EE 235, Winter 2018, Homework 7: Fourier Transforms
Due Friday February 16, 2018 in class
Write down *ALL* steps for full credit

HW7 Topics:

- Fourier Transforms: Analyze (Transform) and Synthesize (Inverse Transform)
- Fourier Transforms: Periodic Signals
- Parseval's Theorem

HW7 Course Learning Goals Satisfied:

- Goal 1: Describe signals in different domains (time, frequency) and map characteristics in one domain to those in another.
- Goal 3: Perform convolutions for arbitrary and closed-form continuous-time signals
- Goal 4: Analyze LTI system given different system representations (including input-output equations, impulse response, frequency response) and translate between these representations.

HW7 References: OWN Sections 4.1 - 4.6

HW7 Problems (Total = 106 pts):

1. *Review* (20 pts)

   (a) *Partial Fraction Expansion.* (4 pts)
   Let $H(s) = \frac{s+2}{(s+3)(s^2+2s+1)}$. Write H(s) as a sum of partial fractions.

   Solution. $H(s) = \frac{s+2}{(s+3)(s^2+2s+1)} = \frac{s+2}{(s+3)(s+1)^2} = \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{s+3}$
   $A(s+3) + B(s+1)(s+3) + C(s+1)^2 = s+2$
   By comparing coefficients,
   $B + C = 0$
   $A + 4B + 2C = 1$
   $3A + 3B + C = 2$
   $A = \boxed{0.5}, B = \boxed{0.25}, C = \boxed{-0.25}$

   (b) *Unit Impulse.* (4 pts)
   Suppose we have an impulse train signal $h(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$.
   Given an arbitrary signal $x(t)$, find $x(t)h(t)$ and $x(t) * h(t)$ in terms of $x(t)$.

   Solution. *According to the* sampling property *of unit impulse, we have*

   $$x(t)h(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT) = \sum_{k=-\infty}^{\infty} x(t)\delta(t-kT) = \boxed{\sum_{k=-\infty}^{\infty} x(kT)\delta(t-kT)}$$

   According to the echo property *of unit impulse, we have*

   $$x(t) * h(t) = x(t) * [\sum_{k=-\infty}^{\infty} \delta(t-kT)] = \sum_{k=-\infty}^{\infty} [x(t) * \delta(t-kT)] = \boxed{\sum_{k=-\infty}^{\infty} x(t-kT)}$$

   (c) *System Properties.* (3 pts)

      i. (1 pt) In general, if an LTI system has a causal impulse response, does it mean the system is also stable? Justify your response (with a proof if "true", or a counter-example if "false").

Solution. *No, this is not true. Consider the system with impulse response $h(t) = u(t)$. This system is causal, but* not *stable.*

ii. (1 pt) What about the reverse of the previous question: if an LTI system has a stable impulse response, does it mean the system is causal? Justify your response (with a proof if "true", or a counter-example if "false").

Solution. *No. Consider the system with impulse response $h(t) = e^t u(-t)$. This is stable, because $\int_{t=-\infty}^{\infty} |h(t)|dt = \int_{-\infty}^{0} e^t dt = 1$. However, it is* not *causal, because $h(t) \neq 0$ for $t < 0$.*

iii. (1 pt) In general, if an LTI system has a periodic impulse response, what can you say about its stability?

Solution. *Not stable, since it won't be absolutely summable, by the nature of the periodic impulse response.*

(d) *Fourier Series.* (9 pts)
Consider an LTI system with frequency response

$$H(jw) = \begin{cases} -2, & -2 \leq w \leq 3 \\ 0, & otherwise \end{cases}$$

i. (3 pts) Suppose the input is a periodic signal $x(t)$ with the Fourier Series representation $w_o = \frac{1}{2}$, $a_0 = 1$, $a_2 = a_{-2}^* = e^{-j\frac{\pi}{2}}$, $a_4 = 2$, $a_{-4} = -1$, $a_7 = 5$, and $a_k = 0$ for k otherwise. Find the output Fourier coefficients b_k, and specify your answer **for all k**. Show that $b_0 = -2$, $b_2 = -2e^{-j\frac{\pi}{2}}$, $b_{-2} = -2e^{j\frac{\pi}{2}}$, $b_4 = -4$, $b_{-4} = 2$, $b_7 = 0$, and $b_k = 0$ otherwise.

Solution. $b_k = a_k H(jkw_o) = a_k H(j\frac{k}{2})$
$b_0 = (1)H(j0) = (1)(-2) = -2$
$b_2 = (e^{-j\frac{\pi}{2}})H(j1) = (e^{-j\frac{\pi}{2}})(-2) = -2e^{-j\frac{\pi}{2}}$
$b_{-2} = (e^{j\frac{\pi}{2}})H(-j1) = (e^{j\frac{\pi}{2}})(-2) = -2e^{j\frac{\pi}{2}}$
$b_4 = (2)H(j2) = (2)(-2) = -4$
$b_{-4} = (-1)H(-j2) = (-1)(-2) = 2$
$b_7 = (5)H(-j3.5) = (5)(0) = 0$

ii. (3 pts) Using your results from (A), find the output $y(t)$. Simplify $y(t)$ as much as possible. Show that $y(t) = -2 - 4cos(t - \frac{\pi}{2}) - 4jsin(2t) - 2e^{j2t}$.

Solution. $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jkw_o t} = -2e^{j0t} - 2e^{-j\frac{\pi}{2}}e^{j1t} - 2e^{j\frac{\pi}{2}}e^{-j1t} - 4e^{j2t} + 2e^{-j2t}$
$= -2 - 2(e^{-j(\frac{\pi}{2}-t)} + e^{j(\frac{\pi}{2}-t)}) - 2(e^{j2t} - e^{-j2t}) - 2e^{j2t}$
$y(t) = -2 - 4cos(\frac{\pi}{2} - t) - 4jsin(2t) - 2e^{j2t}$

iii. (3 pts) If $x(t)$ is delayed by 2, then what are the output coefficients b_k now? Show that $b_0 = -2$, $b_2 = -2e^{-j(\frac{\pi}{2}+2)}$, $b_{-2} = -2e^{j(\frac{\pi}{2}+2)}$, $b_4 = -4e^{-j4}$, $b_{-4} = 2e^{j4}$, $b_k = 0$ otherwise.

Solution. *According to the Fourier Series property of shifting, we have*
$y(t) = x(t - t_o) \leftrightarrow c_k^y = c_k^x e^{-jkw_o t_o}$
$b_0 = -2e^{-jk(\frac{1}{2})(2)} = -2e^{-j0} = -2$
$b_2 = -2e^{-j\frac{\pi}{2}}e^{-jk(\frac{1}{2})(2)} = -2e^{-j(\frac{\pi}{2}+k)} = -2e^{-j(\frac{\pi}{2}+2)}$
$b_{-2} = -2e^{j\frac{\pi}{2}}e^{-jk(\frac{1}{2})(2)} = -2e^{j(\frac{\pi}{2}+2)}$
$b_4 = -4e^{-jk(\frac{1}{2})(2)} = -4e^{-j4}$
$b_{-4} = 2e^{-jk(\frac{1}{2})(2)} = 2e^{j4}$
$b_7 = 0e^{-jk(\frac{1}{2})(2)} = 0$

2. *Fourier Transform: Analysis* (27 pts)

(a) (3 pts) $x(t) = 2 + e^{-|t|}$.
Show that $X(j\omega) = 4\pi\delta(\omega) + \frac{2}{1+\omega^2}$

Solution. $x(t) = 2 + e^{-t}u(t) + e^{t}u(-t)$
$X(j\omega) = 2(2\pi\delta(\omega)) + \frac{1}{1+j\omega} + \frac{1}{1-j\omega}$
$X(j\omega) = 4\pi\delta(\omega) + \frac{2}{1+\omega^2}$

(b) (5 pts) $x(t) = rect(2(t+\frac{3}{4})) - rect(2(t+\frac{1}{4}))$.
Show that $Xj\omega = je^{j\frac{\omega}{2}}\sin(\frac{\omega}{4})\text{sinc}(\frac{\omega}{4})$.

Solution. $X(j\omega) = e^{j\omega\frac{3}{4}}(\frac{1}{2}sinc(\frac{\omega}{4})) - e^{j\omega\frac{1}{4}}(\frac{1}{2}sinc(\frac{\omega}{4}))$
$X_{j\omega} = \frac{1}{2}(e^{j\omega\frac{3}{4}} - e^{j\omega\frac{1}{4}})sinc(\frac{\omega}{4})$
$X_{j\omega} = \frac{1}{2}e^{j\frac{\omega}{2}}(e^{j\frac{\omega}{4}} - e^{-j\frac{\omega}{4}})sinc(\frac{\omega}{4})$
$X_{j\omega} = \frac{1}{2}e^{j\frac{\omega}{2}}(2j)\frac{e^{j\frac{\omega}{4}} - e^{-j\frac{\omega}{4}}}{2j}sinc(\frac{\omega}{4})$
$X_{j\omega} = je^{j\frac{\omega}{2}}\sin(\frac{\omega}{4})sinc(\frac{\omega}{4})$

(c) (3 pts) $x(t) = \frac{1}{2}e^{-j\frac{\pi}{4}}\delta(t-3) + \frac{1}{2}e^{j\frac{\pi}{4}}\delta(t+3)$.
Show that $X(j\omega) = \cos(\frac{\pi}{4} + 3\omega)$.

Solution. $X(j\omega) = \frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j3\omega} + \frac{1}{2}e^{j\frac{\pi}{4}}e^{j3\omega}$
$X(j\omega) = \frac{1}{2}\left(e^{-j(\frac{\pi}{4}+3\omega)} + e^{j(\frac{\pi}{4}+3\omega)}\right)$
$X(j\omega) = \cos(\frac{\pi}{4} + 3\omega)$

(d) (3 pts) $x(t) = \frac{j}{\pi}\sin t + \frac{1}{\pi}\cos(3t)$.
Show that $X(j\omega) = \delta(\omega-1) - \delta(\omega+1) + \delta(\omega-3) + \delta(\omega+3)$.

Solution. $X(j\omega) = \frac{j}{\pi}\frac{\pi}{j}(\delta(\omega-1) - \delta(\omega+1)) + \frac{1}{\pi}\pi(\delta(\omega-3) + \delta(\omega+3))$
$X(j\omega) = \delta(\omega-1) - \delta(\omega+1) + \delta(\omega-3) + \delta(\omega+3)$

(e) (3 pts) $x(t)$ is given as
$x(t) = (e^{-t} - e^{-2t})u(t)$.
Find $X(j\omega)$. Show that $X(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)}$.

Solution. $x(t) = e^{-t}u(t) - e^{-2t}u(t)$

$$\boxed{X(j\omega) = \frac{1}{1+j\omega} - \frac{1}{2+j\omega} = \frac{2+j\omega-1-j\omega}{(1+j\omega)(2+j\omega)} = \frac{1}{(1+j\omega)(2+j\omega)}}$$

(f) (5 pts) $x(t)$ is given as
$x(t) = e^{-3|t|}\sin t$.
Find $X(j\omega)$. Show that $X(j\omega) = -\frac{3j}{9+(\omega-1)^2} + \frac{3j}{9+(\omega+1)^2}$.
Hint: $X(j\omega) = \frac{1}{2\pi}(X_1(j\omega) * X_2(j\omega))$
Solution. *Let* $x(t) = x_1(t)x_2(t)$, *such that* $X(j\omega) = \frac{1}{2\pi}(X_1(j\omega) * X_2(j\omega))$
$x_1(t) = e^{-3|t|}$ *and* $x_2(t) = \sin t$
$X_1(j\omega) = \frac{6}{9+\omega^2}$
$X_2(j\omega) = \frac{\pi}{j}(\delta(\omega-1) - \delta(\omega+1))$
$X(j\omega) = \frac{1}{2\pi}(X_1(j\omega) * X_2(j\omega))$

$$\boxed{X(j\omega) = -\frac{3j}{9+(\omega-1)^2} + \frac{3j}{9+(\omega+1)^2}}$$

(g) (5 pts) $x(t)$ is given as
$x(t) = 4\pi\text{sinc}(4\pi t)\cos(4\pi t)$.

Find $X(j\omega)$.

Solution. *If* $x(t) = x_1(t)x_2(t)$, $X(j\omega)$ *can be represented as* $X(j\omega) = \frac{1}{2\pi}(X_1(j\omega) * X_2(j\omega))$.
$x_1(t) = 4\pi sinc(4\pi t)$ *and* $x_2(t) = \cos(4\pi t)$
$X_1(j\omega) = 4\pi\left(\frac{\pi}{4\pi} rect\left(\frac{\omega}{(2)(4\pi)}\right)\right) = \pi rect\left(\frac{\omega}{8\pi}\right)$
$X_2(j\omega) = \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$
$X(j\omega) = \frac{1}{2\pi}X_1(j\omega) * X_2(j\omega)$

$$\boxed{X(j\omega) = \frac{\pi}{2} rect\left(\frac{\omega - 4\pi}{8\pi}\right) + \frac{\pi}{2} rect\left(\frac{\omega + 4\pi}{8\pi}\right)}$$

3. *Fourier Transform: Synthesize (Inverse Transform).* (25 pts)
Using common Fourier Transform pairs and properties, find the signal $x(t)$ given:

(a) (2 pts) $X(jw) = 3[\delta(w-1) + \delta(w+1)] + 2[\delta(w-2\pi) + \delta(w+2\pi)]$.
Show that $\frac{3}{\pi}cos(t) + \frac{2}{\pi}cos(2\pi t)$.

Solution. $x(t) = \frac{3}{\pi}\pi[\delta(w-1) + \delta(w+1)] + \frac{2j}{\pi}\frac{\pi}{j}[\delta(w-2\pi) + \delta(w+2\pi)]$
$= \frac{3}{\pi}cos(t) + \frac{2}{\pi}cos(2\pi t)$

(b) (2 pts) $X(jw) = \delta(w) + 2\delta(w+3) + 2\delta(w-3)$.
Show that $x(t) = \frac{1}{2\pi} + \frac{2}{\pi}cos(3t)$.

Solution. $x(t) = \frac{1}{2\pi} + 2\frac{1}{2\pi}e^{j(-3)t} + 2\frac{1}{2\pi}e^{j(3)t}$
$= \frac{1}{2\pi} + \frac{2}{\pi}cos(3t)$

(c) (2 pts) $X(jw) = cos(w + \frac{\pi}{6})$.
Hint: Write $X(jw)$ *as a sum of complex exponentials first.* Show that $x(t) = \frac{1}{2}e^{j\frac{\pi}{6}}\delta(t+1) + \frac{1}{2}e^{-j\frac{\pi}{6}}\delta(t-1)$.

Solution. *Rewriting* $X(jw)$ *using Euler's relation*
$X(jw) = \frac{1}{2}e^{j(w+\frac{\pi}{6})} + \frac{1}{2}e^{-j(w+\frac{\pi}{6})}$
$X(jw) = \frac{1}{2}e^{j\frac{\pi}{6}}e^{jw} + \frac{1}{2}e^{-j\frac{\pi}{6}}e^{-jw}$
$x(t) = \frac{1}{2}e^{j\frac{\pi}{6}}\delta(t+1) + \frac{1}{2}e^{-j\frac{\pi}{6}}\delta(t-1)$

(d) (2 pts) $X(jw) = \frac{1}{2}e^{j2w} + \frac{1}{3}$.
Show that $x(t) = \frac{1}{2}\delta(t+2) + \frac{1}{3}\delta(t)$.

Solution. $X(jw) = \frac{1}{2}e^{-jw(-2)} + \frac{1}{3}$
$x(t) = \frac{1}{2}\delta(t+2) + \frac{1}{3}\delta(t)$

(e) (5 pts) $X(jw) = \frac{2sin(3(w-2\pi))}{w-2\pi}$.
Show that $x(t) = rect(\frac{t}{6})e^{j2\pi t}$.
Hint: represent $X(jw)$ using a sinc function.

Solution. *We already knew that when* $x(t) = rect(\frac{t}{T})$, $X(jw) = Tsinc(wT/2)$.
when $x(t) = x(t - t_o)$, $X(jw) \to e^{-jwt_o}X(jw)$
T *is 6, and this was delayed* 2π
So $x(t) = e^{j2\pi t}$ *for* $|t| < 3$
And $x(t) = 0$ *for otherwise.*
Thus, $x(t) = rect(\frac{t}{6})e^{j2\pi t}$.

(f) (5 pts) $X(jw) = \frac{2-jw}{12-7jw-w^2}$.
Hint: Use partial fractions. Show that $x(t) = 2e^{4t}u(-t) - e^{3t}u(-t)$.

Solution. $X(jw) = \frac{2-jw}{12-7jw+(jw)^2}$
$X(jw) = \frac{2-jw}{(4-jw)(3-jw)} = \frac{A}{4-jw} + \frac{B}{3-jw}$
$A = \frac{2-jw}{3-jw}|_{jw=4} = \frac{2-4}{3-4} = 2$
$B = \frac{2-jw}{4-jw}|_{jw=3} = \frac{2-3}{4-3} = -1$

$X(jw) = \frac{2}{4-jw} - \frac{1}{3-jw}$
$x(t) = 2e^{4t}u(-t) - e^{3t}u(-t)$

(g) (5 pts) $X(jw) = \begin{cases} 1, & -2 < w < 0 \\ -1, & 0 \leqslant w < 2 \\ 0, & otherwise \end{cases}$.

Hint: Write this as a sum of two shifted rect functions. Show that $x(t) = -\frac{2j}{\pi} sinc(t) sin(t)$.

Solution. $X(jw) = rect(\frac{w+1}{2}) - rect(\frac{w-1}{2})$
Using the properties of Fourier transform, we have
$x(t) = \frac{1}{\pi} sinc(t) e^{-jt} - \frac{1}{\pi} sinc(t) e^{jt} = -\frac{2j}{\pi} sinc(t) \frac{e^{jt} - e^{-jt}}{2j} = -\frac{2j}{\pi} sinc(t) sin(t)$
Another way to solve this problem:
$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw) e^{jwt} dw$
$x(t) = \frac{1}{2\pi} \int_{-2}^{0} e^{jwt} dw + \frac{1}{2\pi} \int_{0}^{2} -e^{jwt} dw$
$x(t) = \frac{1}{2\pi} (\frac{1}{jt} - \frac{e^{-j2t}}{jt}) + \frac{1}{2\pi} (-\frac{e^{j2t}}{jt} + \frac{1}{jt}) = -\frac{2j}{\pi} sinc(t) sin(t)$

(h) (2 pts) $X(jw) = e^{-j3w}[u(w+\pi) - u(w-\pi)]$.
Show that $x(t) = sinc(\pi(t-3))$.

Solution. $X(jw) = e^{-jw(3)} X_1(jw)$
$x(t) = x_1(t-3)$
$X_1(jw) = rect(\frac{w}{2\pi})$
$x_1(t) = \frac{\pi}{\pi} sinc(\pi t) = sinc(\pi t)$
$x(t) = sinc(\pi(t-3))$

4. *Fourier Transform: Periodic Signals* (8 pts)

(a) (4 pts) Find the Fourier Transform of $x(t) = e^{j\pi t} + \sin(2\pi t)$ without using the table of pairs (applying the formula of Fourier Transform in Continuous Time).

Solution. $x(t) = e^{j\pi t} + \frac{e^{j2\pi t} - e^{-j2\pi t}}{2j}$
$\omega_0 = \pi, a_1 = 1, a_2 = \frac{1}{2j}, a_{-2} = \frac{-1}{2j}$
Hence, $X(jw) = (2\pi)(1)\delta(w-\pi) + (2\pi)(\frac{1}{2j})\delta(w-2\pi) + (2\pi)(\frac{-1}{2j})\delta(w+2\pi)$
$= 2\pi\delta(w-\pi) + \frac{\pi}{j}[\delta(w-2\pi) - \delta(w+2\pi)]$

(b) (4 pts) Find the Fourier Transform of $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k)$.

Hint: Find the Fourier series coefficients a_k of the signal first, and then apply the formula $X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_o)$.

Solution. *The Fourier series coefficients for this signal were computed by:*
$a_k = \frac{1}{2} \int_{-1}^{1} \delta(t) e^{-jkw_o t} dt = \frac{1}{2}$
That is, every Fourier coefficient of the periodic impulse train has the same value, $\frac{1}{2}$*. Substituting this value for a_k in* $X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_o)$ *yields*

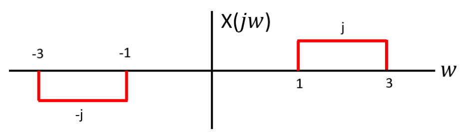
$$X(jw) = \frac{2\pi}{2} \sum_{k=-\infty}^{\infty} \delta(w - \frac{2\pi k}{2}) = \boxed{\pi \sum_{k=-\infty}^{\infty} \delta(w - \pi k)}$$

5. *Fourier Transforms: Other Properties* (16 pts)

(a) (4 pts) Suppose $x(t)$ has the Fourier series representation $w_o = \frac{\pi}{8}$ and nonzero coefficients $a_1 = 2j$, $a_{-1} = -2j$, $a_3 = 3e^{j\frac{\pi}{4}}$, $a_{-3} = 3e^{-j\frac{\pi}{4}}$. Without synthesizing and calculating $x(t)$, what is P_∞?

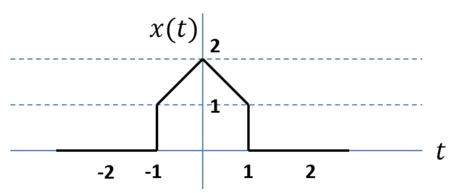
Solution. $P_\infty = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$
$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/8} = 16$
$P_\infty = \lim_{T\to\infty} \frac{1}{16}\int_{-8}^{8} |x(t)|^2 dt$
By Parseval's Theorem:
$P_\infty = \sum_{k=-\infty}^{\infty} |a_k|^2$
$P_\infty = |a_1|^2 + |a_{-1}|^2 + |a_3|^2 + |a_{-3}|^2$
$P_\infty = (2j)(-2j) + (-2j)(2j) + (3e^{j\frac{\pi}{4}})(3e^{-j\frac{\pi}{4}}) + (3e^{-j\frac{\pi}{4}})(3e^{j\frac{\pi}{4}})$
$P_\infty = 4+4+9+9 = \boxed{26}$

(b) (4 pts) Given $X(jw)$ below, what is E_∞ of $x(t)$?



Solution. *In the frequency domain, energy of signal can be expressed in terms of the Fourier Transform of the time domain representation:*
$E_\infty = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$
$E_\infty = \frac{1}{2\pi}\left[\int_{-3}^{-1} |-j|^2 d\omega + \int_1^3 |j|^2 d\omega\right]$
$E_\infty = \left[\frac{1(-1-(-3))+1(3-1)}{2\pi}\right]$
$E_\infty = \boxed{\frac{2}{\pi}}$

(c) Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ shown in the figure. *Note: The top part in the figure is a triangle.*



Using the properties of the Fourier transform (and without explicitly evaluating $X(j\omega)$),

i. (4 pts) Find $X(0)$.
Hint: Apply the definition of the Fourier transform formula.
Solution. *Setting $\omega = 0$ in the definition of the Fourier transform,*

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$
$$X(0) = X(jw)\big|_{w=0} = \int_{-\infty}^{\infty} x(t)e^{-j0t}dt$$
$$= \int_{-\infty}^{\infty} x(t)dt$$

so $X(0)$ is the area under $x(t)$. From the figure this is the sum of the areas of a rectangle and a triangle, $\boxed{2+1=3}$.

ii. (4 pts) Find $\int_{-\infty}^{\infty} X(j\omega)d\omega$.
Hint: Apply the definition of the inverse Fourier transform formula.

Solution. *Setting $t=0$ in the inverse Fourier transform formula,*

$$\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{jwt}d\omega \\ x(0) = x(t)\big|_{t=0} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{jw0}d\omega \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)d\omega \\ 2\pi x(0) &= \int_{-\infty}^{\infty} X(j\omega)d\omega \end{aligned}$$

so we get $\int_{-\infty}^{\infty} X(j\omega)d\omega = 2\pi x(0) = \boxed{4\pi}$.

6. *Homework Self-Reflection*
(10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

`http://bit.ly/2s2mQ1o`