EE 235, Winter 2018, Homework 7: Fourier Transforms
Due Friday February 16, 2018 in class
Write down *ALL* steps for full credit

HW7 Topics:

- Fourier Transforms: Analyze (Transform) and Synthesize (Inverse Transform)
- Fourier Transforms: Periodic Signals
- Parseval's Theorem

HW7 Course Learning Goals Satisfied:

- Goal 1: Describe signals in different domains (time, frequency) and map characteristics in one domain to those in another.
- Goal 3: Perform convolutions for arbitrary and closed-form continuous-time signals
- Goal 4: Analyze LTI system given different system representations (including input-output equations, impulse response, frequency response) and translate between these representations.

HW7 References: OWN Sections 4.1 - 4.6

HW7 Problems (Total = 106 pts):

1. *Review* (20 pts)

   (a) *Partial Fraction Expansion.* (4 pts)
   Let $H(s) = \frac{s+2}{(s+3)(s^2+2s+1)}$. Write H(s) as a sum of partial fractions.

   (b) *Unit Impulse.* (4 pts)
   Suppose we have an impulse train signal $h(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$.
   Given an arbitrary signal $x(t)$, find $x(t)h(t)$ and $x(t) * h(t)$ in terms of $x(t)$.

   (c) *System Properties.* (3 pts)

   i. (1 pt) In general, if an LTI system has a causal impulse response, does it mean the system is also stable? Justify your response (with a proof if "true", or a counter-example if "false").

   ii. (1 pt) What about the reverse of the previous question: if an LTI system has a stable impulse response, does it mean the system is causal? Justify your response (with a proof if "true", or a counter-example if "false").

   iii. (1 pt) In general, if an LTI system has a periodic impulse response, what can you say about its stability?

   (d) *Fourier Series.* (9 pts)
   Consider an LTI system with frequency response

$$H(jw) = \begin{cases} -2, & -2 \leq w \leq 3 \\ 0, & otherwise \end{cases}$$

   i. (3 pts) Suppose the input is a periodic signal $x(t)$ with the Fourier Series representation $w_o = \frac{1}{2}$, $a_0 = 1$, $a_2 = a_{-2}^* = e^{-j\frac{\pi}{2}}$, $a_4 = 2$, $a_{-4} = -1$, $a_7 = 5$, and $a_k = 0$ for k otherwise. Find the output Fourier coefficients b_k, and specify your answer **for all k**. Show that $b_0 = -2$, $b_2 = -2e^{-j\frac{\pi}{2}}$, $b_{-2} = -2e^{j\frac{\pi}{2}}$, $b_4 = -4$, $b_{-4} = 2$, $b_7 = 0$, and $b_k = 0$ otherwise.

ii. (3 pts) Using your results from (A), find the output $y(t)$. Simplify $y(t)$ as much as possible. Show that $y(t) = -2 - 4cos(t - \frac{\pi}{2}) - 4jsin(2t) - 2e^{j2t}$.

iii. (3 pts) If $x(t)$ is delayed by 2, then what are the output coefficients b_k now? Show that $b_0 = -2$, $b_2 = -2e^{-j(\frac{\pi}{2}+2)}$, $b_{-2} = -2e^{j(\frac{\pi}{2}+2)}$, $b_4 = -4e^{-j4}$, $b_{-4} = 2e^{j4}$, $b_k = 0$ otherwise.

2. *Fourier Transform: Analysis* (27 pts)

(a) (3 pts) $x(t) = 2 + e^{-|t|}$.
Show that $X(j\omega) = 4\pi\delta(\omega) + \frac{2}{1+\omega^2}$

(b) (5 pts) $x(t) = rect(2(t + \frac{3}{4})) - rect(2(t + \frac{1}{4}))$.
Show that $Xj\omega = je^{j\frac{\omega}{2}} \sin(\frac{\omega}{4})\text{sinc}(\frac{\omega}{4})$.

(c) (3 pts) $x(t) = \frac{1}{2}e^{-j\frac{\pi}{4}}\delta(t-3) + \frac{1}{2}e^{j\frac{\pi}{4}}\delta(t+3)$.
Show that $X(j\omega) = \cos(\frac{\pi}{4} + 3\omega)$.

(d) (3 pts) $x(t) = \frac{j}{\pi}\sin t + \frac{1}{\pi}\cos(3t)$.
Show that $X(j\omega) = \delta(\omega-1) - \delta(\omega+1) + \delta(\omega-3) + \delta(\omega+3)$.

(e) (3 pts) $x(t)$ is given as
$x(t) = (e^{-t} - e^{-2t})u(t)$.
Find $X(j\omega)$. Show that $X(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)}$.

(f) (5 pts) $x(t)$ is given as
$x(t) = e^{-3|t|}\sin t$.
Find $X(j\omega)$. Show that $X(j\omega) = -\frac{3j}{9+(\omega-1)^2} + \frac{3j}{9+(\omega+1)^2}$.
Hint: $X(j\omega) = \frac{1}{2\pi}(X_1(j\omega) * X_2(j\omega))$

(g) (5 pts) $x(t)$ is given as
$x(t) = 4\pi\text{sinc}(4\pi t)\cos(4\pi t)$.
Find $X(j\omega)$.

3. *Fourier Transform: Synthesize (Inverse Transform).* (25 pts)
Using common Fourier Transform pairs and properties, find the signal $x(t)$ given:

(a) (2 pts) $X(jw) = 3[\delta(w-1) + \delta(w+1)] + 2[\delta(w-2\pi) + \delta(w+2\pi)]$.
Show that $\frac{3}{\pi}cos(t) + \frac{2}{\pi}cos(2\pi t)$.

(b) (2 pts) $X(jw) = \delta(w) + 2\delta(w+3) + 2\delta(w-3)$.
Show that $x(t) = \frac{1}{2\pi} + \frac{2}{\pi}cos(3t)$.

(c) (2 pts) $X(jw) = cos(w + \frac{\pi}{6})$.
Hint: Write $X(jw)$ as a sum of complex exponentials first. Show that $x(t) = \frac{1}{2}e^{j\frac{\pi}{6}}\delta(t+1) + \frac{1}{2}e^{-j\frac{\pi}{6}}\delta(t-1)$.

(d) (2 pts) $X(jw) = \frac{1}{2}e^{j2w} + \frac{1}{3}$.
Show that $x(t) = \frac{1}{2}\delta(t+2) + \frac{1}{3}\delta(t)$.

(e) (5 pts) $X(jw) = \frac{2sin(3(w-2\pi))}{w-2\pi}$.
Show that $x(t) = rect(\frac{t}{6})e^{j2\pi t}$.
Hint: represent $X(jw)$ using a sinc function.

(f) (5 pts) $X(jw) = \frac{2-jw}{12-7jw-w^2}$.
Hint: Use partial fractions. Show that $x(t) = 2e^{4t}u(-t) - e^{3t}u(-t)$.

(g) (5 pts) $X(jw) = \begin{cases} 1, & -2 < w < 0 \\ -1, & 0 \leqslant w < 2 \\ 0, & otherwise \end{cases}$.

Hint: Write this as a sum of two shifted rect functions. Show that $x(t) = -\frac{2j}{\pi} sinc(t) sin(t)$.

(h) (2 pts) $X(jw) = e^{-j3w}[u(w+\pi) - u(w-\pi)]$.
Show that $x(t) = sinc(\pi(t-3))$.

4. *Fourier Transform: Periodic Signals* (8 pts)

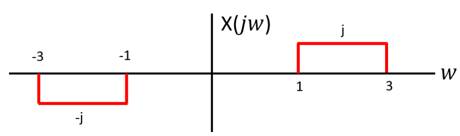
   (a) (4 pts) Find the Fourier Transform of $x(t) = e^{j\pi t} + \sin(2\pi t)$ without using the table of pairs (applying the formula of Fourier Transform in Continuous Time).

   (b) (4 pts) Find the Fourier Transform of $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-2k)$.
   Hint: Find the Fourier series coefficients a_k of the signal first, and then apply the formula $X(jw) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(w - kw_o)$.

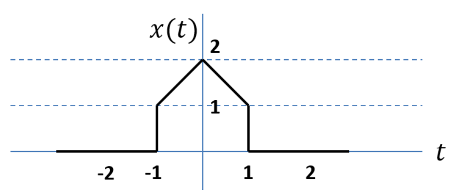
5. *Fourier Transforms: Other Properties* (16 pts)

   (a) (4 pts) Suppose $x(t)$ has the Fourier series representation $w_o = \frac{\pi}{8}$ and nonzero coefficients $a_1 = 2j$, $a_{-1} = -2j$, $a_3 = 3e^{j\frac{\pi}{4}}$, $a_{-3} = 3e^{-j\frac{\pi}{4}}$. Without synthesizing and calculating $x(t)$, what is P_∞?

   (b) (4 pts) Given $X(jw)$ below, what is E_∞ of $x(t)$?



   (c) Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ shown in the figure. *Note: The top part in the figure is a triangle.*



   Using the properties of the Fourier transform (and without explicitly evaluating $X(j\omega)$),

   i. (4 pts) Find $X(0)$.
   Hint: Apply the definition of the Fourier transform formula.

   ii. (4 pts) Find $\int_{-\infty}^{\infty} X(j\omega) d\omega$.
   Hint: Apply the definition of the inverse Fourier transform formula.

6. *Homework Self-Reflection*
(10 pts) After completing your homework, go to the following link to rate your skill or concept understanding level for each item listed. Your self-reflection must be completed by the due date. All submissions are time-stamped, so please give yourself plenty of time to complete and submit your self-reflection.

`http://bit.ly/2s2mQ1o`