11. $H_0: \xi \sim p_0(x) = \begin{cases} 1, x \in (0,1) \\ 0, x \notin (0,1) \end{cases}$

$H_1: \xi \sim p_1(x) = \begin{cases} \frac{e}{e-1} e^{-x}, x \in (0,1) \\ 0, x \notin (0,1) \end{cases}$

a) $n=1$ α

$$l = \frac{L_1}{L_0} = \frac{\frac{e}{e-1} e^{-x}}{1} \geq C$$

$$e^{-x} \geq B \Rightarrow x \leq A$$

$$P(x \leq A \mid H_0) = \alpha$$

$$\int_0^A 1dx = A = \alpha \quad \Rightarrow \quad G: x \le \alpha$$

$$\alpha_1 = \alpha$$

$$W = P(x \le A | H_1) = \int_0^\alpha \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}(1-e^{-\alpha})$$

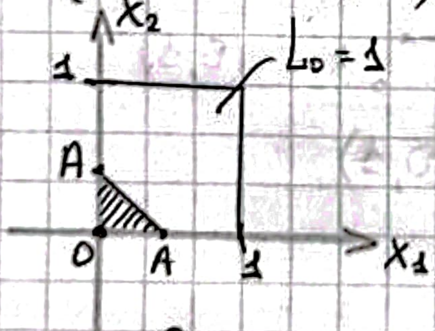
$$\alpha_2 = 1 - W = \frac{e-1-e(1-e^{-\alpha})}{e-1} = \frac{e^{-\alpha}-1}{e-1}$$

b) $n=2 \quad \alpha$

$$l = \frac{L_1}{L_0} = \frac{\left(\frac{e}{e-1}\right)^2 e^{-x_1} \cdot e^{-x_2}}{1 \cdot 1} \ge C$$

$$e^{-(x_1+x_2)} \ge B \qquad x_1 + x_2 \le A$$

$$P(x_1 + x_2 \le A | H_0) = \alpha$$



$$\frac{A^2}{2} = \alpha$$

$$A = \sqrt{2\alpha}$$

$$G: x_1 + x_2 \le \sqrt{2\alpha}$$

$$\alpha_1 = \alpha$$

$$W = \iint\limits_{x_1+x_2 \le \sqrt{2\alpha}} \left(\frac{e}{e-1}\right)^2 e^{-x_1} \cdot e^{-x_2} dx_1 dx_2 =$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1} \left(\int_0^{A-x_1} e^{-x_2} dx_2 \right) dx_1 = \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1} (1 - e^{x_1 - A}) dx_1 =$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A (e^{-x_1} - e^{-A}) dx_1 = \left(\frac{e}{e-1}\right)^2 \cdot (1 - e^{-A} - A e^{-A}), \; A = \sqrt{2\alpha}$$

$$\alpha_2 = 1 - W$$

c)

$$l = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \geq C$$

$$\ln l = \sum_{i=1}^{n} \underbrace{\ln\left(\frac{p_1}{p_0}\right)}_{\eta_i} \geq \ln C$$

$$\frac{\sum \eta_i - nM[\eta_i]}{\sqrt{nD[\eta_i]}} \sim N(0,1)$$

$$P(\ln l \geq \ln C \mid H_0) = \alpha$$

$$\eta_i = \ln\left(\frac{e}{e-1} e^{-x_i}\right) = \ln\left(\frac{e}{e-1}\right) - x_i$$

$$\ln l = n \ln\left(\frac{e}{e-1}\right) - \sum_{i=1}^{n} x_i \geq \ln C$$

$$G: \sum_{i=1}^{n} x_i \leq A$$

$$P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \leq \frac{A - nMx}{\sqrt{nDx}} \Big| H_0\right) = \alpha$$

$$Mx = \frac{1}{2} \qquad Dx = \frac{1}{12}$$

$$\frac{A - \frac{n}{2}}{\sqrt{\frac{n}{12}}} = u_\alpha$$

$$A = \frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$G: \sum_{i=1}^{n} x_i \le \frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$\alpha_1 = \alpha$$

$$W = P\left(\sum_{i=1}^{n} x_i \le A \mid H_1\right) = P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \le \frac{A - nMx}{\sqrt{nDx}} \mid H_1\right)$$

$$Mx = \int_0^1 \frac{e}{e-1} e^{-x} x\,dx = \frac{-exe^{-x}}{e-1}\Big|_0^1 + \int_0^1 \cancel{\frac{e}{e-1}} e^{-x} dx =$$

$$= \frac{-1}{e-1} + 1 = \frac{e-2}{e-1}$$

$$Dx = Mx^2 - M^2x$$

$$Mx^2 = \int_0^1 \frac{e}{e-1} e^{-x} x^2 dx = \frac{-ex^2e^{-x}}{e-1}\Big|_0^1 + 2\int_0^1 \frac{e}{e-1} e^{-x} x\,dx =$$

$$= \frac{-1}{e-1} + 2\,\frac{e-2}{e-1} = \frac{2e-5}{e-1}$$

$$Dx = \frac{2e-5}{e-1} - \left(\frac{e-2}{e-1}\right)^2 = \frac{2e^2-7e+5-e^2+4e-4}{(e-1)^2} =$$

$$= \frac{e^2-3e+1}{(e-1)^2}$$

$$W = \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \xrightarrow[n\to\infty]{?} 1$$

$$B = \frac{\frac{n}{2} + u_{\alpha}\sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n \frac{e^2-3e+1}{(e-1)^2}}} = \sqrt{n}\frac{\left(\frac{1}{2} - \frac{e-2}{e-1}\right)}{C_1} + C_2,$$

$C_1 > 0$

$$\frac{1}{2} \vee \frac{e-2}{e-1}$$

$$e-1 \vee 2e-4$$

$$3 > e \Rightarrow \frac{1}{2} - \frac{e-2}{e-1} > 0$$

$B \to +\infty,\ n \to +\infty \Rightarrow W \xrightarrow[n\to\infty]{} 1 \Rightarrow$

$\Rightarrow$ Критерий состоятелен

d) $G: X_{min} < C$ $\qquad H_0: \xi \sim R(0,1)$

$$P(X_{min} < C | H_0) = \alpha$$

$$\xi \sim F(x) \Rightarrow \xi_{min} \sim 1 - (1 - F(x))^n$$

$$P(X_{min} < C | H_0) = 1 - (1 - C)^n = \alpha$$

$$(1-C)^n = 1 - \alpha$$

$$C = 1 - \sqrt[n]{1-\alpha}$$

$\alpha_1 = \alpha$

$$W = P(X_{min} < C | H_1) = \Big\} H_1: \xi \sim \frac{e}{e-1} e^{-x} \{[0,1]\}$$

$$F_1(x) = \frac{e}{e-1}\left(1 - e^{-x}\right)$$

$$= 1 - \left(1 - \frac{e}{e-1}\left(1 - e^{-c}\right)\right)^n$$

$$\alpha_2 = 1 - W$$

$$W \xrightarrow[n\to\infty]{?} 1$$

$$c = 1 - (1-\alpha)^{1/n} = 1 - 1 - \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)$$

$$e^{-c} = e^{\frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)} = 1 + \frac{1}{n}\ln(1-\alpha) + o\left(\frac{1}{n}\right)$$

$$W = 1 - \left(1 + \frac{1}{n}\cdot\frac{e}{e-1}\ln(1-\alpha) + o\left(\frac{1}{n}\right)\right)^n \xrightarrow[n\to\infty]{} 1 - e^{\frac{e\ln(1-\alpha)}{e-1}} =$$

$$= 1 - (1-\alpha)^{\frac{e}{e-1}} \neq 1$$

Критерий не состоятелен