EE 235, Winter 2018, Homework 8: Fourier Transforms, LTI Systems, and Filters
Due Wednesday February 21, 2018 in class via Canvas Submission
Write down *ALL* steps for full credit

HW8 Topics:

- Fourier Transforms: LTI
- LTI Filters

HW8 Course Learning Goals Satisfied:

- Goal 1: Perform convolutions for arbitrary and closed-form continuous-time signals
- Goal 2: Analyze LTI system given different system representations (including input-output equations, impulse response, frequency response) and translate between these representations.
- Goal 3: Use and understand standard EE terminology associated with filtering and LTI systems (e.g. LPF, HPF, impulse response, step response, etc.)

HW8 References: OWN Sections 4.4, 4.7, 3.9.2, 6.1, 6.2.0

HW8 Problems (Total = 84 pts):

1. *Review* (15 pts)

   (a) (5 pts) *LTI Systems.*
   Consider two LTI subsystems that are connected in cascade, where system T1 has step response $s_1(t) = u(t-2) - u(t-6)$ and system T2 has impulse response $h_2(t) = e^{-4t}u(t)$. Find the overall impulse response $h(t)$.

   (b) (5 pts) *Fourier Series.*
   The input signal $x(t)$ and the impulse response $h(t)$ of the system is given as follows:
   $x(t) = sin(2t)cos(t) - e^{j3t} + 2$ and $h(t) = \frac{sin(2t)}{t}$
   Using Fourier Series, find the output $y(t)$.

   (c) (5 pts) *Parseval's Theorem.*
   Let's consider the system in Problem 1-(b). Using Parseval's Theorem, compute the power P_∞ of the output $y(t)$ and the energy E_∞ of the impulse response $h(t)$.

2. *Fourier Transform: Frequency Response* (15 pts)

   (a) (5 pts) Let's consider the LTI system with the impulse response $h(t) = 5e^{-3t}u(t)$. And the input to this LTI system is $x(t) = e^{-2t}u(t)$. Find $Y(jw)$ and then take the inverse transform to find $y(t)$.

   (b) (5 pts) The impulse response and the output are given as follows:
   $H(jw) = \frac{1}{5+jw}$ and $y(t) = e^{-4t}u(t) - e^{-5t}u(t)$.
   Find input $x(t)$.

   (c) (5 pts) Let's consider the LTI system with the impulse response $h(t) = \frac{4}{\pi}sinc(2(t-1))$.
      i. Find the frequency response $H(jw)$.

      ii. Find the output $y(t)$ when input is $x(t) = \sin(t)$.

3. *Fourier Transform: LTI Systems Described by LCCDE.* (32 pts)

(a) Consider the causal LTI system represented by its input-output relationship: $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = -x(t)$.

i. (4 pts) Find the frequency response $H(jw)$.

ii. (4 pts) Find the impulse response $h(t)$.

iii. (4 pts) Find the output $y(t)$ when $x(t) = e^{-2t}u(t)$.

(b) A causal LTI system is described by the following differential equation:

$$\frac{dy(t)}{dt} + 4y(t) = 9x(t).$$

i. (4 pts) Find the frequency response $H(j\omega)$ of this system.

ii. (4 pts) Find the magnitude of the frequency response, $|H(j\omega)|$.

iii. (4 pts) Sketch the magnitude of the frequency response (for both positive and negative ω).

iv. (4 pts) Classify this system as low-pass/high-pass/band-pass/band-stop.

v. (4 pts) Find the impulse response $h(t)$ of this system.

4. *Fourier Transforms: LTI Filters.* (12 pts)

An LTI system is described by the following equation: $y(t) = x(t) - x(t) * h_1(t)$, where $h_1(t)$ is an ideal BPF with gain A = 1 and cutoff frequencies $w_l = 5$ and $w_u = 6$.

(a) (4 pts) What is the overall frequency response $H(jw)$ in terms of $H_1(jw)$?

(b) (4 pts) Sketch $|H(jw)|$.

(c) (4 pts) Classify filter type of this system. Show, or explain why, this is a bandstop filter (BSF).

5. *Homework Self-Reflection*
(10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.
http://bit.ly/2G8x0zL