EE 235, Winter 2018
Homework 2: Continuous-Time Signals
Due Friday January 12, 2018 by 12:30pm via Canvas Submission
Write down *ALL* steps for full credit

HW2 Topics:

- Continuous Time Signal Properties: Periodic, Even/Odd, Energy/Power
- Operations on Signals
- Exponential and Sinusoidal Signals
- Unit Impulse and Unit Step Functions

HW2 Course Learning Goals Satisfied:

- Goal 1: Describe signals in different domains (time).

HW2 References: OWN Sections 1.1-1.4
HW2 Problems (Total = 80 pts):

1. *Review*

   (a) *Complex Numbers.* Prove each of the following properties below. These will be useful when we start using complex numbers more heavily in Ch.3.
   Hint: For any integer k, $e^{j\theta k} = (e^{j\theta})^k$

   i. (2 pts) For all integers k, $e^{j2\pi k} = 1$

   ii. (2 pts) For the case that k is an even integer, $e^{j\pi k} = 1$

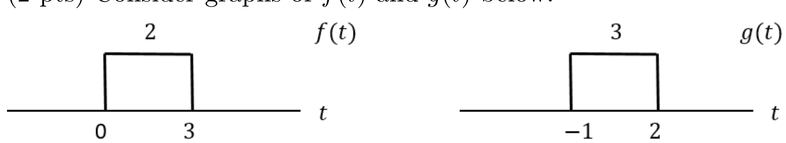
   iii. (2 pts) For the case that k is an odd integer, $e^{j\pi k} = -1$

   (b) *Integration I.* In Chapter 2, you will need to be comfortable multiplying graphs together and then finding the area of the resulting curve. We will start practicing this now to get you ready for this. For each of the signals below, evaluate the integral

$$\int_{-\infty}^{\infty} f(t)g(t)dt$$

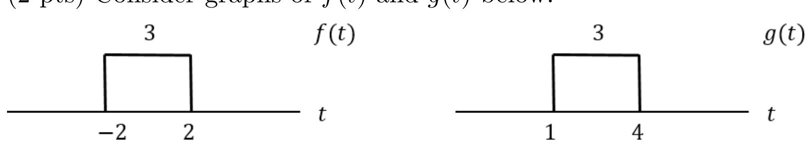
   Show all your work and thought process to receive full credit.

   i. (2 pts) Consider graphs of $f(t)$ and $g(t)$ below:

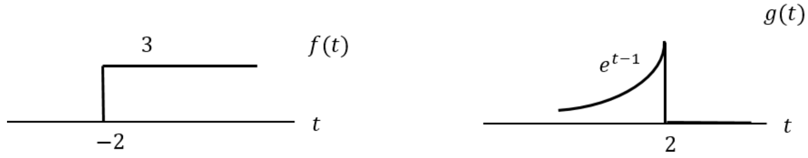


   Show that the integral evaluates to 12

   ii. (2 pts) Consider graphs of $f(t)$ and $g(t)$ below:

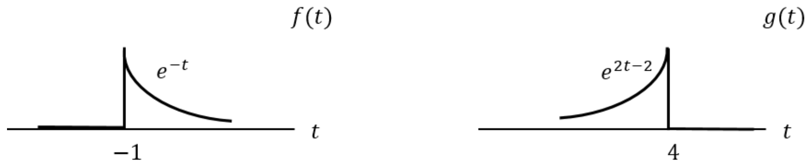


iii. (2 pts) Consider graphs of $f(t)$ and $g(t)$ below:



Show that the integral evaluates to $3e^1 - 3e^{-3}$

iv. (4 pts) Consider graphs of $f(t)$ and $g(t)$ below:



(c) *Integration II.* In Chapter 2, you will need to be comfortable performing an integration for an expression consisting of multiple variables. We will start practicing this now to get you ready for this. For each problem below, evaluate the integral and reduce as much as possible. Refer to the HW2 Supplementary Notes for some comments on this topic.

i. (2 pts) $y(t) = \int_{0}^{t-3} (3)(e^t e^{-\tau})d\tau$

Show that $y(t) = 3[e^t - e^3]$.

ii. (4 pts) $y(t) = \int_{0}^{t+1} (e^{\tau})(e^{2t}e^{-2\tau})d\tau$

iii. (2 pts) $y(t) = \int_{t-1}^{t+1} (2)(1)d\tau$

iv. (4 pts) $y(t) = \int_{t-2}^{t} (t-\tau)(1)d\tau$

v. (4 pts) $y(t) = \int_{-\infty}^{t} (2)\left(\frac{1}{2}e^{-t}e^{\tau}\right)d\tau$

(d) *Integration III.* Since some students had trouble with u-substitution in HW1, we will review that again. See the HW2 Supplementary Notes for some comments on this topic.

i. (2 pts) We will need to use u-substitution in this class for a couple topics including later in Ch.1 when we need to decide whether two integral statements are equivalent. Consider this statement:

$$\int_{t+t_o-1}^{t+t_o} x(\tau)d\tau = \int_{t-1}^{t} x(\tau + t_o)d\tau$$

On a first inspection, they do not look equal because the bounds are not the same. We will prove, however, they are using a u-substitution on the right integral so that its integrand $x(\tau + t_o)$ is in the same form as the left integrand $x(\tau)$. Rewrite the integral on the right using u-substitution so that it is in the form $\int_{u_0}^{u_1} x(u)du$. Show that after u-substitution you get the right integral in the same form as the left but with a different variable of integration.

ii. (4 pts) Let us apply what you learned in the last part to another problem. Consider the following two integrals:

$$y_1(t) = \int_{-\infty}^{2t-2t_o-1} x(\tau)d\tau$$

$$y_2(t) = \int_{-\infty}^{2t} x(\tau - t_o)d\tau$$

Is $y_1(t) = y_2(t)$? Full work is needed to receive full credit.

2. *Operations on Signals*

   Two continuous-time signals, $x(t)$ and $y(t)$, are shown in Figure 1. Sketch (labeling all key points) the signals

   (a) (2 pts) $x(-2t + 1)$

   (b) (2 pts) $y(4t)$

   (c) (2 pts) $x(-2t + 1) + y(4t)$

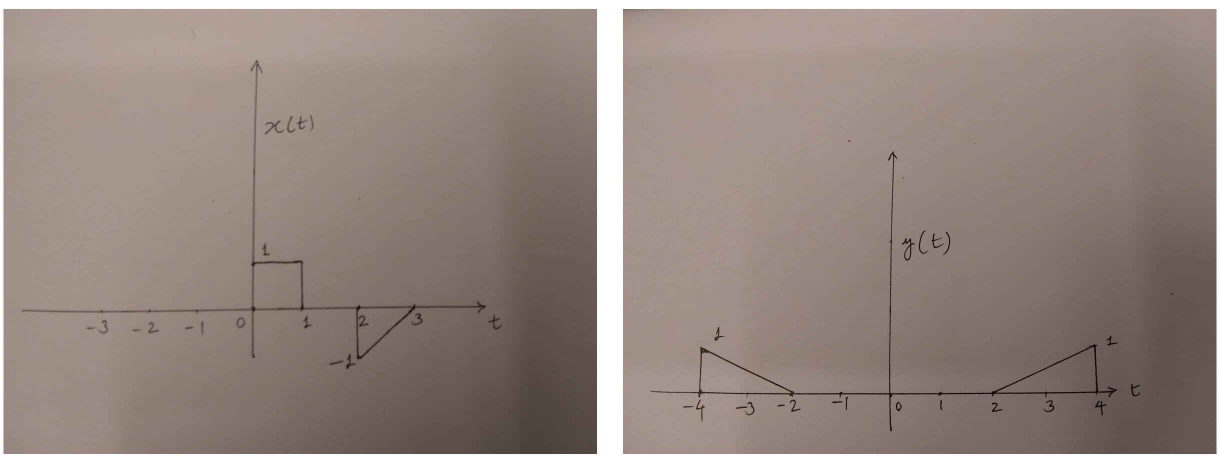


Figure 1: Problem 2: Signal transformations

3. *Signal Properties - Periodicity* Are the following signals periodic? Justify your response and if periodic, derive the period.

   (a) (2 pts) $x(t) = \sin(6t) + \cos^2(t)$

   (b) (2 pts) $x(t) = \sin(6t) + \cos^2(2\pi t)$

   (c) (2 pts) $x(t) = \sin(6\pi t) + \cos^2(2\pi t)$

   (d) (4 pts) $x(t) = e^{5t}\sin(6t)$

   (e) (4 pts) $x(t) = e^{j5t}\sin(6t)$

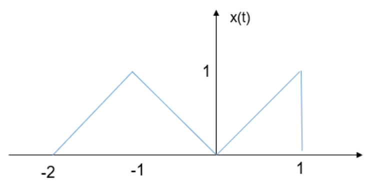
4. *Signal Properties - Even/Odd Signals*

(a) (2 pts) $x(t) = u(t+3) - u(t-3)$. Is $x(t)$ an even signal or an odd signal?
Show / explain why $x(t)$ is an even signal.

(b) (4 pts) Let $x(t) = e^{-j\pi t} - 5t\cos(3\pi t)$. Is $x(t)$ even, odd, or neither? Justify your answer.

(c) (2 pts) Consider $x(t)$ in Problem 4-Part (b) (i.e., $x(t) = e^{-j\pi t} - 5t\cos(3\pi t)$.). Find even and odd parts, $x_e(t)$ and $x_o(t)$. Show that $x_e(t) = \cos(\pi t)$ and $x_o(t) = -j\sin(\pi t) - 5t\cos(3\pi t)$.

(d) (4 pts) $x(t)$ is depicted in the figure below. Sketch the odd $x_o(t)$ and the even $x_e(t)$ parts of $x(t)$.



5. *Homework Self-Reflection* (10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

`http://bit.ly/2m4UMEU`