

IIT HYDERABAD

Arithmetic Progression Problem

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Question 10.5.2-9: If the 3rd and the 9th terms of an AP are 4 and -8, respectively, which term of this AP is zero?

Solution:

TABLE I: Input Parameters

Parameter	Value	Description
T_3	4	Third term of the AP
T_9	-8	Ninth term of the AP
$x(n)$	$x(0) + nd$	n^{th} term of the AP

From the values given in Table ??:

$$x(0) + 2d = 4 \quad (1)$$

$$x(0) + 8d = -8 \quad (2)$$

On subtracting equation 1 from equation 2 :

$$6d = -12 \quad (3)$$

$$d = -2 \quad (4)$$

Substitute $d = -2$ into:

$$x(0) = 4 - 2d \quad (5)$$

$$x(0) = 4 - 2(-2) = 8 \quad (6)$$

Substitute $x(0) = 8$ and $d = -2$ into:

$$T_n = x(0) + (n - 1)d = 0 \quad (7)$$

$$8 + (n - 1)(-2) = 0 \quad (8)$$

$$n - 1 = 4 \quad (9)$$

$$n = 5 \quad (10)$$

The term where the value is zero in the given arithmetic progression is the 5th term.

1) Finding $x(n)$

The series is an arithmetic progression.

$$x(n) = (x(0) + nd)(u(n)) \quad (11)$$

as $x(n) = 0 \quad \forall \quad n < 0$.

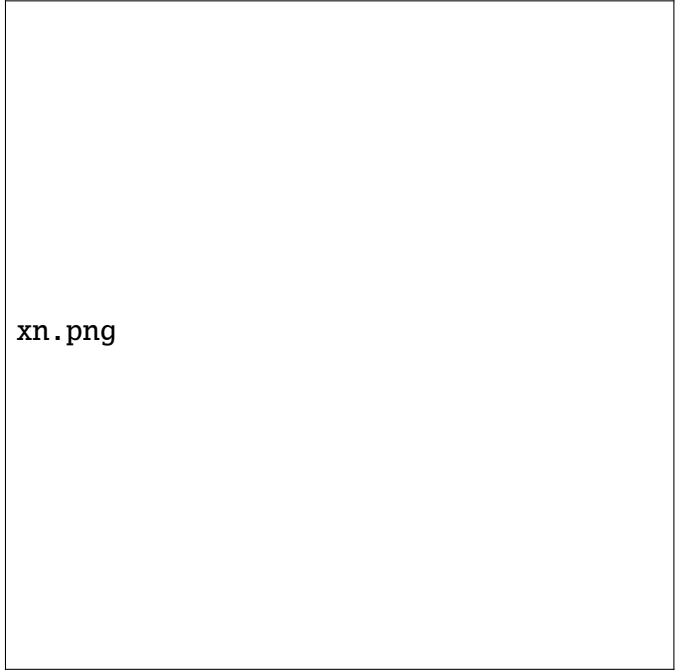


Fig. 1: Plot of $x(n)$ vs n ; Refer to Table ?? for values of $x(0)$ and d

2) Z-transform of $x(n)$

Let Z-transform of $x(n)$ be $X(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd)(u(n))(z^{-n}) \quad (12)$$

$$= (x(0))(U(z)) + d \sum_{n=0}^{\infty} nz^{-n} \quad (13)$$

$$= (x(0))(U(z)) + d \left(\frac{z^{-1}}{(1 - z^{-1})^2} \right) \quad (14)$$

$$= (x(0))(U(z)) + d \left(\frac{z}{(z - 1)^2} \right) \quad (15)$$

$$= \frac{x(0)(z)}{z - 1} + \frac{dz}{(z - 1)^2} \quad \forall \quad |z| > 1 \quad (16)$$

Using the values from Table ??:

$$X(z) = \frac{8z}{z - 1} + \frac{-2z}{(z - 1)^2} \quad \forall \quad |z| > 1 \quad (17)$$