IIT HYDERABAD
Arithmetic Progression Problem

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Question 10.5.2-9: If the 3rd and the 9th terms of an AP are 4 and -8, respectively, which term of this AP is zero?

Solution:

TABLE I: Input Parameters

Parameter	Value	Description
$x(2)$	4	Third term of the AP
$x(8)$	-8	Ninth term of the AP
$x(n)$	$x(0)+(n)d$	$(n+1)^{th}$ term of the AP

From the values given in Table **??**:

$$x(0) + 2d = 4 \tag{1}$$
$$x(0) + 8d = -8 \tag{2}$$

Subtracting equation 1 from equation 2 :

$$\begin{pmatrix} x(0) & -x(0) \\ 8d & -2d \\ -8 & -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 6d \\ -12 \end{pmatrix} \tag{3}$$

$$6d = -12 \tag{4}$$
$$d = -2 \tag{5}$$

Substitute d = -2 into:

$$x(0) = 4 - 2d \tag{6}$$
$$x(0) = 4 - 2(-2) = 8 \tag{7}$$

Substitute x(0) = 8 and d = -2 into:

$$x(n) = x(0) + (n)d = 0 \tag{8}$$
$$8 + (n)(-2) = 0 \tag{9}$$
$$n = 4 \tag{10}$$

Term number = n + 1 = 5
The term where the value is zero in the given arithmetic progression is the 5th term.

1) Finding $x(n)$
The series is an arithmetic progression.

$$x(n) = (x(0) + nd)(u(n)) \tag{11}$$

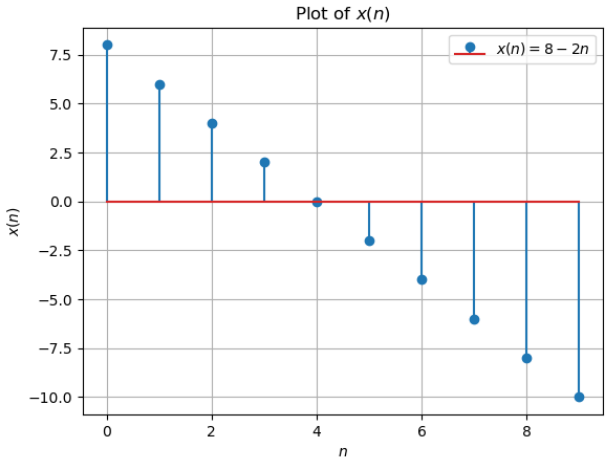

Fig. 1: Plot of $x(n)$ vs n; Refer to Table **??** for values of $x(0)$ and d

as $x(n) = 0 \quad \forall \quad n < 0$.

2) Z-transform of $x(n)$
Let Z-transform of $x(n)$ be $X(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

$$X(z) = \sum_{n=-\infty}^{\infty} (x(0) + nd)(u(n))(z^{-n}) \tag{12}$$
$$= (x(0))(U(z)) + d\sum_{n=0}^{\infty} nz^{-n} \tag{13}$$
$$= (x(0))(U(z)) + d\left(\frac{z^{-1}}{(1-z^{-1})^2}\right) \tag{14}$$
$$= (x(0))(U(z)) + d\left(\frac{z}{(z-1)^2}\right) \tag{15}$$
$$= \frac{x(0)(z)}{z-1} + \frac{dz}{(z-1)^2} \quad \forall \quad |z| > 1 \tag{16}$$

Using the values from Table **??**:

$$X(z) = \frac{8z}{z-1} + \frac{-2z}{(z-1)^2} \quad \forall \quad |z| > 1 \tag{17}$$