

Preface

There is a lot here and this R notebook is busy. I was trying to compare the advice and examples available from different sources. There is a lot to be said about regression diagnostics. Needless to say, don't worry about trying to understand all of this in one week. See if you can connect a couple dots between what is said in the texts and what is in this notebook and call it good.

That being said, this is a great resource for future learning and analysis should you choose to take it on. Keep it as an aid for understanding regression in the future.

Regression Diagnostics

The regression diagnostics available in Jamovi are still limited. It would be a good idea to augment what is available in Jamovi with some regression diagnostics in R. This R markdown document provides an example of performing a multiple regression using the lm() function in R and following up with some regression diagnostics to evaluate bias and generalizability of the model.

Additional resources

- Field has a good example of fitting a regression model and getting regression diagnostics in his book Discovering Statistics Using R.
<https://archive.org/details/discoveringstatisticsusingr/page/n307/mode/2up>
- Applied Statistics with R Ch. 13 (optional) available at:
<https://daviddalpiaz.github.io/appliedstats/model-diagnostics.html>
- R for Researchers OLS (optional) available at:
https://ssc.wisc.edu/sscc/pubs/RFR/RFR_Regression.html
- R for Researchers regression diagnostics (optional) available at:
https://ssc.wisc.edu/sscc/pubs/RFR/RFR_Diagnostics.html
- Diagnostics in multiple linear regression (optional) available at:
https://web.stanford.edu/class/stats191/notebooks/Diagnostics_for_multiple_regression.html
- Introduction to regression diagnostics (optional) available at:
[https://stats.idre.ucla.edu/wp-content/uploads/2019/02/R_reg_part2.html#\(1\)](https://stats.idre.ucla.edu/wp-content/uploads/2019/02/R_reg_part2.html#(1))
- Regression model diagnostics (optional) available at:
<http://www.sthda.com/english/articles/39-regression-model-diagnostics/161-linear-regression-assumptions-and-diagnostics-in-r-essentials/>

- Regression diagnostics (optional) available at:
<https://www.statmethods.net/stats/riagnostics.html>
- Outlier detection with Mahalanobis distance (optional) available at:
<https://www.r-bloggers.com/outlier-detection-with-mahalanobis-distance/>
- Multivariate outlier (optional) available at:
https://en.wikiversity.org/wiki/Multivariate_outlier

Package management in R

```
``` r
keep a list of the packages used in this script
packages <-
c("tidyverse", "rio", "jmv", "boot", "car", "QuantPsyc", "lmtest", "faraway", "broom")
```
```

This next code block has eval=FALSE because you don't want to run it when knitting the file. Installing packages when knitting an R notebook can be problematic.

```
``` r
check each of the packages in the list and install them if they're not
installed already
for (i in packages){
 if(! i %in% installed.packages()){
 install.packages(i, dependencies = TRUE)
 }
 # show each package that is checked
 print(i)
}
```
```

```
``` r
load each package into memory so it can be used in the script
for (i in packages){
 library(i, character.only=TRUE)
 # show each package that is loaded
 print(i)
}
```
```

```
## -- Attaching packages ----- tidyverse
1.3.0 --
```

```
## v ggplot2 3.3.3      v purrr   0.3.4
## v tibble  3.0.6      v dplyr    1.0.4
## v tidyr   1.1.2      v stringr  1.4.0
## v readr   1.4.0      vforcats  0.5.1
```

```
## -- Conflicts -----
tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
```

```
## x dplyr::lag()      masks stats::lag()

## [1] "tidyverse"
## [1] "rio"
## [1] "jmv"
## [1] "boot"

## Loading required package: carData

##
## Attaching package: 'car'

## The following object is masked from 'package:boot':
##
##     logit

## The following object is masked from 'package:dplyr':
##
##     recode

## The following object is masked from 'package:purrr':
##
##     some

## [1] "car"

## Warning: package 'QuantPsyc' was built under R version 4.0.4

## Loading required package: MASS

##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select

##
## Attaching package: 'QuantPsyc'

## The following object is masked from 'package:base':
##
##     norm

## [1] "QuantPsyc"

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## [1] "lmtest"
```

```
## Warning: package 'faraway' was built under R version 4.0.4

## Registered S3 methods overwritten by 'lme4':
##   method           from
##   cooks.distance.influence.merMod car
##   influence.merMod      car
##   dfbeta.influence.merMod car
##   dfbetas.influence.merMod car

##
## Attaching package: 'faraway'

## The following objects are masked from 'package:car':
## 
##   logit, vif

## The following objects are masked from 'package:boot':
## 
##   logit, melanoma

## [1] "faraway"
## [1] "broom"
```

Regression Diagnostics

Regression diagnostics should be performed when fitting a linear model to assess for bias and generalizability of the model.

Open data file

The rio package works for importing several different types of data files. We're going to use it in this class. There are other packages which can be used to open datasets in R. You can see several options by clicking on the Import Dataset menu under the Environment tab in RStudio. (For a csv file like we have this week we'd use either From Text(base) or From Text (readr). Try it out to see the menu dialog.)

```
``` r
Using the file.choose() command allows you to select a file to import from
another folder.
dataset <- rio::import(file.choose())
dataset <- rio::import("Album Sales.sav")
````
```

lm() function in R

Many linear models are calculated in R using the lm() function. We'll look at how to perform a multiple regression using the lm() function since it's so common.

Visualization

```

``` r
This code creates a scatter matrix
library(GGally)
```

## Registered S3 method overwritten by 'GGally':
##   method from
##   +.gg   ggplot2

##
## Attaching package: 'GGally'

## The following object is masked from 'package:faraway':
## 
##   happy

```
GGally:::ggpairs(dataset, columns=c('Sales','Adverts','Airplay','Image'), lower = list(continuous = "smooth"))
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-chunk-5-1.png)

``` r
This code creates a scatterplot between a single pair of variables
ggplot(dataset, aes(x = Adverts, y = Sales)) +
 geom_point() +
 stat_smooth(method = lm)
```

## `geom_smooth()` using formula 'y ~ x'

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-chunk-6-1.png)

#### Computation

Multiple models adding variables in steps can be created in R using the lm() and update() functions.

``` r
model.1 <- lm(formula = Sales ~ Adverts, data = dataset)
model.2 <- update(model.1, .~. + Airplay)
model.3 <- update(model.2, .~. + Image)
```

#### Model assessment

``` r
summary(model.1)
```

## 
## Call:
## lm(formula = Sales ~ Adverts, data = dataset)

```

```

## 
## Residuals:
##   Min     1Q Median     3Q    Max
## -152.949 -43.796 -0.393  37.040 211.866
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 1.341e+02 7.537e+00 17.799 <2e-16 ***
## Adverts     9.612e-02 9.632e-03  9.979 <2e-16 ***  
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 65.99 on 198 degrees of freedom
## Multiple R-squared:  0.3346, Adjusted R-squared:  0.3313 
## F-statistic: 99.59 on 1 and 198 DF,  p-value: < 2.2e-16

``` r
summary(model.2)
```
## 
## Call:
## lm(formula = Sales ~ Adverts + Airplay, data = dataset)
## 
## Residuals:
##   Min     1Q Median     3Q    Max
## -112.121 -30.027  3.952  32.072 155.498
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 41.123811  9.330952  4.407 1.72e-05 ***
## Adverts     0.086887  0.007246 11.991 < 2e-16 ***  
## Airplay      3.588789  0.286807 12.513 < 2e-16 ***  
## ---        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 49.38 on 197 degrees of freedom
## Multiple R-squared:  0.6293, Adjusted R-squared:  0.6255 
## F-statistic: 167.2 on 2 and 197 DF,  p-value: < 2.2e-16

``` r
summary(model.3)
```
## 
## Call:
## lm(formula = Sales ~ Adverts + Airplay + Image, data = dataset)
## 
## Residuals:
##   Min     1Q Median     3Q    Max
## -121.324 -28.336 -0.451  28.967 144.132
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -26.612958 17.350001 -1.534  0.127    
## Adverts      0.084885  0.006923 12.261 < 2e-16 ***

```

```

## Airplay      3.367425  0.277771 12.123 < 2e-16 ***
## Image       11.086335  2.437849  4.548 9.49e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 47.09 on 196 degrees of freedom
## Multiple R-squared:  0.6647, Adjusted R-squared:  0.6595
## F-statistic: 129.5 on 3 and 196 DF,  p-value: < 2.2e-16

#### Standardized residuals from lm()

```

You might notice `lm()` does not provide the standardized residuals. Those must be calculated separately.

```

``` r
standardized = lm(scale(Sales) ~ scale(Adverts) + scale(Airplay) +
scale(Image), data=dataset)
summary(standardized)
```

##
## Call:
## lm(formula = scale(Sales) ~ scale(Adverts) + scale(Airplay) +
##     scale(Image), data = dataset)
##
## Residuals:
##     Min      1Q  Median      3Q      Max
## -1.50342 -0.35113 -0.00559  0.35895  1.78605
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.586e-17 4.126e-02  0.000    1
## scale(Adverts) 5.108e-01 4.166e-02 12.261 < 2e-16 ***
## scale(Airplay) 5.120e-01 4.223e-02 12.123 < 2e-16 ***
## scale(Image)   1.917e-01 4.215e-02  4.548 9.49e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5835 on 196 degrees of freedom
## Multiple R-squared:  0.6647, Adjusted R-squared:  0.6595
## F-statistic: 129.5 on 3 and 196 DF,  p-value: < 2.2e-16

```

Another option for getting standardized residuals is in the `QuantPsyc` package.

```

``` r
QuantPsyc::lm.beta(model.3)
```

##
## Adverts     Airplay     Image
## 0.5108462  0.5119881  0.1916834

#### Confidence intervals for the parameters

```

The `lm()` function didn't provide confidence intervals for the model parameters. This can be done using the `confint()` function.

```

``` r
confint(model.3)
```

##          2.5 %      97.5 %
## (Intercept) -60.82960967  7.60369295
## Adverts      0.07123166  0.09853799
## Airplay       2.81962186  3.91522848
## Image         6.27855218 15.89411823

#### AIC & BIC

From
<a href="https://ssc.wisc.edu/sscc/pubs/RFR/RFR_Regression.html"
class="uri">https://ssc.wisc.edu/sscc/pubs/RFR/RFR_Regression.html</a>

``` r
Smaller values indicate a better model
AIC(model.1)
```

## [1] 2247.375

``` r
AIC(model.2)
```

## [1] 2132.398

``` r
AIC(model.3)
```

## [1] 2114.337

``` r
BIC(model.1)
```

## [1] 2257.27

``` r
BIC(model.2)
```

## [1] 2145.592

``` r
BIC(model.3)
```

## [1] 2130.828

#### Compare models
```

Hierachal models can be compared using ANOVA.

```
``` r
anova(model.1, model.2, model.3)
```

## Analysis of Variance Table
##
## Model 1: Sales ~ Adverts
## Model 2: Sales ~ Adverts + Airplay
## Model 3: Sales ~ Adverts + Airplay + Image
##   Res.Df   RSS Df Sum of Sq      F    Pr(>F)
## 1     198 862264
## 2     197 480428  1    381836 172.214 < 2.2e-16 ***
## 3     196 434575  1     45853  20.681 9.492e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Evaluating for bias in the model

```
#### Add observation statistics to data (missing in Jamovi)
```

```
``` r
add ID column
dataset$id = 1:nrow(dataset)

Outliers
dataset$m3resid = round(resid(model.3), digits = 3)
dataset$m3stdres = round(rstandard(model.3), digits = 3)
dataset$m3studres = round(rstudent(model.3), digits = 3)

Influential cases
dataset$m3cooks = round(cooks.distance(model.3), digits = 3)
dataset$m3dfbetas = round(dfbetas(model.3), digits = 3)
dataset$m3dffits = round(dffits(model.3), digits = 3)
dataset$m3lev = round(hatvalues(model.3), digits = 3)
dataset$m3covrat = round(covratio(model.3), digits = 3)
dataset$m3mahal =
mahalanobis(dataset[,c("Adverts","Airplay","Image")], colMeans(dataset[,c("Adverts","Airpla
```

```
fitted values
dataset$fitted <- round(model.3$fitted.values, digits = 3)
```
```

After I created a lot of this code, I ran across a function to get a lot of regression diagnostics quickly. The site seems to give some examples using more tidyverse notation. From

```
<a href="http://www.sthda.com/english/articles/39-regression-model-
diagnostics/161-linear-regression-assumptions-and-diagnostics-in-r-
essentials/" class="uri">http://www.sthda.com/english/articles/39-regression-
model-diagnostics/161-linear-regression-assumptions-and-diagnostics-in-r-
essentials/</a>
```

```
``` r
model.diag.metrics <- broom::augment(model.3)
```

```

head(model.diag.metrics)
```

## # A tibble: 6 x 10
##   Sales Adverts Airplay Image .fitted .resid     .hat .sigma .cooksdi
.std.resid
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
<dbl>
## 1    330    10.3     43    10    230.   100.    0.0472   46.6  5.87e-2
2.18
## 2    120    986.     28     7    229.  -109.    0.00801   46.6  1.09e-2
-2.32
## 3    360   1446.     35     7    292.   68.4    0.0207   46.9  1.14e-2
1.47
## 4    270   1188.     33     7    263.    7.02   0.0126   47.2  7.17e-5
0.150
## 5    220    575.     44     5    226.  -5.75   0.0261   47.2  1.03e-4
-0.124
## 6    170    569.     19     5    141.   28.9    0.0142   47.2  1.38e-3
0.618

#### Flag cases for investigation

``` r
studentized residuals greater than 2 (or 3)
No more than 5% of cases above 2. No more than 1% above 2.5.
print("Studentized residuals greater than 2 or 3")
```

## [1] "Studentized residuals greater than 2 or 3"

``` r
dataset$largeRes <- dataset$m3studres > 3 | dataset$m3studres < -3
dataset[dataset$largeRes, c("id")]
```

## [1] 169

``` r
Cook's greater than 1
Any case
print("Cook's value greater than 1")
```

## [1] "Cook's value greater than 1"

``` r
dataset$largeCook <- dataset$m3cooks > 1 | dataset$m3cooks < -1
dataset[dataset$largeCook, c("id")]
```

## integer(0)

``` r
Mahalanobis distance greater than chi-square critical value df=number
predictors
```

```

```

# related to leverage
print("Mahalanobis distance greater than chi-square cutoff")
```

[1] "Mahalanobis distance greater than chi-square cutoff"

``` r
dataset$largeMahal <- dataset$m3mahal > qchisq(.99, df=3)
dataset[dataset$largeMahal, c("id")]
```

[1] 7 12 138 181 184

``` r
# leverage greater than 2 (or 3) times average
print("Leverage greater than 2 or 3 times the average")
```

[1] "Leverage greater than 2 or 3 times the average"

``` r
averageLeverage = (3 + 1)/200
dataset$largeLev <- dataset$m3lev > 3*averageLeverage
dataset[dataset$largeLev, c("id")]
```

[1] 7 12 138 181 184

``` r
# covariance ratio out of bounds (1 +/- (k*(K+1)/n))
print("Covariance ratio out of bounds")
```

[1] "Covariance ratio out of bounds"

``` r
dataset$COVcheck <- dataset$m3covrat > (1 + (3*(3+1)/200)) | dataset$m3covrat
< (1 - (3*(3+1)/200))
dataset[dataset$COVcheck, c("id")]
```

[1] 2 7 12 23 42 43 47 55 61 68 87 88 93 116 138 164
169 181 184
[20] 199

``` r
# standardized dfbeta greater than 1 - b change when exclude cases
print("Standardized dfbeta greater than 1")
```

[1] "Standardized dfbeta greater than 1"

``` r
dataset$largedfbeta <- dataset$m3dfbetas > 1 | dataset$m3dfbetas < -1
dataset[dataset$largedfbeta, c("id")]
```

```

```

integer(0)

``` r
# standardized dffit greater than 1 - predicted values change when exclude
cases
print("Standardized dffit greater than 1")
```

[1] "Standardized dffit greater than 1"

``` r
dataset$largeddffit <- dataset$m3dffits > 1 | dataset$m3dffits < -1
dataset[dataset$largeddffit, c("id")]
```

integer(0)

```

There is another function in R to help flag potentially influential cases

```

``` r
# https://web.stanford.edu/class/stats191/notebooks/
Diagnostics_for_multiple_regression.html
influence.measures(model.3)
```

Influence measures of
lm(formula = Sales ~ Adverts + Airplay + Image, data = dataset) :
##
dfb.1_ dfb.Advr dfb.Arpl dfb.Imag dffit cov.r cook.d
hat inf
1 -3.16e-01 -2.42e-01 0.157739 0.353292 0.48929 0.971 5.87e-02
0.04719 *
2 1.26e-02 -1.26e-01 0.009421 -0.018683 -0.21110 0.920 1.09e-02
0.00801 *
3 -3.81e-02 1.75e-01 0.046574 -0.005385 0.21418 0.997 1.14e-02
0.02070
4 -2.58e-03 1.22e-02 0.003444 0.000129 0.01689 1.033 7.17e-05
0.01256
5 -8.58e-03 1.09e-03 -0.014254 0.013554 -0.02020 1.048 1.03e-04
0.02607
6 6.58e-02 2.24e-03 -0.020820 -0.051246 0.07411 1.027 1.38e-03
0.01421
7 -1.45e-01 -1.00e-03 -0.005086 0.147724 -0.15389 1.117 5.94e-03
0.09104 *
8 -2.81e-02 -5.86e-03 -0.019167 0.045204 0.05366 1.040 7.23e-04
0.02092
9 1.12e-02 -8.96e-03 -0.028957 0.014483 0.06154 1.016 9.49e-04
0.00691
10 -1.26e-02 -1.56e-01 0.167721 0.006723 0.26896 0.944 1.78e-02
0.01541
11 8.75e-03 -5.17e-02 -0.003237 0.001001 -0.05723 1.051 8.23e-04
0.03118
12 -2.41e-01 -1.95e-02 0.000943 0.243196 -0.25743 1.069 1.66e-02
0.06410 *

```



|         |       |           |           |           |           |          |       |          |
|---------|-------|-----------|-----------|-----------|-----------|----------|-------|----------|
|         | ## 41 | 3.78e-03  | -1.21e-02 | -0.009467 | -0.000679 | -0.02647 | 1.027 | 1.76e-04 |
| 0.00812 |       |           |           |           |           |          |       |          |
|         | ## 42 | -3.58e-02 | -9.83e-02 | 0.206271  | -0.018746 | 0.23182  | 1.065 | 1.34e-02 |
| 0.05823 | *     |           |           |           |           |          |       |          |
|         | ## 43 | -1.04e-02 | 6.63e-02  | 0.000144  | -0.001539 | 0.07067  | 1.068 | 1.25e-03 |
| 0.04593 | *     |           |           |           |           |          |       |          |
|         | ## 44 | -7.70e-04 | 2.14e-02  | -0.007418 | 0.003410  | 0.03230  | 1.028 | 2.62e-04 |
| 0.00967 |       |           |           |           |           |          |       |          |
|         | ## 45 | 1.29e-02  | 4.09e-02  | -0.083959 | 0.000910  | -0.11410 | 1.015 | 3.26e-03 |
| 0.01380 |       |           |           |           |           |          |       |          |
|         | ## 46 | 5.46e-02  | -2.30e-01 | -0.077306 | 0.011464  | -0.28216 | 0.978 | 1.97e-02 |
| 0.02380 |       |           |           |           |           |          |       |          |
|         | ## 47 | 6.64e-02  | 1.96e-01  | 0.048288  | -0.178574 | -0.31469 | 0.915 | 2.41e-02 |
| 0.01568 | *     |           |           |           |           |          |       |          |
|         | ## 48 | 1.05e-02  | -4.39e-02 | -0.004067 | 0.012615  | 0.06877  | 1.019 | 1.19e-03 |
| 0.00888 |       |           |           |           |           |          |       |          |
|         | ## 49 | -4.57e-03 | 1.34e-02  | 0.010421  | -0.000924 | 0.02186  | 1.036 | 1.20e-04 |
| 0.01504 |       |           |           |           |           |          |       |          |
|         | ## 50 | -4.22e-02 | 9.52e-04  | 0.173808  | -0.012978 | 0.21041  | 0.983 | 1.10e-02 |
| 0.01649 |       |           |           |           |           |          |       |          |
|         | ## 51 | 1.51e-02  | 1.10e-02  | 0.026222  | -0.023993 | 0.04854  | 1.027 | 5.91e-04 |
| 0.01033 |       |           |           |           |           |          |       |          |
|         | ## 52 | 3.53e-01  | -2.88e-02 | -0.136669 | -0.269650 | 0.36742  | 0.960 | 3.32e-02 |
| 0.02921 |       |           |           |           |           |          |       |          |
|         | ## 53 | 4.81e-03  | -4.32e-02 | -0.123515 | 0.074557  | 0.15430  | 1.048 | 5.96e-03 |
| 0.03793 |       |           |           |           |           |          |       |          |
|         | ## 54 | -1.73e-02 | -2.36e-03 | -0.000888 | 0.024897  | 0.03786  | 1.027 | 3.60e-04 |
| 0.00895 |       |           |           |           |           |          |       |          |
|         | ## 55 | 1.74e-01  | -3.26e-01 | -0.023068 | -0.124349 | -0.40736 | 0.925 | 4.04e-02 |
| 0.02610 | *     |           |           |           |           |          |       |          |
|         | ## 56 | 5.12e-03  | -4.60e-02 | 0.003598  | -0.004287 | -0.06477 | 1.023 | 1.05e-03 |
| 0.01035 |       |           |           |           |           |          |       |          |
|         | ## 57 | 3.51e-02  | 2.42e-02  | -0.050804 | -0.012286 | 0.06839  | 1.034 | 1.17e-03 |
| 0.01791 |       |           |           |           |           |          |       |          |
|         | ## 58 | -2.19e-02 | 3.40e-02  | 0.073443  | -0.001318 | 0.12487  | 0.996 | 3.89e-03 |
| 0.00938 |       |           |           |           |           |          |       |          |
|         | ## 59 | -4.05e-02 | 1.08e-02  | 0.005168  | 0.033754  | -0.04241 | 1.048 | 4.52e-04 |
| 0.02713 |       |           |           |           |           |          |       |          |
|         | ## 60 | -4.52e-02 | -5.95e-02 | 0.064342  | 0.022212  | -0.10409 | 1.028 | 2.71e-03 |
| 0.01874 |       |           |           |           |           |          |       |          |
|         | ## 61 | 8.19e-04  | -1.54e-02 | 0.027926  | 0.020543  | 0.15562  | 0.937 | 5.95e-03 |
| 0.00535 | *     |           |           |           |           |          |       |          |
|         | ## 62 | -3.73e-02 | 5.94e-02  | 0.019305  | 0.021939  | 0.07937  | 1.043 | 1.58e-03 |
| 0.02597 |       |           |           |           |           |          |       |          |
|         | ## 63 | 2.15e-02  | -9.71e-03 | 0.027330  | -0.041342 | -0.06376 | 1.024 | 1.02e-03 |
| 0.01109 |       |           |           |           |           |          |       |          |
|         | ## 64 | -2.82e-02 | 3.43e-02  | 0.004978  | 0.012469  | -0.04889 | 1.031 | 6.00e-04 |
| 0.01357 |       |           |           |           |           |          |       |          |
|         | ## 65 | -3.07e-02 | -1.57e-02 | -0.021028 | 0.052067  | 0.06276  | 1.040 | 9.89e-04 |
| 0.02206 |       |           |           |           |           |          |       |          |
|         | ## 66 | -2.83e-02 | 6.60e-02  | 0.051708  | -0.029031 | -0.12539 | 0.999 | 3.92e-03 |
| 0.01006 |       |           |           |           |           |          |       |          |
|         | ## 67 | -3.54e-02 | 2.17e-02  | 0.022872  | 0.013776  | -0.05296 | 1.026 | 7.04e-04 |
| 0.01068 |       |           |           |           |           |          |       |          |
|         | ## 68 | -2.81e-03 | 2.11e-01  | -0.147655 | -0.017599 | -0.30216 | 0.924 | 2.23e-02 |
| 0.01571 | *     |           |           |           |           |          |       |          |





```

125 -1.02e-01 7.02e-02 0.134428 0.011323 -0.18024 1.026 8.12e-03
0.02788
126 -1.68e-02 -9.47e-02 -0.002248 0.033586 -0.10949 1.040 3.01e-03
0.02692
127 -1.36e-02 -6.77e-02 0.105225 -0.000936 0.14475 1.012 5.23e-03
0.01670
128 -7.77e-03 7.88e-02 -0.014248 0.003901 0.09226 1.032 2.13e-03
0.01942
129 1.30e-04 4.13e-03 -0.000137 0.003108 0.02172 1.024 1.19e-04
0.00533
130 2.85e-02 -7.23e-03 -0.020753 -0.025406 -0.05111 1.027 6.56e-04
0.01109
131 7.60e-02 -1.04e-02 0.038077 -0.082520 0.10634 1.021 2.83e-03
0.01509
132 2.16e-03 -7.39e-03 -0.004958 -0.000300 -0.01457 1.029 5.33e-05
0.00867
133 3.66e-04 -8.24e-03 0.016324 0.012010 0.09129 0.994 2.08e-03
0.00534
134 -3.25e-03 -1.09e-02 0.016951 -0.004657 -0.02464 1.034 1.53e-04
0.01348
135 -9.34e-03 2.25e-02 0.014838 -0.011282 -0.05087 1.021 6.49e-04
0.00741
136 -1.28e-02 1.47e-02 0.000722 0.012566 0.02598 1.033 1.70e-04
0.01319
137 7.80e-02 -4.58e-02 -0.010743 -0.055318 0.09254 1.029 2.15e-03
0.01796
138 -1.41e-01 3.37e-02 -0.031591 0.147462 -0.15770 1.127 6.24e-03
0.09871 *
139 1.08e-03 1.05e-02 -0.009229 0.002497 0.01687 1.035 7.15e-05
0.01379
140 9.24e-05 -1.78e-03 0.000106 -0.000654 -0.00517 1.027 6.72e-06
0.00583
141 5.27e-02 -4.72e-02 -0.031523 -0.040181 -0.09674 1.022 2.34e-03
0.01425
142 6.32e-03 6.36e-03 -0.000790 -0.005866 0.01322 1.029 4.39e-05
0.00850
143 -1.04e-02 -1.49e-02 0.002184 0.017705 0.03001 1.031 2.26e-04
0.01184
144 -1.11e-02 -2.57e-02 0.053137 -0.013211 -0.06695 1.035 1.13e-03
0.01896
145 -2.34e-02 5.89e-02 0.039273 -0.024562 -0.10848 1.005 2.94e-03
0.00959
146 7.79e-02 2.92e-02 0.084972 -0.145121 -0.17879 1.019 7.98e-03
0.02438
147 -1.08e-02 1.81e-02 0.025799 -0.010021 -0.04087 1.033 4.19e-04
0.01465
148 -4.14e-02 1.45e-01 0.078695 -0.008187 0.20400 0.990 1.03e-02
0.01745
149 -3.38e-02 -3.23e-02 -0.059904 0.055814 -0.11380 1.007 3.23e-03
0.01066
150 2.74e-03 1.27e-02 -0.030423 -0.009358 -0.09254 0.997 2.14e-03
0.00584
151 9.66e-02 -3.49e-02 -0.065682 -0.051438 0.11554 1.034 3.34e-03
0.02372
152 -1.37e-01 1.39e-01 -0.143011 0.152493 -0.26114 1.001 1.69e-02
0.02809

```

|         |        |           |           |           |           |          |       |          |
|---------|--------|-----------|-----------|-----------|-----------|----------|-------|----------|
|         | ## 153 | 1.08e-02  | -4.31e-02 | -0.007982 | 0.010878  | 0.05827  | 1.028 | 8.52e-04 |
| 0.01247 | ## 154 | -5.40e-03 | -4.87e-02 | 0.056299  | -0.000433 | 0.08047  | 1.040 | 1.63e-03 |
| 0.02411 | ## 155 | 1.01e-02  | 1.24e-01  | 0.131336  | -0.124390 | -0.23484 | 1.003 | 1.37e-02 |
| 0.02529 | ## 156 | -1.24e-01 | -2.18e-02 | 0.108623  | 0.057443  | -0.19290 | 0.953 | 9.17e-03 |
| 0.00962 | ## 157 | 3.44e-03  | -1.59e-02 | -0.005729 | -0.001072 | -0.02745 | 1.028 | 1.89e-04 |
| 0.00863 | ## 158 | 1.16e-02  | 2.40e-02  | -0.007368 | -0.011078 | 0.03305  | 1.034 | 2.74e-04 |
| 0.01458 | ## 159 | 7.36e-02  | 1.03e-03  | -0.124567 | -0.008452 | 0.14107  | 1.039 | 4.98e-03 |
| 0.03035 | ## 160 | -2.90e-02 | 1.84e-02  | -0.021230 | 0.030998  | -0.04581 | 1.039 | 5.27e-04 |
| 0.01999 | ## 161 | -5.79e-02 | 1.07e-02  | 0.000795  | 0.041824  | -0.09080 | 1.002 | 2.06e-03 |
| 0.00663 | ## 162 | -7.26e-02 | 7.90e-02  | 0.035295  | 0.025457  | -0.12046 | 1.015 | 3.63e-03 |
| 0.01466 | ## 163 | -2.70e-04 | 5.65e-02  | -0.020592 | 0.015578  | 0.11768  | 0.986 | 3.44e-03 |
| 0.00683 | ## 164 | 1.80e-01  | 2.90e-01  | -0.400884 | -0.117064 | -0.54029 | 0.920 | 7.08e-02 |
| 0.03935 | *      |           |           |           |           |          |       |          |
| 0.01530 | ## 165 | -4.30e-02 | -1.06e-01 | 0.005805  | 0.088875  | 0.16433  | 1.000 | 6.73e-03 |
| 0.00955 | ## 166 | 1.83e-02  | -1.00e-01 | 0.010106  | 0.023524  | 0.14783  | 0.984 | 5.43e-03 |
| 0.00986 | ## 167 | -1.36e-01 | 3.87e-02  | 0.106300  | 0.055201  | -0.20094 | 0.949 | 9.94e-03 |
| 0.01301 | ## 168 | 1.88e-02  | 3.73e-02  | 0.009157  | -0.042344 | -0.07011 | 1.026 | 1.23e-03 |
| 0.02082 | *      |           |           |           |           |          |       |          |
| 0.01342 | ## 169 | -1.68e-01 | -2.58e-01 | 0.257392  | 0.169684  | 0.46132  | 0.853 | 5.09e-02 |
| 0.01101 | ## 170 | 3.47e-02  | 3.93e-02  | -0.037480 | -0.020355 | 0.07260  | 1.026 | 1.32e-03 |
| 0.01975 | ## 171 | -3.76e-02 | -1.85e-02 | 0.029661  | 0.041013  | 0.07692  | 1.021 | 1.48e-03 |
| 0.01182 | ## 172 | -8.76e-03 | -4.09e-02 | 0.064870  | -0.001562 | 0.08583  | 1.033 | 1.85e-03 |
| 0.01641 | ## 173 | -8.34e-04 | -1.09e-02 | 0.008443  | -0.002527 | -0.01764 | 1.032 | 7.81e-05 |
| 0.02662 | ## 174 | 3.10e-02  | -6.87e-02 | -0.063318 | 0.027601  | 0.11809  | 1.020 | 3.49e-03 |
| 0.00743 | ## 175 | 5.08e-02  | -1.09e-01 | 0.016771  | -0.044737 | -0.13408 | 1.035 | 4.50e-03 |
| 0.01514 | ## 176 | -7.34e-02 | 4.08e-02  | -0.009538 | 0.051372  | -0.11874 | 0.990 | 3.51e-03 |
| 0.02723 | ## 177 | -4.63e-02 | 4.60e-02  | -0.118434 | 0.114010  | 0.19277  | 0.987 | 9.22e-03 |
| 0.00957 | ## 178 | 3.09e-02  | -1.26e-02 | -0.007721 | -0.034182 | -0.05838 | 1.023 | 8.55e-04 |
| 0.02966 | ## 179 | -5.65e-02 | -2.12e-02 | 0.046994  | 0.050514  | 0.08517  | 1.044 | 1.82e-03 |
| 0.02966 | ## 180 | 3.27e-03  | 5.73e-03  | -0.006343 | -0.002787 | -0.00979 | 1.052 | 2.41e-05 |

```

181 4.33e-02 -6.01e-03 0.001171 -0.042352 0.04508 1.124 5.11e-04
0.09260 *
182 3.25e-02 4.86e-02 0.047904 -0.085889 -0.13351 1.004 4.45e-03
0.01248
183 8.87e-02 5.09e-02 -0.108397 -0.066751 -0.15256 1.054 5.83e-03
0.04232
184 -7.16e-03 -3.44e-02 -0.002086 0.015372 -0.03822 1.104 3.67e-04
0.07582 *
185 -1.54e-02 1.47e-02 0.011049 0.010396 0.02841 1.037 2.03e-04
0.01645
186 2.26e-02 1.58e-03 -0.012204 -0.012922 0.03390 1.025 2.89e-04
0.00750
187 1.53e-02 -2.77e-02 0.020912 -0.014033 0.04238 1.036 4.51e-04
0.01713
188 2.92e-03 7.08e-03 -0.012487 0.005095 0.02527 1.026 1.60e-04
0.00734
189 -2.79e-02 2.45e-02 0.099888 -0.007776 0.13080 1.012 4.27e-03
0.01464
190 -2.31e-03 1.19e-02 -0.000326 -0.002549 -0.01581 1.033 6.28e-05
0.01196
191 -7.29e-02 -2.61e-02 0.072042 0.033489 -0.11989 1.004 3.59e-03
0.01068
192 -3.98e-02 6.36e-02 -0.036081 0.031150 -0.09417 1.024 2.22e-03
0.01487
193 9.35e-03 -2.76e-03 -0.003433 -0.010068 -0.01742 1.030 7.62e-05
0.00958
194 -4.73e-03 4.22e-03 0.011991 -0.006051 -0.02565 1.026 1.65e-04
0.00695
195 1.11e-03 -1.62e-05 -0.000258 -0.001204 -0.00153 1.040 5.86e-07
0.01837
196 4.23e-04 -2.41e-02 0.008180 -0.005901 -0.04672 1.022 5.48e-04
0.00716
197 1.04e-01 8.38e-02 -0.127903 -0.047769 0.20005 0.979 9.92e-03
0.01422
198 2.02e-02 1.57e-02 0.025375 -0.028588 0.05704 1.023 8.16e-04
0.00930
199 -2.78e-03 1.06e-02 -0.009404 0.005674 0.01544 1.062 5.99e-05
0.03885 *
200 1.66e-01 -4.64e-02 0.142132 -0.259070 -0.31985 0.954 2.51e-02
0.02254

```

Some more plots looking for outliers.

```

``` r
car::outlierTest(model.3)
```


```

## No Studentized residuals with Bonferroni p < 0.05
## Largest |rstudent|:
##      rstudent unadjusted p-value Bonferroni p
## 169 3.163622          0.0018077        0.36154
```
``` r
qqPlot(model.3, main="QQ Plot")
```

```


```

```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-18-1.png)

## [1] 164 169

``` r
leveragePlots(model.3)
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-18-2.png)

#### Cook's distance plot

This site is the worst to navigate, but they have some plots and tests
the others don't.
https://stats.idre.ucla.edu/wp-content/
uploads/2019/02/R\_reg\_part2.html#\(4\)

``` r
https://stats.idre.ucla.edu/wp-content/uploads/2019/02/R_reg_part2.html#(4)
plot(model.3, which = 4, cook.levels = cutoff)
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-19-1.png)

``` r
influencePlot(model.3, main="Influence Plot", sub="Circle size proportional to
Cook's Distance")
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-19-2.png)

## #>      StudRes      Hat      CookD
## #> 1    2.1985963 0.04719053 0.0587038821
## #> 138 -0.4765186 0.09871169 0.0062419532
## #> 164 -2.6695844 0.03934866 0.0707658818
## #> 169  3.1636219 0.02082115 0.0508669997
## #> 181  0.1411258 0.09259768 0.0005106578

``` r
infIndexPlot(model.3)
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-19-3.png)

``` r
standardized residuals with lines at cutoff points
res.std <- rstandard(model.3)
plot(res.std, ylab = "Standardized Residual", ylim=c(-3.5, 3.5))
abline(h = c(-3, 0, 3), lty=2)
index <- which(res.std > 3 | res.std < -3)

```

```

text(index-20, res.std[index], labels = dataset$id[index])
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-19-4.png)

``` r
print(index)
```

## 169
## 169

Nice plot to identify high leverage values.

``` r
#a vector containing the diagonal of the 'hat' matrix
h <- influence(model.3)$hat
#half normal plot of leverage from package faraway
halfnorm(influence(model.3)$hat, ylab = "leverage")
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-20-1.png)

#### Added variable plots

May be helpful in identifying influential cases.

``` r
https://web.stanford.edu/class/stats191/notebooks/Diagnostics_for_multiple_regression.html
If the partial regression relationship is linear, this plot should look
linear.
avPlots(model.3, 'Adverts')
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-21-1.png)

``` r
avPlots(model.3, 'Airplay')
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-21-2.png)

``` r
avPlots(model.3, 'Image')
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-21-3.png)

#### Component + residual plots

```

Similar to the Added Variable plot, but may be more helpful in identifying non-linear relationships.

```
``` r
https://web.stanford.edu/class/stats191/notebooks/
Diagnostics_for_multiple_regression.html
The green line is a non-parametric smooth of the scatter plot that may
suggest
relationship other than linear.
crPlots(model.3, 'Adverts')
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-22-1.png)
```

```
``` r
crPlots(model.3, 'Airplay')
```
```

```
! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-22-2.png)
```

```
``` r
crPlots(model.3, "Image")
```
```

```
! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-22-3.png)
```

Evaluating generalizability of the model

This is based on how well the model meets statistical assumptions.

Additivity and linearity

Check scatterplots for linear relationships of predictors with outcome.

```
``` r
no curve in the graph
first plot - plot fitted values against residuals
plot(model.3)
```
```

```
! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-23-1.png) ! [] (Regression-Diagnostics-Assignment_files/figure-
markdown_github/unnamed-chunk-23-2.png) ! [] (Regression-Diagnostics-
Assignment_files/figure-markdown_github/unnamed-chunk-23-3.png) ! [] (Regression-
Diagnostics-Assignment_files/figure-markdown_github/unnamed-chunk-23-4.png)
```

Residual plots can help check for linearity. From
[https://stats.idre.ucla.edu/wp-content/
uploads/2019/02/R_reg_part2.html#\(6\)](https://stats.idre.ucla.edu/wp-content/uploads/2019/02/R_reg_part2.html#(6))

```
``` r
```

```

car::residualPlots(model.3)
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-24-1.png)

##           Test stat Pr(>|Test stat|)
## Adverts      -0.1368    0.8913
## Airplay       0.9238    0.3567
## Image        -1.1047    0.2706
## Tukey test   -0.1142    0.9091

#### Independent errors

``` r
value less than 1 or greater than 3 problematic
close to 2 is best
durbinWatsonTest(model.3)
```

##   lag Autocorrelation D-W Statistic p-value
##     1      0.0026951    1.949819    0.734
## Alternative hypothesis: rho != 0

``` r
dwt(model.3)
```

##   lag Autocorrelation D-W Statistic p-value
##     1      0.0026951    1.949819    0.726
## Alternative hypothesis: rho != 0

You can also plot residuals with ID From
<a href="https://stats.idre.ucla.edu/wp-content/uploads/2019/02/
R_reg_part2.html#(7)" class="uri">https://stats.idre.ucla.edu/wp-content/
uploads/2019/02/R_reg_part2.html#(7)</a>

``` r
plot(model.3$resid ~ dataset$id)
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-26-1.png)

#### Homoscedasticity

``` r
no funneling
first plot - plot fitted values against residuals
plot(model.3)
```

```

```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-27-1.png) ! [] (Regression-Diagnostics-Assignment_files/figure-
markdown_github/unnamed-chunk-27-2.png) ! [] (Regression-Diagnostics-
Assignment_files/figure-markdown_github/unnamed-chunk-27-3.png) ! [] (Regression-
Diagnostics-Assignment_files/figure-markdown_github/unnamed-chunk-27-4.png)

``` r
Breusch-Pagan test https://daviddalpiaz.github.io/appliedstats/model-diagnostics.html
bptest(model.3)
```

##
## studentized Breusch-Pagan test
##
## data: model.3
## BP = 6.1906, df = 3, p-value = 0.1027

#### Normally distributed errors

``` r
Q-Q plot of residuals for normality
second plot - points follow the line
plot(model.3)
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-28-1.png) ! [] (Regression-Diagnostics-Assignment_files/figure-
markdown_github/unnamed-chunk-28-2.png) ! [] (Regression-Diagnostics-
Assignment_files/figure-markdown_github/unnamed-chunk-28-3.png) ! [] (Regression-
Diagnostics-Assignment_files/figure-markdown_github/unnamed-chunk-28-4.png)

``` r
can also plot a histogram of the standardized residuals
hist(dataset$m3studres)
```

! [] (Regression-Diagnostics-Assignment_files/figure-markdown_github/unnamed-
chunk-28-5.png)

``` r
Shapiro-Wilk test on the residuals
https://daviddalpiaz.github.io/appliedstats/model-diagnostics.html
shapiro.test(resid(model.3))
```

##
## Shapiro-Wilk normality test
##
## data: resid(model.3)
## W = 0.99483, p-value = 0.7253

#### Predictors uncorrelated with external variables (that should have been
included)

Variables which correlate with both the predictor and outcome that are

```

left out of the model will cause conclusions from the model to be potentially less valid.

Correct variable types

All variables should be quantitative. Categorical variables should only have 2 categories. The outcome should be continuous with no ceiling or floor effects.

No perfect multicollinearity

You can check pair-wise correlations between predictors before fitting the model. Correlations above .8 could be problematic.

```
``` r
largest VIF greater than 10 problematic
Average VIF greater than 1 regression may be biased
tolerance less than .1 a serious problem
tolerance less than .2 potential problems
vif(model.3)
````
```

```
##   Adverts   Airplay     Image
## 1.014593 1.042504 1.038455
```

```
``` r
1/vif(model.3)
````
```

```
##   Adverts   Airplay     Image
## 0.9856172 0.9592287 0.9629695
```

```
``` r
mean(vif(model.3))
````
```

```
## [1] 1.03185
```

Non-zero variance

That can be checked with descriptive statistics before the model is fit.

Cross-validation

How well does the model predict outcome from a different sample?

Save new data

Save the dataset with your new regression diagnostics variables.

```
``` r
can use with .sav, .rds, .csv files and more
see documentation for options https://www.rdocumentation.org/packages/rio/
versions/0.5.16
rio::export(dataset, "Album Sales Diagnostics.sav")
```

~ ~ ~

Turn in your dataset with your new influence diagnostics variables with your other assignment files.