Sample Exam (**??** points)
September 26, 2019 Name: ________________________________

No notes, calculators, or other aids are allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work.

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0, otherwise

Compute the average power of the signal x(t).

(c) (4 pts) Suppose a signal with Fourier Series representation o = 2, a5 = a5 =  1/3 , a3 = a3





3. LTI System Interconnection and Properties (14 pts)

Consider the same LTI systems in Problem 2. Answer the following questions.

(a) (2 pts) Is system T2 BIBO stable? Using the impulse response test that we discussed in lec

show that T2 is BIBO stable with   |h(t)|dt = 2.

(b) (4 pts) Suppose T1 and T3 are connected in parallel. Is the overall system causal? Use the

(c) (4 pts) Suppose T2 and T3 are connected in parallel. Find the overall impulse response h(t

then find the overall output y(t) when the input is x(t) = sin(t^2).

(d) (4 pts) Suppose T1 and T3 are connected in series. Is the system causal? Is the system BIB

stable? Use the impulse response tests.





1. Review (16 pts)

(a) Partial Fraction Expansion. (2 pts)

X(s) = 5/(s^2+2s+2). Using the cover-up method, show that X(s) = (5/2*j)/(s+1+j) + (5/2*j)/(s+

(b) Partial Fraction Expansion. (4 pts)

X(s) = (s+2)/(s(s+1)2(s+5)). Using the cover-up method.
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(c) Magnitude and Phase Equation. (2 pts)

Let X = 1j , where > 0. Evaluate the magnitude and phase of X. Show that |X| =1^2+^2and X = ta

(d) Signal Properties. (4 pts)

x(t) = cos(t+ 3). Evaluate P. Is x(t) a power signal?

Show that the power of the signal is P = 1/2 , thus making x(t) a power signal.

Hint: If x(t) is periodic with fundamental period To, the power of x(t), P is equivalent to the

average power of x(t) over any interval of length To: P = 1/To To 0 |x(t)|^2 dt.

(e) Convolution. (4 pts)

Let T denote an LTI system with impulse response p(t + 1) 2p(t 1), then find and sketch

y(t) = T [p(t/2)].

Hint: The pulse signal p(t) = u(t) u(t 1).

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2. Fourier Transform: Analysis (27 pts)

(a) (3 pts) x(t) = 2 + e^|t|.

Show that X(j) = 4() + 2/ 1 + 2

(b) (5 pts) x(t) = rect(2(t+ 3/4)) rect(2(t+1/4)).

Show that X(j) = j^e^j 2 sin(/4)sinc(/4).

(c) (3 pts) x(t) = 1/2e^j /4 (t 3) + 1/2e^j /4 (t+ 3).

Show that X(j) = cos(/4 + 3).

(d) (3 pts) x(t) = j sin t+1/ cos(3t).

Show that X(j) = (1) (+ 1) + (3) + (+ 3).

(e) (3 pts) x(t) is given as x(t) = (e^t e^2t)u(t).

Find X(j). Show that X(j) = 1(1+j)(2+j).

(f) (5 pts) x(t) is given as x(t) = e^3|t| sin t.

Find X(j). Show that X(j) = 3j9+(1)2 + 3j9+(+1)2.

Hint: X(j) = 12 (X1(j) X2(j))

(g) (5 pts) x(t) is given as

x(t) = 4sinc(4t) cos(4t).

Find X(j).

0, t < 0

1 et, 0 < t < 1

2e(t1) et 1, 1 < t < 3

2e(t1) et e(t3), t > 3

ii. (4 pts) x(t) = k=

(t k /2) and h(t) = 4sinc(8t)e^j(6t). Using Fourier Series, find y(t).

Hint: You will need to find H(j) to solve this problem.

Show that y(t) = 1 + ej4t + ej8t + ej12t.

iii. (4 pts) x(t) = e3tu(t)2te3tu(t) and h(t) = 4etu(t)2e2tu(t). Using Fourier Transforms, find y(t).

1. Review (20 pts)

(a) Partial Fraction Expansion. (4 pts)

Let H(s) = s+2(s+3)(s^2+2s+1) . Write H(s) as a sum of partial fractions.

(b) Unit Impulse. (4 pts)

Suppose we have an impulse train signal h(t) = k= (t kT).

Given an arbitrary signal x(t), find x(t)h(t) and x(t) h(t) in terms of x(t).

(c) System Properties. (3 pts)

i. (1 pt) In general, if an LTI system has a causal impulse response, does it mean the system i

also stable? Justify your response (with a proof if \true", or a counter-example if \false").

ii. (1 pt) What about the reverse of the previous question: if an LTI system has a stable impul

response, does it mean the system is causal? Justify your response (with a proof if \true", or

a counter-example if \false").

iii. (1 pt) In general, if an LTI system has a periodic impulse response, what can you say abou

its stability?

(d) Fourier Series. (9 pts)

Consider an LTI system with frequency response

H(jw) ={2, 2 w 3

6. Fourier Series: Properties (6 pts)

2

(a) Let x(t) be a continuous-time periodic signal with fundamental frequency and Fourier coef

ak. Express the Fourier series coefficients of the following signals (call them bk) in terms o

Justify all your statements. Use the properties from Table 3.1 in the textbook.

i. (2 pts) y(t) = x(2 t) + x(t 1).

ii. (2 pts) y(t) = Even(x(t)).

iii. (2 pts) y(t) = x(3t 1).

0, otherwise

Find the output Fourier Series coefficients bk and the corresponding output signal y(t)

Show that the final output you get is y(t) = 4 cos(6t) 8 sin(4t)

(d) (4 pts) Suppose a signal with Fourier Series representation o = 2 and ak = {1/2jk, |k| < 3