EE 235, Winter 2018, Homework 10: Laplace Transforms
Due Friday March 9, 2018 via Canvas Submission
Write down *ALL* steps for full credit

HW10 Topics:

- Laplace Transform and Inverse Laplace Transform
- Laplace Transform ROC and Signal Properties
- Laplace Transform ROC and LTI System Properties

HW10 Course Learning Goals Satisfied:

- Goal 1: Describe signals in different domains and map characteristics in one domain to those in another.
- Goal 2: Understand the implications of different system properties and how to test for them.
- Goal 4: Analyze LTI systems given different system representations, and translate between these different representations.

HW10 References: OWN Sections 9.1 - 9.2, 9.5 - 9.6, 9.7

HW10 Problems (Total = 60 pts):

1. *Laplace Transform.* (10 pts)
   Find the Laplace Transform of the following signals and sketch the corresponding pole-zero plot for each signal. In the plot, indicate the regions of convergence (ROC). Write $X(s)$ as a single fraction in the form of $\frac{N(s)}{D(s)}$.

   (a) (2 pts) $x(t) = e^{-4t}u(t) + e^{-6t}u(t)$. Show that $X(s) = \frac{2s+10}{(s+4)(s+6)}$. with ROC of $Re\{s\} > -4$.

   (b) (4 pts) $x(t) = e^{4t}u(-t) + e^{8t}u(-t)$.

   (c) (4 pts) $x(t) = \delta(t) - u(-t)$.

2. *Inverse Laplace Transform.* (14 pts)
   Find $x(t)$ for given $X(s)$ and ROC. Plot pole-zero plots.

   (a) (2 pts) $X(s) = \frac{1}{s^2+5s+6}$, ROC: $Re\{s\} > -2$. Show that $x(t)$ is $x(t) = e^{-2t}u(t) - e^{-3t}u(t)$.

   (b) (4 pts) $X(s) = \frac{s-3}{s^2+5s+6}$, ROC: $-3 < Re\{s\} < -2$.

   (c) (4 pts) $X(s) = \frac{s+2}{s^2+4s+20}$, ROC: $Re\{s\} < -2$.

   (d) (4 pts) $X(s) = \frac{s}{s^2+9}$, ROC: $Re\{s\} < 0$.

3. *ROC and Signal Properties.* (10 pts)

   (a) (5 pts) Suppose $x(t)$ is a real signal with rational Laplace transform $X(s)$ with the following properties:

   i. $X(s)$ has two poles and one zero, with one pole at $s = -1 - 2j$,
   ii. the Fourier transform of $e^{2t}x(t)$ does not exist,
   iii. $\int_{-\infty}^{\infty} x(t)dt = -2$,
   iv. $\int_{-\infty}^{\infty} e^{-t}x(t)dt = 0$.

Find $x(t)$ (the time-domain signal).

(b) (5 pts) Signal $x(t)$ has the following properties:

i. $X(s)$ is rational with 1 zero and 2 poles

ii. $x(t)$ is real

iii. $X(s)$ has its zero at s = 1 and a known pole at s = 1 - 2j

iv. Area under $x(t)$ is equal to 1

v. $e^{-3t}x(t)$ is absolutely integrable (so Fourier Transform of $e^{-3t}x(t)$ does exist)

Deduce the expression for signal $x(t)$.

4. *ROC and LTI Systems.* (16 pts)

(a) (5 pts) Consider another system described by: $-2y(t) - \frac{dy(t)}{dt} + \frac{d^2y(t)}{dt} = 3x(t) + \frac{dx(t)}{dt}$.

i. (1 pt) Specify the ROC corresponding to H(s) if it is known the system is causal. Also, sketch the pole-zero plot and associated ROC.

ii. (2 pts) Specify the ROC corresponding to H(s) if it is known the system is stable. Also, sketch the pole-zero plot and associated ROC.

iii. (2 pts) Specify the ROC corresponding to H(s) if it is known the system is left-sided. Also, sketch the pole-zero plot and associated ROC.

(b) (5 pts) Let $H(s)$ represent the system function for a causal, stable LTI system. The input to the system consists of the sum of three terms, one of which is an impulse $\delta(t)$ and the second term is a complex exponential $e^{s_0 t}$, where s_0 is a complex constant. The output of the system is:

$$y(t) = e^{-t}u(t) + \frac{10}{34}e^{4t}\cos(3t) + \frac{6}{34}e^{4t}\sin(3t).$$

Determine $h(t)$ and s_0 consistent with this information.
Hint1: What happens when a complex exponential is passed through an LTI system?
Hint2: One thing to note is that the output is real even if s_0 was complex. This indicates that the third term consists of $e^{s_0^* t}$, where s_0^* is the complex conjugate of s_0.

(c) For each of the cases below, draw a pole-zero plot and ROC for a system that matches the description:

i. (2 pts) The system is stable; the impulse response is two-sided; and the system frequency response approaches a non-zero constant at high frequencies.

ii. (2 pts) The system is not stable and contains complex poles in the left half plane.

iii. (2 pts) The impulse-response is two-sided and absolutely integrable, and the system behaves like a lowpass filter.

5. *Homework Self-Reflection*
(10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Y our self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.
http://bit.ly/2FFYTAd