Justify your answer.

  i. (2 pts) $x(t) = sinc(\omega_x t)$
    Show that we need $\omega_x < 100$.
  ii. (2 pts) $x(t) = sinc^2(\omega_x t)$
  iii. (2 pts) $x(t)$ is generated from the system below:

$$s(t) \longrightarrow \otimes \longrightarrow x(t)$$

$$S(j\omega)$$
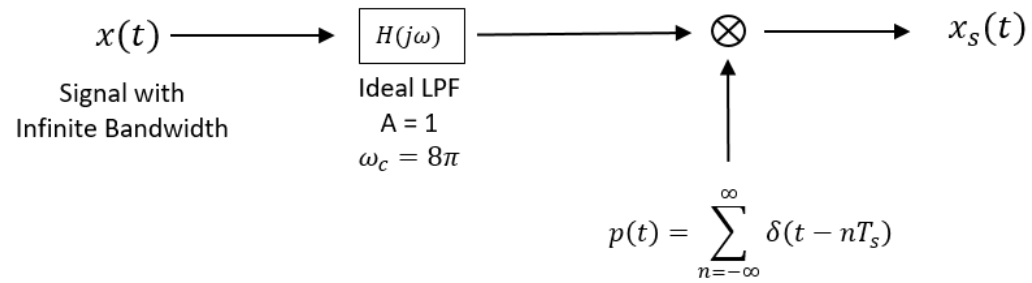
1

$$cos(\omega_x t)$$

-2   2 ω
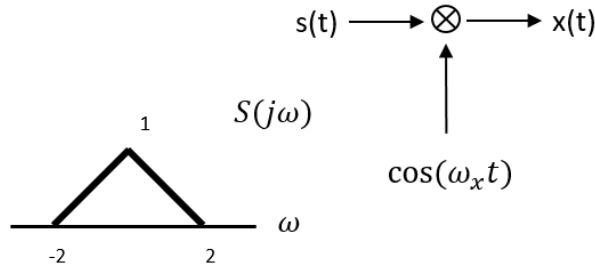
(d) In the real-world, signals are not bandlimited in frequency ω and hence must undergo pre-filtering before sampling for perfect reconstruction. Answer the following questions for each ideal pre-filter and impulse train sampling system below:

  i. (4 pts) System #1: Consider an ideal pre-filter with unknown cutoff frequency ω_c and an impulse train sampling system with a fixed sampling period T_s:

$$x(t) \longrightarrow \boxed{H(j\omega)} \longrightarrow \otimes \longrightarrow x_s(t)$$

Signal with
Infinite Bandwidth

Ideal LPF
A = 1
Unknown ω_c

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - 5n)$$

What range of cutoff frequencies ω_c in the pre-filter are allowed that will prevent aliasing? Justify your answer.

  ii. (4 pts) System #2: Consider an idea pre-filter with known cutoff frequency ω_c and an impulse train sampling system with unknown paramters:

7

$$x(t) \longrightarrow \boxed{H(j\omega)} \longrightarrow \otimes \longrightarrow x_s(t)$$

Signal with
Infinite Bandwidth

Ideal LPF
A = 1
$\omega_c = 8\pi$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

What should the sampling period T_s of the impulse train sampling system be to avoid aliasing? Justify your answer.

6. *Homework Self-Reflection*

(10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Y our self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

http://bit.ly/2CGRqxF