EE 235, Winter 2018, Homework 6: Fourier Series
Due Friday February 9, 2018 via Canvas Submission
Write down *ALL* steps for full credit

HW6 Topics:

- Fourier Series: Analysis, Synthesis, Properties, and LTI

HW6 Course Learning Goals Satisfied:

- Goal 1: Describe signals in different domains (time, frequency, and Laplace) and map characteristics in one domain to those in another.
- Goal 4: Analyze LTI system given different system representations (including input-output equations, impulse response, frequency response) and translate between these representations.

HW6 References: OWN Sections 3.2 - 3.5, 3.8

HW6 Problems (Total = 104 pts):

1. *Review* (16 pts)

   (a) *Partial Fraction Expansion.* (2 pts)
   $X(s) = \frac{-5}{s^2+2s+2}$. Using the cover-up method, show that $X(s) = -\frac{\frac{5}{2}j}{s+1+j} + \frac{\frac{5}{2}j}{s+1-j}$.

   (b) *Partial Fraction Expansion.* (4 pts)
   $X(s) = \frac{s+2}{s(s+1)^2(s+5)}$. Using the cover-up method.

   (c) *Magnitude and Phase Equation.* (2 pts)
   Let $X = \frac{1}{\alpha - j\omega}$, where $\alpha > 0$. Evaluate the magnitude and phase of X. Show that $|X| = \frac{1}{\sqrt{\alpha^2+\omega^2}}$ and $\angle X = tan^{-1}(\frac{\omega}{\alpha})$.

   (d) *Signal Properties.* (4 pts)
   $x(t) = \cos(t + \frac{\pi}{3})$. Evaluate P_∞. Is $x(t)$ a power signal?
   Show that the power of the signal is $P_\infty = \frac{1}{2}$, thus making $x(t)$ a power signal.
   Hint: If $x(t)$ is periodic with fundamental period T_o, the power of $x(t)$, P_∞ is equivalent to the average power of $x(t)$ over any interval of length T_o: $P = \frac{1}{T_o}\int_0^{T_o} |x(t)|^2 dt$.

   (e) *Convolution.* (4 pts)
   Let T denote an LTI system with impulse response $p(t+1) - 2p(t-1)$, then find and sketch $y(t) = T[p(-t/2)]$.
   Hint: The pulse signal $p(t) = u(t) - u(t-1)$.

2. *LTI System Description* (14 pts)
   Consider the following three LTI systems:

   - T_1: Has impulse response $h_1(t) = \begin{cases} 2, & -1 < t < 1 \\ 0, & otherwise \end{cases}$
   - T_2: Has step response $s_2(t) = 2u(t-1)$
   - T_3: Has input-output relationship $y_3(t) = x_3(t+2)$

   (a) (4 pts) What is the step response $s_1(t)$ of system T_1?

   (b) (4 pts) What is impulse response $h_2(t)$ of system T_2?

   (c) (2 pts) What is the corresponding input-output relationship for system T_2? Show that $y_2(t) = 2x_2(t-1)$.

   (d) (4 pts) What is the impulse response $h_3(t)$ of system T_3? What is the corresponding step response $s_3(t)$?

3. *LTI System Interconnection and Properties* (14 pts)
   Consider the same LTI systems in Problem 2. Answer the following questions.

   (a) (2 pts) Is system T_2 BIBO stable? Using the impulse response test that we discussed in lecture, show that T_2 is BIBO stable with $\int_{-\infty}^{\infty} |h(t)|dt = 2$.

   (b) (4 pts) Suppose T_1 and T_3 are connected in parallel. Is the overall system causal? Use the impulse response test that we discussed in lecture.

   (c) (4 pts) Suppose T_2 and T_3 are connected in parallel. Find the overall impulse response $h(t)$ and then find the overall output $y(t)$ when the input is $x(t) = \sin(t^2)$.

   (d) (4 pts) Suppose T_1 and T_3 are connected in series. Is the system causal? Is the system BIBO stable? Use the impulse response tests.

4. *Fourier Series: Synthesis.* (4 pts)

   (a) (2 pts) $w_o = \frac{\pi}{4}$ and $a_0 = -2$, $a_1 = a_{-1} = 1$, $a_2 = a_{-2}^* = \frac{1}{j}$, $a_3 = 2$, and $a_{-3} = -2$.
   Using the synthesis equation, convert back to the time domain and show that:
   $x(t) = -2[1 - \cos(\frac{\pi}{4}t) - \sin(\frac{\pi}{2}t) - 2j\sin(\frac{3\pi}{4}t)]$

   (b) (2 pts) $w_o = \frac{\pi}{4}$ and $a_1 = \frac{1}{2j}e^{j\frac{\pi}{4}}$, $a_{-1} = -\frac{1}{2j}e^{-j\frac{\pi}{4}}$.
   Using the synthesis equation, convert back to the time domain and show that:
   $x(t) = \sin(\frac{\pi}{4}t + \frac{\pi}{4})$

5. *(Fourier Series: Analysis)* (20 pts) In the following problems, we practice analyzing signals and computing their Fourier Series coefficients:

   (a) Consider the continuous-time signal $x(t) = 2cos(3t)sin(t) - je^{-j8t} + e^{j6(t-3)}$.

      i. (4 pts) Show the Fourier Series representation of $x(t)$ by finding the fundamental frequency ω_0 and its Fourier Series coefficients a_k.
      Hint: $cos(A)sin(B) = \frac{1}{2}[sin(A+B) - sin(A-B)]$

      ii. (2 pts) Using the result of the previous part, find the power of $x(t)$ using Parseval's Theorem.

   (b) Consider a signal $x(t) = \sin(3\pi t) + \cos(2\pi t)$. Answer the following questions about $x(t)$.

      i. (1 pt) What the fundamental frequency ω_0 of this signal?

      ii. (1 pt)Express $x(t)$ as a sum of complex exponentials using Euler's formula.

      iii. (2 pts) What are the coefficients a_k in the Fourier Series representation of $x(t)$?

      iv. (1 pt) The DC component of a signal is defined as its mean value. What is the DC component of this signal?

   (c) Consider the periodic signal described by $x(t) = \sum\limits_{k=-\infty}^{\infty} \delta(t-2k) - \frac{1}{4}\sum\limits_{k=-\infty}^{\infty} \delta(t+2k-1)$. Answer the following questions about this signal.

      i. (1 pt) Find the fundamental frequency, ω_0.

      ii. (4 pts) Find the DC value c_0, and the Fourier series coefficients c_k for $k \neq 0$. Specify the value of c_k for even k and odd k.

   (d) (4 pts) Consider a periodic signal of the form $x(t) = \begin{cases} 1 & \text{if } 0 \leq t < 4 \\ -1 & 4 \leq t < 8 \end{cases}$, with period $T = 8$.
   Compute the Fourier series coefficients of this signal *using analysis formulas.*

6. *Fourier Series: Properties* (6 pts)

(a) Let $x(t)$ be a continuous-time periodic signal with fundamental frequency ω and Fourier coefficients a_k. Express the Fourier series coefficients of the following signals (call them b_k) in terms of a_k. *Justify all your statements.* Use the properties from Table 3.1 in the textbook.

i. (2 pts) $y(t) = x(2-t) + x(t-1)$.

ii. (2 pts) $y(t) = \text{Even}(x(t))$.

iii. (2 pts) $y(t) = x(3t-1)$.

7. *Fourier Series: Parseval's Relation and LTI* (30 pts)

(a) (2 pts) Suppose a signal $x(t)$ has Fourier Series representation $\omega_o = 2, a_5 = a_{-5} = -\frac{1}{3}, a_3 = a_{-3} = \frac{1}{3}, a_2 = a^*_{-2} = \frac{j}{\pi}$. Compute the average power of the signal $x(t)$.
Show that $P = \frac{4}{9} + \frac{2}{\pi^2}$

(b) (4 pts) Suppose a signal $x(t)$ has Fourier Series representation $\omega_o = 2$ and $a_k = \begin{cases} \frac{1}{2}jk, & |k| < 3 \\ 0, & otherwise \end{cases}$

Compute the average power of the signal $x(t)$.

(c) (4 pts) Suppose a signal with Fourier Series representation $\omega_o = 2, a_5 = a_{-5} = -\frac{1}{3}, a_3 = a_{-3} = \frac{1}{3}, a_2 = a^*_{-2} = \frac{j}{\pi}$ is input to an LTI system with frequency response

$$H(\omega) = \begin{cases} \omega, & 0 < \omega < 8 \\ -\omega, & -8 < \omega < 0 \\ 0, & otherwise \end{cases}$$

Find the output Fourier Series coefficients b_k and the corresponding output signal $y(t)$
Show that the final output you get is $y(t) = 4\cos(6t) - \frac{8}{\pi}\sin(4t)$

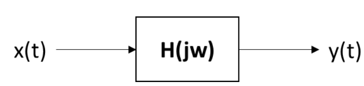
(d) (4 pts) Suppose a signal with Fourier Series representation $\omega_o = 2$ and $a_k = \begin{cases} \frac{1}{2}jk, & |k| < 3 \\ 0, & otherwise \end{cases}$

is input to an LTI system with frequency response

$$H(\omega) = \begin{cases} 2, & 10 < |\omega| < 14 \\ 0, & otherwise \end{cases}$$

Find the output Fourier Series coefficients b_k and the corresponding output signal $y(t)$

(e) *Fourier Series.*
Consider the following LTI system:



Suppose the input to the system is given by: $x(t) = 2cos(4t) + je^{j2t}$.
Also, suppose the LTI system has the frequency response $H(jw) = \left\{ \begin{array}{ll} 2 & -2 < w < 10 \\ 0 & otherwise \end{array} \right.$.

i. (4 pts) Transform the input signal $x(t)$ into its Fourier Series representation by finding the fundamental frequency w_o and the Fourier Series coefficients a_k.

ii. (4 pts) Find the output Fourier Series coefficients b_k.

iii. (4 pts) Using your results from (b), find the output signal $y(t)$.

iv. (4 pts) What is the average power of the input, which we can denote as P_{in}? What is the average power of the output, which we can denote as P_{out}?

8. *Homework Self-Reflection* (10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

   http://bit.ly/2DNFX0i