EE 235, Winter 2018, Homework 4: LTI Systems and Convolution
Due Wednesday January 24, 2018 via Canvas Submission
Write down *ALL* steps for full credit

HW4 Topics:

- LTI Systems and Impulse Response
- Echo Property of Convolution
- Convolution Integral

HW4 Course Learning Goals Satisfied:

- Goal 3: Perform convolutions for arbitrary and closed-form continuous-time signals.
- Goal 4: Analyze LTI systems given different system representations (including input-output equations and impulse response).
- Goal 6: Use and understand standard EE terminology associated with LTI systems (e.g. impulse response, step response).

HW4 References: OWN Sections 2.2

HW4 Problems (Total = 72 pts):

1. *Review*

   (a) *Complex Numbers I.* In Ch.3, we will need to be able to perform integration of complex exponential functions. Here are some practice problems to help you review that.

      i. (2 pts) Evaluate: $\frac{1}{\pi}\int\limits_{0}^{\pi} e^{-jk2t}dt$ where k is an integer not equal to 0.
      Use your results from HW2 Problem 1 to reduce your answer.
      Show that this integral evaluates to 0

      ii. (4 pts) Evaluate: $j(2-k)\cdot\int\limits_{0}^{\pi} e^{j2t}e^{-jkt}dt$ where k is an integer.
      Use your results from HW2 Problem 1 to reduce your answer.
      Hint: The answer will have two parts, one when k is even and when k is odd.

   (b) *Signal Properties.*
   Let $x(t) = \cos(\frac{\pi}{2}t) + 4\cos(\frac{3\pi}{2}t) + \cos(3t)$.

      i. (4 pts) Is $x(t)$ periodic? If so, find the fundamental period T_o.
      ii. (4 pts) Is $x(t)$ even or odd?

   (c) *System Properties.*

      i. Consider the system described by $y(t) = x(e^t)$. For this system, state if the following properties hold. Justify your assertion in all cases, giving a proof if “true” and a counter-example if “false”.

      (1 pt) Linearity.
      (1 pt) Time-invariance.
      (1 pt) Stability.
      (1 pt) Causality.

ii. Consider the system described by $y(t) = \text{Odd}(x(t))$. For this system, state if the following properties hold. Justify your assertion in all cases, giving a proof if "true" and a counter-example if "false".

(1 pt) Linearity.
(1 pt) Time-invariance.
(1 pt) Causality.
(1 pt) Stability.

(d) *Unit Impulse and Unit Step.*

i. (4 pts) Signal $x(t)$ is described below. Write an expression for $x(t)$ in terms of the step function $u(t)$.

$$x(t) = \begin{cases} 1 & \text{if } -2 \le t < -1 \quad \text{or} \quad t > 5 \\ 2 & \text{if } 1 < t \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Hint: Plot the function first, then try to express each piece in terms of step functions.

ii. (4 pts) Evaluate the integral $\int_{t-4}^{\infty} \delta(\tau) d\tau$.

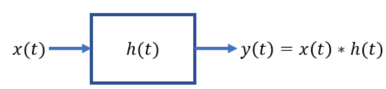
2. *Linearity* Consider the following input-output relationships of systems and justify if they are linear or not.

(a) (2 pts) $y(t) = 5x(\sin^2(t)) + \cos(t^3)$. Is this system linear?
Show that it is *not* linear.

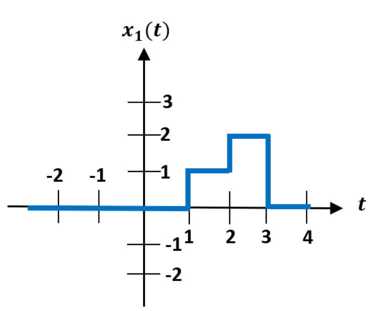
(b) (2 pts) $y(t) = \frac{d^2}{dt^2}(x(t))^2$. Is this system linear?
Show that it is *not* linear.

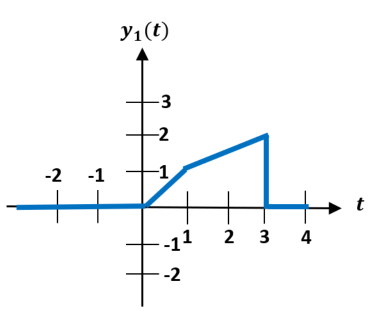
(c) (4 pts) $y(t) = \cos(5x(t) + 3)$. Is the system linear?

3. *LTI Systems.*
You have an LTI system, but you don't know the system equation as shown in the figure below.

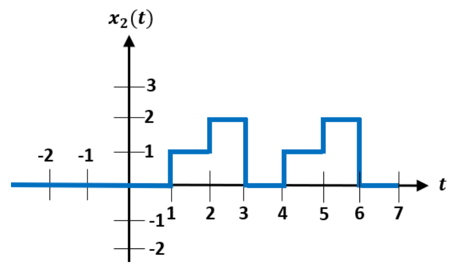


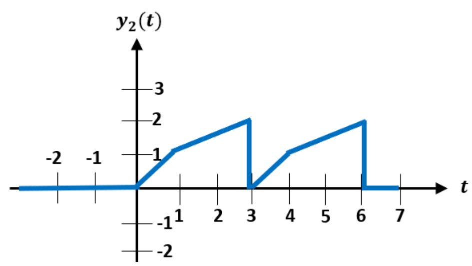
However, you know that $y_1(t)$ is the output of the system when the input is $x_1(t)$. $x_1(t)$ and $y_1(t)$ are given as below.



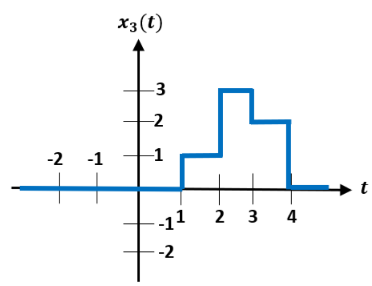


(a) (2 pts) Sketch $y_2(t)$ when the input is $x_2(t)$ where $x_2(t)$ is given as below. Show that sketch of $y_2(t)$ is as shown in the figure below.





(b) (4 pts) Sketch $y_3(t)$ which is the output of the system when the input is $x_3(t)$ where $x_3(t)$ is given as below.



4. *Impulse Response.*

(a) Consider the LTI system with input-output relationship given by $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau - 2)d\tau$.

i. (2 pts) Derive the impulse response of this system.

ii. (1 pt) Also sketch the impulse response, labeling all the key points.

(b) i. (2 pts) The impulse response of a system is given by $h(t) = \delta(t+1) + \delta(t-1)$. What is the system's input-output relationship relating output $y(t)$ to input $x(t)$?

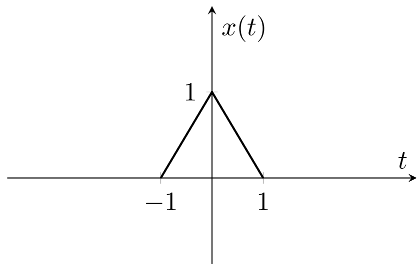


Figure 1: Input signal for Problem 5, part *a*

ii. (1 pt) Based on this input-output relationship, state if the system is causal or not.
Hint: The relationship between $x(t)$, $y(t)$, and $h(t)$ is given by convolution: $y(t) = x(t) * h(t)$.

(c) (2pts) You got an awesome bluetooth speaker as Christmas present. Unfortunately your cat played with it, and it's now damaged. The speaker plays music amplified by whatever volume you choose; but in *addition* to this, you can hear distortion in the form of music *from two seconds ago*, *amplified* at a tenth of the selected volume. Assuming the speaker to be a simple LTI system, what is the input-output relationship? Express this in terms of the selected volume, v.

(2 pts) Also compute the impulse response of the system from this relationship.
Hint: The output is a *sum* of two signals, one good, one distorted. What is another word for 'amplified' in the language of signal operations you learnt in class? Note that the good signal only has amplification. The distorted signal, on the other hand, has amplification (at a different level!), but also, it is from some *time before the current time* - so what transformation was applied to the input to generate this distortion?

(d) (2 pts) The input-output relationship of an RL circuit (which is an LTI system) is given by $y(t) = \frac{R}{L}\int_{\tau=-\infty}^{t} x(\tau)e^{-\frac{R}{L}(t-\tau)}d\tau$, where R and L are the circuit resistance and inductance, respectively. Compute the impulse response of the system.

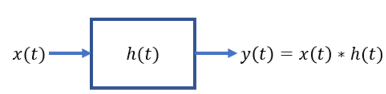
5. *Convolution.*

(a) (4 pts) Consider an LTI system with impulse response $h(t) = \sum_{k=-2}^{2} \delta(t - kT)$, for some positive value of T. Suppose the input to this system is the signal shown in Figure 1. Sketch the output for $T = 1$ and $T = 2$.

(b) (2 pts) Consider an LTI system with input $x(t) = \frac{1}{2}(u(t) - u(t-2))$ and impulse response $h(t) = 3\delta(t+1) - \delta(t-3)$.
Show that $y(t) = \frac{3}{2}u(t+1) - \frac{3}{2}u(t-1) - \frac{1}{2}u(t-3) + \frac{1}{2}u(t-5)$

(c) (4 pts) Consider an LTI system with input $x(t) = \delta(t) + 2\delta(t-4)$ and impulse response $h(t) = \cos(t)$. What is the output?

6. *Convolution II.* Consider the system as shown below.



(a) (2 pts) Consider $x(t) = e^{-(t-2)}u(t)$ and $h(t) = u(t+3)$. Find $y(t)$. Represent $y(t)$ in terms of $u(t)$. Show that $y(t) = e^2(1 - e^{-(t+3)})u(t+3)$.

(b) (4 pts) $x(t) = e^t u(-4-t)$ and $h(t) = e^{-2t}u(t+2)$. Find $y(t)$ where $y(t) = x(t) * h(t)$. Represent $y(t)$ in terms of $u(t)$. Also plot $h(t-\tau)$.

7. *Homework Self-Reflection* (10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

   http://bit.ly/2mFStbQ