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Question 10.5.2-9: If the 3rd and the 9th terms of an AP are 4 and -8, respectively, which term of this AP is zero?

Solution:

TABLE I: Input Parameters

Parameter	Value	Description
$x(n)$	$x(0) + (n)d$	$(n+1)^{th}$ term of the AP
$x(0) + 2d$	4	Third term of the AP
$x(0) + 8d$	-8	Ninth term of the AP
$x(0)$	-	First term of the AP
d	-	Common difference of the AP

$$\begin{pmatrix} x(0) \\ 8d \\ -8 \end{pmatrix} - \begin{pmatrix} x(0) \\ 2d \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 6d \\ -12 \end{pmatrix} \tag{1}$$

$$6d = -12 \tag{2}$$

$$\implies d = -2 \tag{3}$$

Substitute d = -2 into:

$$x(0) = 4 - 2d \tag{4}$$

$$= 4 - 2(-2) \tag{5}$$

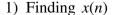$$= 8 \tag{6}$$

Substitute x(0) = 8 and d = -2 into:

$$x(n) = x(0) + (n)d = 0 \tag{7}$$

$$8 + (n)(-2) = 0 \tag{8}$$

$$n = 4 \tag{9}$$

Term number = n + 1 = 5
The term where the value is zero in the given arithmetic progression is the 5th term.

1) Finding $x(n)$
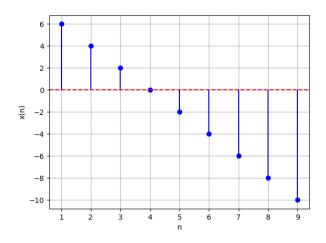   The series is an arithmetic progression.

$$x(n) = (x(0) + n(-2))(u(n)) \tag{10}$$

2) Z-transform of $x(n)$

$$\frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \tag{11}$$

Using the values from Table **??**:

$$\frac{8}{1 - z^{-1}} + \frac{-2z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \tag{12}$$

Fig. 1: Plot of $x(n)$ vs n