**EE 235, Winter 2018**
**Homework 3: Continuous-Time Systems**
**(Due Friday January 19, 2018 by 12:30pm via Canvas Submission)**
**Write down *ALL* steps for full credit**

**HW3 Topics**:

- System Properties: C, S, I, TI

**HW3 Course Learning Goals Satisfied**:

- Goal 1: Describe signals in different domains (time).
- Goal 2: Understand the implications of different system properties and how to test for them.

**HW3 References**:

- OWN Sections 1.4, 1.5, 1.6

**HW3 Problems (Total = 106 pts)**:

1. *Review*

   (a) *Complex Numbers.* (2 pts) In Ch.3, you will need to be comfortable working with equations with complex numbers and terms. Here is one problem for you to practice your skills. Consider the following equation where $s$ is complex:

   $$H(s) = 3e^{2s}$$

   Using $H(s)$ above, evaluate and simplify as much as possible the following equation $y(t) = \frac{1}{2}H(j5)e^{j5t} + \frac{1}{2}H(-j5)e^{-j5t}$. *Hint: Use Euler's formulas.*

   (b) *Integration* We will use this problem to continue practice doing multivariable integration. In Ch.2, you will need to evaluate a so-called running integral where the upper bound is a variable as shown below:

   $$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

   Because $t$ is a variable and changing values, the answer for $s(t)$ will also be a function of $t$ − that is, for different regions of $t$, you will get a different equation for $s(t)$. See Supplementary Notes for an example. Evaluate the integral above for the following cases:

   i. (2 pts) $h(\tau) = \begin{cases} 3, & \tau > -1 \\ 0, & \tau < -1 \end{cases}$

   Show that $s(t) = \begin{cases} 0, & t < -1 \\ 3(t+1), & t > -1 \end{cases}$

   ii. (4 pts) $h(\tau) = \begin{cases} 0, & \tau > 0 \\ e^{2\tau}, & \tau < 0 \end{cases}$

   (c) *Periodicity and Even/Odd.* (4 pts)
   Let $z(t) = \sin^3(4t) + e^{-j6t}$. Find the odd $z_0(t)$ part of $z(t)$. Is the odd part of $z(t)$ periodic?

2. *Energy and Power*

   (a) (2 pts) $x(t) = e^{-at}u(-t), a > 0$. Evaluate $E_\infty$ and $P_\infty$. Is $x(t)$ an energy signal or a power signal? Show that $x(t)$ is not an energy signal and not a power signal.

(b) (2 pts) Find the energy $E_\infty$ and power $P_\infty$ of the signal $x(t) = \cos^2(2t)$. Is this an energy signal or a power signal? Show that $x(t)$ has infinite energy but finite power with $P_\infty = \frac{3}{8}$, so hence it is a power signal .
*Hint: The power of a periodic signal can be calculated from the power in one period.*

(c) (4 pts) Find the energy $E_\infty$ and power $P_\infty$ of the signal $x(t)$ is defined as below. Is this an energy signal or a power signal?

$$x(t) = \begin{cases} t+1 & -1 < t < 0 \\ -t+1 & 0 \le t < 1 \\ 0 & otherwise \end{cases}$$

3. *Unit Step and Unit Impulse*

(a) Graph the following sum of unit step functions. See HW3 Supplementary Notes for extra assistance, if needed.

i. (2 pts) $f(t) = u(t) - u(t-3)$

ii. (2 pts) $f(t) = u(t+2) - u(t-2) + 3u(t-3) - 3u(t-4)$

(b) Consider the following signal:

$$f(t) = -u(t-3) - 3u(t-1) + 2u(t+2) + 2u(t-2)$$

i. (2 pts) Graph $f(t)$

ii. (2 pts) Graph $f(3-t)$

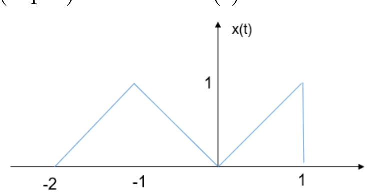
(c) Given the following piecewise function, rewrite each function in terms of the unit step function. See HW3 Supplementary Notes for extra assistance, if needed.

i. (2 pts) $f(t) = \begin{cases} 3, & 2 < t < 4 \\ 0, & otherwise \end{cases}$

ii. (2 pts) $f(t) = \begin{cases} -2, & -3 < t < -1 \\ 4, & 5 < t < 7 \\ 0, & otherwise \end{cases}$

(d) (2 pts) Show that the following statement is true. $\int_{-\infty}^{0} [2\delta(t+3) + 3\delta(t-1)]dt = 2$

(e) (4 pts) Consider $x(t)$ below:




Simplify the following: $x(t)x(-t)[\delta(t-0.5) + \delta(t+1.5)]$.

(f) (2 pts) Evaluate $\int_{-\infty}^{\infty} \tau^2 \delta(\tau+1) d\tau$. Show that the answer is 1.

4. *Causal.*
Consider the following input-output relationships of a system:

(a) (2 pts) $y(t) = x(t-3) - x(3-t)$. Is the system causal? Show that the system is *not* causal.

(b) (4 pts) $y(t) = x(t/4)$. Is the system causal?

(c) (2 pts) $y(t) = \dfrac{dx(t)}{dt}$. Is the system causal? Show that the system is causal.

(d) (4 pts) $y(t) = \cos(2t)x(t-7) + \sin(t)x(t-1)$. Is the system causal?

(e) (4 pts) The input-output relationship is given as:
$y(t) = x(t + 3a - 2b + 4)$. Given input-output relationship, determine the values of $b$ in terms of $a$ that will make the system causal.

5. *Stable.*
Consider the following input-output relationships of a system:

(a) (2 pts) $y(t) = x(t)(x(t-k))$ where $k$ is a real number. Is the system stable?
Show that the system is stable by showing that it has a constant upper bound.

(b) (2 pts) $y(t) = \int_t^{2t} x(\tau)d\tau$. Is the system stable?
Show that the system is *not* stable by showing that it has no constant upper bound.

(c) (4 pts) $y(t) = \int_{10}^{300} x(\tau)d\tau$. Is the system stable?

(d) (4 pts) $y(t) = \dfrac{d}{dt}(x(t) - t^2)$. Is the system stable? Hint: find the counter-example.

6. *Invertible.*
Given the following input-output relationships of different systems find out if the system is invertible or not; if so, derive the inverse. Justify your answer in both cases.

(a) (2 pts) $y(t) = x(t/3)$. Is this system invertible?
Show that it is.

(b) (2 pts) $y(t) = \begin{cases} x^3(t) & \text{if } t > 2 \\ 0 & otherwise \end{cases}$. Is this system invertible?
Show that it is NOT invertible.

(c) (4 pts) $y(t) = tx(t)$. Is this system invertible?

(d) (4 pts) $y(t) = \big(x(t)\big)^k$, where $k$ is an integer. Is the system invertible?
*Hint*: Think of two cases, one when $k$ is odd, second when $k$ is even. It might help to draw a picture of $\big(x(t)\big)^k$ for a couple of small values of $k$ to guess what the answer might be; once you guess it, you can prove invertibility or non-invertibility for each of the two cases.

(e) (2 pts) $y(t) = \begin{cases} x(t-7) & \text{if } t > 1 \\ 8x(t) & t \leq 1 \end{cases}$. Is this system invertible or not?
Show that it is invertible.

7. *Time-Invariance.*
Given input-output relationship of a system, prove whether system is time-invariant.

(a) (2 pts) Consider the system $y(t) = x(5t) + \sin(x(t))$. Is this system time-invariant?

(b) Consider a system $\mathcal{T}$ with input $x(t)$ and output $y(t)$ related by:
$y(t) = x(t)\{g(t) + g(t-1)\}$

i. (2 pts) If $g(t) = 1$ for all $t$, show that $\mathcal{T}$ is time-invariant using the time-invariance test from lecture.

ii. (2 pts) If $g(t) = t$, show that $\mathcal{T}$ is not time-invariant by using the time-invariance test from lecture.

(c) (2 pts) Consider the system $\mathcal{T}$ where $y(t) = \mathcal{T}(x(t)) = x(\sin(t))$. Is this time-invariant?

8. *Homework Self-Reflection* (10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

   `http://bit.ly/2CVDofA`