Sample Exam (0 points)
September 26, 2019 Name: ______________________________

No notes, calculators, or other aids are allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work.

1. Review (15 pts)

(a) (5 pts) LTI Systems.

Consider two LTI subsystems that are connected in cascade, where system T1 has step response

s1(t) = u(t 2)u(t 6) and system T2 has impulse response h2(t) = e4tu(t). Find the overall

impulse response h(t).

(b) (5 pts) Fourier Series.

The input signal x(t) and the impulse response h(t) of the system is given as follows:

x(t) = sin(2t)cos(t) ej3t + 2 and h(t) = sin(2t)t

Using Fourier Series, find the output y(t).

(c) (5 pts) Parseval's Theorem.

Let's consider the system in Problem 1-(b). Using Parseval's Theorem, compute the power P

of the output y(t) and the energy E of the impulse response h(t).

4. LTI System Properties

(a) Suppose you are given the following impulse response of an LTI system. Using your causalit

from Ch.2 and last week's lectures, is this LTI system causal or not? Justify your answer. Hin

it might help to plot out h(t).

i. (2 pts) h(t) = e^3t*u(1 + t)

Show that this system IS NOT causal.

ii. (2 pts) h(t) = u(2 t)u(t)

Show that this system IS causal.

iii. (4 pts) h(t) = e^2t*u(t + 3)

iv. (4 pts) h(t) = e^2t*u(t + 2)u(t 1)

v. (2 pts) h(t) = (t 5) + 3(t 3)

Show that this system IS causal.

vi. (4 pts) h(t) = d/dt [u(t 1) + u(t)]

(b) (4 pts) Suppose an LTI system has the impulse response h(t) = et/2*u(t to 2). For what va

(c) Suppose you are given the following impulse response of an LTI system. Using your stability

|h(t)|dt.

i. (2 pts) h(t) = e^t/2*u(3 t)

Show that|h(t)|dt = 2e3/2, and hence we conclude the system IS stable.

ii. (4 pts) h(t) = e^t*u(t + 1)

iii. (2 pts) h(t) = e^(t1)/3*u(1 t)

Show that

|h(t)|dt, and hence we conclude the system IS NOT stable.

iv. (4 pts) h(t) = e^3t*u(3 t)

v. (4 pts) h(t) = u(6 t) u(1 t)

Hint: sketch out h(t) to help you reduce the integral.

vi. (2 pts) h(t) = 2e^3t sin(5t)[u(t 1) u(t 3)]

Show that |h(t)|dt 23 [e3 e9], and hence we conclude the system IS stable.

vii. (4 pts) h(t) = e^(j1)t cos(2t)u(t 1)

0, otherwise

Find the output Fourier Series coefficients bk and the corresponding output signal y(t)

Show that the final output you get is y(t) = 4 cos(6t) 8 sin(4t)

(d) (4 pts) Suppose a signal with Fourier Series representation o = 2 and ak = {1/2jk, |k| < 3

5. Convolution.

(a) (4 pts) Consider an LTI system with impulse response h(t) = 2k=2(t kT), for some positive

(b) (2 pts) Consider an LTI system with input x(t) = 12 (u(t) u(t 2)) and impulse response h

Show that y(t) = (3/2)u(t+1) 3/2u(t 1) 1/2u(t 3) + 1/2u(t 5)

(c) (4 pts) Consider an LTI system with input x(t) = (t) + 2(t 4) and impulse response h(t) =

0, otherwise

i. (3 pts) Suppose the input is a periodic signal x(t) with the Fourier Series representation

wo =

4. Impulse Train Sampling System. (34 pts) The purpose of this problem is to get you comfortabl

ing through a block diagram consisting of an impulse train sampling system and to visually unde

the concept of aliasing.

(a) x(t) undergoes impulse train sampling through the following system below:

Answer the following questions:

i. (2 pts) What is the sampling frequency s used by this system? What is the equation for the output Fourier Transform Xs(j) in terms of X(j)?

Show that s = 10 and Xs(j) = 5 k= X(j(10k))

ii. (4 pts) Using your equation from (i), sketch the output spectrum Xs(j) vs. . Make sure to label all critical points.

iii. (2 pts) Using your sketch from (ii) and your understanding of the concept of aliasing, ex

why this is an example of sampling with no aliasing.

(b) Now consider the following system diagram with a different impulse train and input x(t):

Answer the following questions:

i. (4 pts) What is the sampling frequency fs (not s!) used by this system? What is the equation for the output Fourier Transform Xs(j) in terms of X(j)?

ii. (4 pts) Using your equation from (i), sketch the output spectrum Xs(j) vs. . Make sure to label all critical points.

Note: you need to first find and sketch X(j).

iii. (2 pts) Using your sketch from (ii), does aliasing of x(t) occur? Justify your answer.

(c) Now consider the following system consisting of both modulation and impulse train sampling

systems:

i. (4 pts) What is the sampling period Ts used by the impulse train sampling system? What is the equation for X1(j) in terms of X(j)? What is the equation for Y (j) in terms of X1(j)?

ii. (4 pts) Sketch the final output spectrum Y (j) vs. . Make sure to label all critical point

iii. (4 pts) Using your sketch, is it possible to recover the original input x(t) from y(t)? I

draw or describe a system diagram that will do this. Make sure to clearly specify all system blocks being used either in words or with a graph.

(d) (4 pts) Using your knowledge of the behavior of impulse train sampling systems, draw the ap

propriate system diagram that will produce the following input-output pair. Justify your desig

choices.

Make sure to clearly specify your systems with an equation, description, or graph.

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3. Demodulation. (26 pts) The purpose of this problem is to apply what you have learned about ulation by cosine to design a demodulation system to recover an input or obtain some other des result.

(a) Consider the following block diagram:

i. (4 pts) Sketch the output spectrum Y (j). Precisely label all points and axes.

ii. X(j) can be recovered from Y (j) using a single modulator followed by an ideal LPF with frequency response Hr(j). Answer the following questions:

A. (4 pts) Draw the block diagram for this system. Clearly specify the modulation signal, as well as the filter gain A and cutoff frequency wc for the ideal LPF.

B. (2 pts) Note that several cutoff frequencies wc are valid for (A). What range of cutoff frequencies wc can be used to fully recover x(t) from y(t)?

(b) (4 pts) Suppose the output of a modulation system is given by Y (j) below. Draw the demodulation system that will recover the desired input again, shown below as Xr(j):

(c) We briefly learned in class that modulation is useful for different radio stations to send signals out simultaneously. This is a process called frequency division multiplexing, which we practice with below.

i. (4 pts) Consider the following frequency division multiplexing system that can be used to transmit multiple signals at the same time:

Draw the overall output Fourier Transform Y (j) produced by this system.

ii. (4 pts) Suppose a receiver picks up a signal r(t) whose Fourier Transform is given below:

We can get back the received triangle signal x1(t) using the system below. The idea is to first filter out the desired signal and then shift it back to the center around frequency = 0:

Sketch the output Fourier Transform Xr1(j) of this system.

iii. (4 pts) You will notice the system has an error in the output amplitude. What change can be made to the receiver system to get the desired amplitude back up to 1? Note: there are several answers to this question.

7. Fourier Series: Parseval's Relation and LTI (30 pts)

(a) (2 pts) Suppose a signal x(t) has Fourier Series representation o = 2, a5 = a5 = 1/3 , a3

Show that P = 4/9 + 2/2

(b) (4 pts) Suppose a signal x(t) has Fourier Series representation o = 2 and ak = {1/2jk, |k|