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Portfolio value-at-risk with two-sided Weibull distribution: Evidence from cryptocurrency markets

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ABSTRACT

This paper extends the univariate two-sided Weibull distribution to a multivariate case for portfolio-value-at-risk estimation. This method allows to capture the stylized facts of the time series of cryptocurrencies, such as extreme volatility, volatility clustering, very heavy tails, and skewness. This new portfolio risk model is applied to a cryptocurrency portfolio consisting of four major coins: Bitcoin, Litecoin, Ripple, and Dash. The predictive performance of the proposed model is compared with several widely used models. We find that the portfolio value-at-risk with two-sided Weibull distribution outperforms the other models.

1. Introduction

Cryptocurrencies have received great attention due to sharp increase in their prices as well as trading volume in recent years. Bitcoin is the first cryptocurrency introduced by Nakamoto (2008), and there are over 3000 coins traded in https://coinmarketcap.com/. Cryptocurrencies were created as an alternative global currency with low transaction costs; however, it is widely accepted as speculative assets. Bouoiyour and Selmi (2015) find that Bitcoin is not a safe haven due to its high uncertainty. Brauneis and Mestel (2018) selected ten largest cryptocurrencies and checked their predictabilities with liquidity constrains. They find that cryptocurrencies are less predictable and inefficient when there is an increasing trend in liquidity. On the other hand, there are studies claiming that Bitcoin can be used as a hedging asset. Dyhrberg (2016) finds that Bitcoin can be hedged against global uncertainty. Guesmi et al. (2019), Kajtazi and Moro (2019), and Akhtaruzzaman et al. (2019) examine the role of Bitcoin in risk reduction for portfolio investments, and they find that Bitcoin can reduce portfolio risk for global portfolios.

Several studies used multivariate GARCH (MGARCH) type models to investigate volatility dynamics of cryptocurrencies. Katsiampa (2019) used diagonal BEKK (Baba et al., 1985) for two cryptocurrency portfolio, Bitcoin and Ether. They find that a time-varying correlation exists between coins, and multivariate student's *t* distribution is superior to the normal distribution. Katsiampa et al. (2019) examine volatility dynamics of three assets portfolio with Bitcoin, Ether, and Litecoin using a BEKK-MGARCH (Engle and Kroner, 1995). They detect bi-directional volatility spillover and suggest using time-varying conditional correlation for portfolio risk analysis. Canh et al. (2019) select seven largest cryptocurrencies and explore volatility dynamics using DCC-MGARCH (Engle, 2002) model. They find that there are volatility spillovers between seven largest cryptocurrencies with very high positive

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correlations. They conclude that these high positive correlations limit portfolio diversification benefits.

In this study, we select four major coins in the cryptocurrency market, namely Bitcoin, Litecoin, Ripple, and Dash and investigate volatility dynamics using the proposed portfolio value-at-risk with the two-sided Weibull distribution and ten other portfolio value-at-risk models. We check the predictive performance of the new model and benchmarked models with actual over expected exceedance ratio, Kupiec (1995) test, Christoffersen (1998) test, and dynamic quantile test (Engle and Manganelli, 2004). The empirical findings reveal that the new method outperforms the other benchmarked methods.

The remainder of the paper is organized as follows: Section 2 presents the two-sided Weibull distribution. Section 3 explains the benchmarked models, and Section 4 shows the backtesting methodology. Section 5 explains the data, Section 6 presents the empirical findings, and Section 7 provides the robustness check with a subsample. Finally, the last section concludes the study.

2. Portfolio value-at-risk with two-sided Weibull distribution

In this section, we present a new approach for risk quantification of portfolios. It is based on a combination of historical simulation (HS) and generalized autoregressive conditional heteroscedasticity (GARCH) model with innovations following a two-sided Weibull distribution proposed by Chen and Gerlach (2013). Our choice of two-sided Weibull distribution is inspired by the extremely volatile price changes of cryptocurrencies. In HS method, the time series of log-returns of the portfolio (X_t) is constructed by using the historical records of the prices of the assets in the portfolio and the selected weights of the assets. Then, GARCH models are fitted to the generated univariate time series. In the existing literature, innovations are generally assumed to be a normal or student's t random variables. In our study, the distribution of innovations is assumed to be a shifted standardized two-sided Weibull (STW) distribution. Chen and Gerlach (2013) proposed to use STW distribution for forecasting financial tail risk and they provided the pdf, cumulative distribution function (cdf) and the inverse cdf (quantile function).

The pdf of an STW random variable $Y \sim STW(\lambda_1,k_1,\lambda_2,k_2)$ is:

$$f_{\mathrm{STW}}(y) = \begin{cases} b_p\left(\dfrac{-b_p y}{\lambda_1}\right)^{k_1-1}\exp\left[-\left(\dfrac{-b_p y}{\lambda_1}\right)^{k_1}\right];\ y<0 \\ b_p\left(\dfrac{b_p y}{\lambda_2}\right)^{k_2-1}\exp\left[-\left(\dfrac{b_p y}{\lambda_2}\right)^{k_2}\right];\ y\geq 0, \end{cases} \tag{1}$$

where $k_1,k_2 > 0$are the shape parameters and $\lambda_1,\lambda_2 > 0$are the scale parameters. The constant b_p is given by

$$b_p^2 = \frac{\lambda_1^3}{k_1}\Gamma\left(1+\frac{2}{k_1}\right)+\frac{\lambda_2^3}{k_2}\Gamma\left(1+\frac{2}{k_2}\right)-\left[-\frac{\lambda_1^2}{k_1}\Gamma\left(1+\frac{1}{k_1}\right)+\frac{\lambda_2^2}{k_2}\Gamma\left(1+\frac{1}{k_2}\right)\right]^2, \tag{2}$$

where $\Gamma(\bullet)$ denotes the gamma function. The condition, that makes the integral of the pdf equal to one is $\lambda_1/k_1+\lambda_2/k_2=1$. Therefore, there are three free parameters, and we can write $Y \sim STW(\lambda_1,k_1,k_2)$where λ_2 is fixed by $\lambda_2 = k_2(1-(\lambda_1/k_1))$. The STW distribution can be symmetric or skewed depending on parameters. The cdf and inverse cdf of$STW(\lambda_1,k_1,k_2)$ are

$$F_{\mathrm{STW}}(y) = \begin{cases} \dfrac{\lambda_1}{k_1}\exp\left[-\left(\dfrac{-b_p y}{\lambda_1}\right)^{k_1}\right];\ y<0 \\ 1-\dfrac{\lambda_2}{k_2}\exp\left[-\left(\dfrac{b_p y}{\lambda_2}\right)^{k_2}\right];\ y\geq 0, \end{cases} \tag{3}$$

$$F_{\mathrm{STW}}^{-1}(\alpha) = \begin{cases} -\dfrac{\lambda_1}{b_p}\left[-\ln\left(\dfrac{k_1}{\lambda_1}\alpha\right)\right]^{\frac{1}{k_1}};0\leq\ \alpha<\dfrac{\lambda_1}{k_1} \\ \dfrac{\lambda_2}{b_p}\left[-\ln\left(\dfrac{k_2}{\lambda_2}(1-\alpha)\right)\right]^{\frac{1}{k_2}};\dfrac{\lambda_1}{k_1}\leq\ \alpha<1. \end{cases} \tag{4}$$

The mean of STW distribution is

$$\mu_{\mathrm{STW}} = \frac{-\lambda_1^2}{b_p k_1}\Gamma\left(1+\frac{1}{k_1}\right)+\frac{\lambda_2^2}{b_p k_2}\Gamma\left(1+\frac{1}{k_2}\right). \tag{5}$$

So, the distribution of $Z = Y-\mu_{\mathrm{STW}}$ is a shifted $STW(\lambda_1,k_1,k_2)$ distribution with mean 0 and variance one. In our model, Innovations are independent and identically distributed (iid) copies of $Z = Y-\mu_{\mathrm{STW}}$, where $Y \sim STW(\lambda_1, k_1, k_2)$.

The conditional one period value-at-risk (VaR) is defined by

$$\alpha = P\left(R_{t+1} < VaR_{\alpha,t+1}\middle|\mathcal{F}_t\right), \tag{6}$$

whereR_{t+1} is the return of portfolio at time t+1, that is $R_{t+1} = e^{X_{t+1}}-1$, and $\mathcal{F}_t$ is the filtration (history) up to time t. Thus, in our setting, we have

Table 1
VaR equations of univariate models

Model	Equation
HS-empirical	$VaR_{\alpha,t+1} = F_n^{-1}(\alpha)$
HS-normal quantile	$VaR_{\alpha,t+1} = \widehat{\mu} + \widehat{\sigma}\Phi^{-1}(\alpha)$
HS-t quantile	$VaR_{\alpha,t+1} = \widehat{\mu} + \widehat{\sigma}t_{\nu}^{-1}(\alpha)$
HS-EWMA	$VaR_{\alpha,t+1} = \widehat{\sigma}_{t+1}\Phi^{-1}(\alpha)$
HS-GARCH-normal	$VaR_{\alpha,t+1} = \exp[\widehat{\sigma}_{t+1}\Phi^{-1}(\alpha)] - 1$
HS-GARCH-t	$VaR_{\alpha,t+1} = \exp[\widehat{\sigma}_{t+1}t_{\nu}^{-1}(\alpha)] - 1$

$$VaR_{\alpha,t+1} = \exp\left(\widehat{\sigma}_{t+1}\left[F_{\mathrm{STW}}^{-1}(\alpha) - \mu_{\mathrm{STW}}\right]\right) - 1, \tag{7}$$

where $\widehat{\sigma}_{t+1}^2$ is the variance forecast from the fitted GARCH (1,1) model.

3. Benchmarked models

In this section, we present the models which are used as benchmarks to compare our new suggested model. There are two classes of models, namely, univariate and multivariate models of which the details are given below.

3.1. Benchmarked univariate models

The univariate models are based on the HS method in which the multivariate time series of the prices of the assets in the portfolio is transformed to a univariate time series of portfolio returns (or log-returns) by using the selected weights of the assets. For a portfolio having m assets, the univariate time series of portfolio returns is given by

$$R_t = \sum_{i=1}^{m} w_{i,t} R_{i,t}, \tag{8}$$

where $w_{i,t}$ is the weight of asset i at time t and $R_{i,t}$ is the return of asset i at time t, i.e $R_{i,t} = (P_{i,t} - P_{i,t-1})/P_{i,t-1}$, where $P_{i,t}$ is the price of asset i at time t. The weights are assumed to be time dependent. $w_{i,t}$ is calculated by using the history up to time t-1. In the benchmarked models which are presented in this section, univariate stochastic models are fitted to the univariate time series of R_t. We selected HS-empirical, HS-normal quantile, HS-t quantile, HS-EWMA, HS-GARCH-normal, and HS-GARCH-t as univariate benchmarked models, and Table 1 shows the VaR equations for these models.

3.2. Benchmarked multivariate models

The multivariate models are based on stochastic modelling of log-returns of individual assets in the portfolio. We selected four multivariate models as variance-covariance method (VC-normal), variance-covariance method with multivariate t distribution (VC-t), multivariate generalization of GARCH model with dynamic conditional correlation (DCC-MGARCH), and multivariate generalization of GARCH model with dynamic conditional correlation with multivariate t distribution (DCC-MGARCH-t).

In VC-normal, the vector of returns of different assets are assumed to follow a multivariate normal distribution. Under multivariate normality assumption, the distribution of portfolio return is normal, $R_t = \sum_{i=1}^{m} w_{i,t}R_{i,t} \sim N(\mu_{p,t},\ \sigma_{p,t}^2)$, where $\mu_{p,t}$ and $\sigma_{p,t}^2$ are the mean and variance of the portfolio return, respectively. The estimates of mean and variance at time t+1 are calculated by using sample mean, sample variance, and sample covariances based on last n observations, and the VaR with VC-normal is therefore given by

$$VaR_{\alpha,t+1} = \widehat{\mu}_{p,t+1} + \widehat{\sigma}_{p,t+1}\Phi^{-1}(\alpha). \tag{9}$$

In the VC-t method, the vectors of returns are assumed to be iid multi-t vectors, $(R_{1,t}, \ldots, R_{m,t}) \sim t(\nu, \mu, \Sigma)$ for all t, where $\nu > 0$ is the degrees of freedom, $\mu \in \Re^m$ is the mean vector and $\Sigma \in \Re^{m \times m}$ is the shape matrix. Like for the multinormal case, the distribution of portfolio return is the same type of distribution of assets in the portfolio. So, $R_t = \sum_{i=1}^{m} w_{i,t}R_{i,t} \sim t(\nu, \mu_{p,t},\ \sigma_{p,t}^2)$, where $\nu > 0$ is the degrees of freedom, $\mu_{p,t}$ is location parameter and $\sigma_{p,t}$is scale parameter of the univariate t distribution. The estimates $\widehat{\nu}$, $\widehat{\mu}_{p,t+1}$, and $\widehat{\sigma}_{p,t+1}$ are obtained by maximum likelihood estimation of the multivariate t distribution parameters ν, μ, and Σ, using last n observations. So, VaR with VC-t is given by

$$VaR_{\alpha,t+1} = \widehat{\mu}_{p,t+1} + \widehat{\sigma}_{p,t+1}t_{\widehat{\nu}}^{-1}(\alpha). \tag{10}$$

The DCC model introduced by Engle (2002) is an extension of constant conditional correlation (CCC) model of Bollersley (1990). In the DCC model, the conditional correlation dynamically changes with time. In the most basic version of the DCC-MGARCH model, the innovations are assumed to follow a multivariate normal distribution. The model estimation consists of three stages. In the first stage, the volatility of asset log-returns is estimated with the univariate GARCH (1,1) model. Then in the second stage, dynamic correlation parameters are estimated by using devolatized data obtained from the fitted GARCH (1,1) models. In the last stage, the remaining parameters are estimated by fitting a multivariate GARCH model. For more details, see pages 174-175 of McNeil et al. (2005). After estimating the needed parameters, the VaR is calculated by using Eq. (9) and variance-covariance matrix coming from the fitted DCC-MGARCH model.

In DCC-MGARCH-t model, the innovations are assumed to follow a multivariate t distribution. Therefore, the VaR is calculated by using Eq. (10) with the quantile of t distribution.

4. Backtesting methodology

The accuracy of the HS-GARCH-Weibull and benchmarked models are checked with four backtesting methods: the actual over expected exceedance ratio (AE), the unconditional coverage, the conditional coverage, and the dynamic quantile tests. Troster et al. (2019) used the same backtesting models to check different GARCH-based models for Bitcoin. The measure of the number-of-violations based method can be calculated as $AE = x/(np^*)$, where x is the number-of-violations, n is the number of observations, and p^* is the nominal exceedance probability. If the analyzed model is correct, in other words, if the nominal probability p^* is close to the true exceedance probability p, then the observed AE should be close to 1.

The unconditional coverage (UC) test of Kupiec (1995) tests the null hypothesis H_0: $p = p^*$ by using the likelihood ratio (LR)

$$LR_{\mathrm{UC}} = -2\ln[(1-p^*)^{n-x}(p^*)^x] + 2\ln\left[\left(1-\frac{x}{n}\right)^{n-x}\left(\frac{x}{n}\right)^x\right], \tag{11}$$

which follows χ_1^2 distribution under null hypothesis. The null hypothesis is rejected, if LR_{UC} is greater than the critical value.

The conditional coverage (CC) test of Christoffersen (1998) uses a LR hypothesis testing method that shows if the VaR models have a correct coverage at each point in time. This test makes it possible to jointly test the randomness and correct coverage while retaining the individual hypotheses as subcomponents. The LR statistic of conditional coverage is given by

$$LR_{\mathrm{CC}} = LR_{\mathrm{UC}} - 2\ln\left[\frac{\left(1-\frac{x}{n}\right)^{n-x}\left(\frac{x}{n}\right)^{x}}{\left(1-\widehat{\pi}_{01}\right)^{T_{00}}\left(\widehat{\pi}_{01}\right)^{T_{01}}}\right] \sim \chi_2^2, \tag{12}$$

where $\widehat{\pi}_{01} = T_{01}/(T_{00} + T_{01})$, T_{01} is the number of days in which violation occurs when no violation occurs on the previous day, and T_{00} is the number of days in which no violation occurs neither on the current day nor the previous day.

The dynamic quantile (DQ) test of Engle and Manganelli (2004) emphasizes the importance of conditioning violation on the VaR for the period t using the history up to time t-1. For this purpose, they use a linear regression model to test the dependence of the current violation on the previous violations. Under null hypothesis, that is the model used for VaR estimation is correct, the regression coefficients should be equal to zero. By using the regression coefficient estimates, they define a test statistic given by

$$DQ = \frac{\widehat{\beta}_{LS}^{\top} X^{\top} X \widehat{\beta}_{LS}}{(p^*)(1-p^*)} \sim \chi_{q+2}^2, \tag{13}$$

where $\widehat{\beta}_{LS}$ is the vector of regression coefficient estimates, X is the matrix of regressors, and q is the number of regressors.

5. Data

Liu et al. (2019) showed that cryptocurrency portfolio diversification can significantly improve portfolio return. In this study, we select four major cryptocurrencies for multivariate portfolio estimation: Bitcoin (BTC), Litecoin (LTC), Ripple (XRP), and Dash coin (DASH). Platanakis et al. (2018) selected these four coins for portfolio optimization since they are the most liquid digital currencies. Besides, these four coins are selected in our study since they have the highest market capitalizations at the beginning of our study period. The dataset, which is taken from coinmarketcap.com, covers 2070 daily prices of all four coins from April 1, 2014 to November 30, 2019. The portfolio weights are calculated by using the first 365 daily returns of coins. Then, the first 365 of 1704 daily portfolio returns are used for estimating the starting parameters as in-samples of forecasting, and the last 1339 daily portfolio returns are used for out-of-sample forecasting. To determine the portfolio weights, we used variance minimizing weights of Markowitz (1952). The weight estimates are dynamically updated each day, based on the updated variance-covariance matrix estimates. The motivation of using dynamically updated portfolio weights is the fact that constant correlation and volatility assumptions are not fulfilled. We update the portfolio weights on a daily base so that the minimum portfolio variance contains a lot of noise (Plerou et al., 2002). Thus, we use random matrix approach to clear correlation matrix in line with Laloux et al. (1999), Plerou et al. (2002) and Eterovic and Eterovic (2013).

Hale et al. (2018) noted that there was no derivatives market for cryptocurrencies before 2018, and short selling wasn't available. Besides, all cryptocurrencies were not liquid in early dates. Therefore, we evaluate the predictive performance of the models with a subsample of March 1, 2018- November 30, 2019 as a robustness check.

Figure 1 shows the plot of daily returns and histogram of the portfolio returns. It can be observed from the time series plot and the

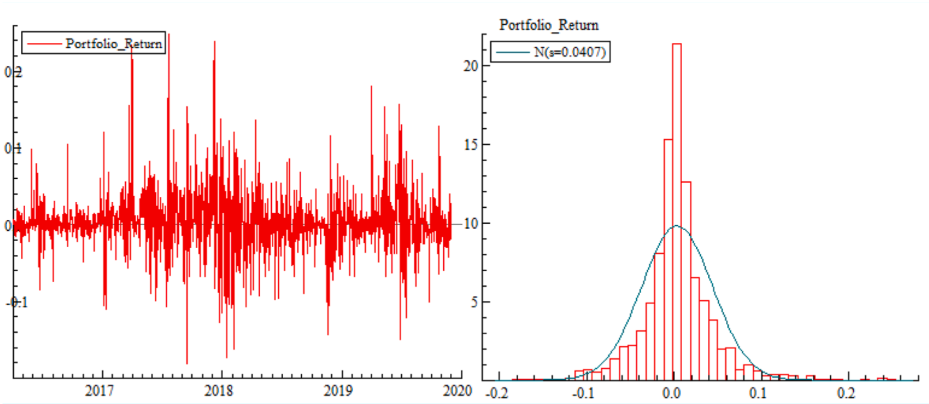

Fig. 1. Daily returns and histogram of the portfolio returns.

Table 2
Descriptive statistics, diagnostic checks, and dynamic correlation tests

Descriptive statistics		*Box-Pierce tests on squared returns**	
Mean	0.0033	Q^2 [2]	98.547 (0.000)
Standard Deviation	0.0382	Q^2 [5]	192.860 (0.000)
Skewness	0.5928	Q^2 [10]	254.760 (0.000)
Excess Kurtosis	6.2645		
Jarque-Bera test	2896.0 (0.000)		
*ARCH tests**		*Dynamic correlation tests*,***	
LM [2]	85.824 (0.000)	E-S [2]	10.711 (0.013)
LM [5]	132.460 (0.000)	E-S [5]	21.142 (0.002)
LM [10]	142.520 (0.000)	E-S [10]	59.850 (0.000)

Numbers in brackets are p-values.
*Lag-lengths are shown in square brackets, **H_0: Constant correlation, H_1: Dynamic correlation.
Note: The calculations are performed by statistical software R (R Core Team, 2019).

histogram that the distribution of the daily returns has a very high variance. In fact, we observe many extreme returns higher than +10% or less than -10%. It can also be observed that the distribution has a very high kurtosis and much heavier tails than normal distribution.

Table 2 shows the descriptive statistics and diagnostic tests for the portfolio returns and dynamic correlation tests for the coin returns. The non-normality of the portfolio returns is confirmed by the skewness, excess kurtosis, and Jarque-Bera test (Jarque and Bera, 1980). This fact suggests using volatility models with non-normal distributions. Also, the heteroscedasticity of portfolio returns is shown by autoregressive conditional heteroscedasticity (ARCH) test of Engle (1982). Furthermore, the Box-Pierce test on squared series indicates that there is a significant serial autocorrelation, namely there exists a volatility persistency. Moreover, constancy of correlations between the coin returns is tested by the dynamic correlation test of Engle and Sheppard (2001). According to this test, the constant correlation hypothesis is rejected at the 5% significance level.

6. Empirical findings

In this section, empirical results are reported for the comparison of the newly suggested HS-GARCH-Weibull method with the existing VaR estimation methods which are HS-empirical, HS-normal quantile, HS-t quantile, HS-EWMA, HS-GARCH-normal, HS-GARCH-t, VC-normal, VC-t, DCC-MGARCH-normal, and DCC-MGARCH-t. We backtested the VaR methods to evaluate the predictive performance of the analyzed methods for one-day ahead forecasts with %5 significance level. The numerical results are shown in Table 3, and comparison of out-of-sample of forecasting graph is shown in Figure 2. According to AE criteria and unconditional coverage test of Kupiec (1995) tests, the best methods are found to be HS-GARCH-Weibull and VC-normal methods, since its AE is closer to one than the other analyzed models and p-value is close to one for Kupiec test. The third-best performance is achieved by the VC-t for both of the backtesting criteria. Moreover, conditional coverage test of Christoffersen (1998) were adopted to estimate tail-loss

Table 3
Backtesting of VaR methods.

Method	AE	R*	LR-UC	R*	LR-CC	R*	DQ	R*
HS-empirical	1.195	7	2.527 (0.112)	7	5.969 (0.051)	7	39.959 (0.000)	10
HS-normal	0.866	4	1.316 (0.251)	4	5.240 (0.073)	5	13.032 (0.011)	6
HS-t	1.195	7	2.527 (0.112)	7	7.607 (0.022)	9	42.202 (0.000)	11
HS-EWMA	0.851	5	1.636 (0.201)	5	1.723 (0.423)	2	5.454 (0.244)	3
HS-GARCH-normal	1.285	9	5.256 (0.022)	9	6.385 (0.041)	8	8.046 (0.090)	4
HS-GARCH-t	0.478	10	23.606 (0.000)	10	23.675 (0.000)	10	19.859 (0.001)	7
VC-normal	0.986	1	0.014 (0.905)	1	5.665 (0.059)	6	12.096 (0.017)	5
VC-t	1.075	3	0.392 (0.531)	3	4.266 (0.119)	4	21.671 (0.000)	8
DCC-MGARCH-normal	0.836	6	1.991 (0.158)	6	2.177 (0.337)	3	2.260 (0.688)	1
DCC-MGARCH-t	0.284	11	49.824 (0.000)	11	50.371 (0.000)	11	36.597 (0.000)	9
HS-GARCH-Weibull	0.986	1	0.014 (0.905)	1	0.917 (0.632)	1	3.747 (0.441)	2

Numbers in brackets are p-values, * Ranking of the VaR models.
Note: The calculations are performed by statistical software R (R Core Team, 2019).

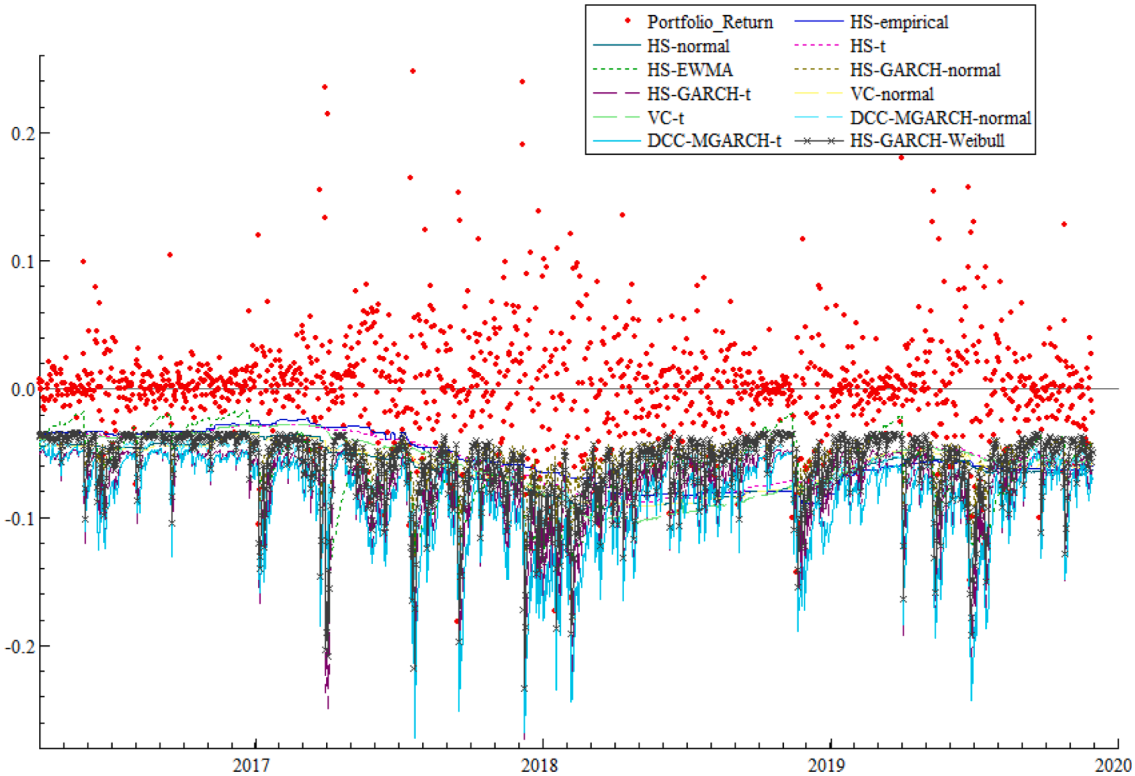

Fig. 2. Comparison of Portfolio VaRs at α=0.05.

performance of the portfolio. HS-GARCH-Weibull is found to be the best method. The second- and the third-best methods are HS-EWMA and DCC-MGARCH-normal, respectively. On the other hand, under DQ test, the best forecasting performance is achieved by DCC-MGARCH-normal method, our HS-GARCH-Weibull method has the second-best performance, and HS-EWMA has the third-best performance. This shows that the proposed HS-GARCH-Weibull method is the best method according to three out of four

Table 4
Backtesting of VaR methods with sub-sample period.

Method	AE	R*	LR-UC	R*	LR-CC	R*	DQ	R*
HS-empirical	0.813	5	1.262 (0.261)	5	1.999 (0.368)	4	13.418 (0.009)	6
HS-normal	0.719	8	2.942 (0.086)	8	6.771 (0.034)	9	20.473 (0.000)	11
HS-t	0.906	2	0.305 (0.581)	2	2.095 (0.351)	5	17.045 (0.002)	8
HS-EWMA	0.875	3	0.549 (0.459)	3	0.597 (0.742)	2	9.454 (0.051)	4
HS-GARCH-normal	1.406	9	4.963 (0.026)	9	6.030 (0.049)	8	7.321 (0.120)	3
HS-GARCH-t	0.531	10	8.861 (0.003)	10	9.791 (0.007)	10	9.465 (0.050)	5
VC-normal	0.844	4	0.867 (0.352)	4	3.229 (0.199)	6	16.731 (0.002)	7
VC-t	0.781	7	1.737 (0.188)	7	4.775 (0.092)	7	19.528 (0.001)	10
DCC-MGARCH-normal	0.813	5	1.262 (0.261)	5	1.265 (0.531)	3	4.817 (0.307)	2
DCC-MGARCH-t	0.281	11	24.026 (0.000)	11	24.283 (0.000)	11	19.377 (0.001)	9
HS-GARCH-Weibull	1.063	1	0.129 (0.719)	1	0.151 (0.927)	1	2.266 (0.687)	1

Numbers in brackets are p-values, * Ranking of the VaR models.
Note: The calculations are performed by statistical software R (R Core Team, 2019).

backtesting criteria.

7. Robustness Check

In this section, we check the predictive performance of the proposed and benchmarked models with a recent subsample period for a robustness check. Table 4 shows similar results for our proposed volatility model with the findings in Section 6. Indeed, HS-GARCH-Weibull method outperforms all other models in all of the backtesting critera. This suggests that our new portfolio value-at-risk model with two-sided Weibull distribution captures well the stylized volatility dynamics of cryptocurrencies in recent periods.

8. Conclusion

In this note, we propose the two-sided Weibull distribution for portfolio value-at-risk estimation by extending Chen and Gerlach's (2013) univariate approach. This method allows to capture the stylized facts of the time series of cryptocurrencies, such as extreme volatility, volatility clustering, very heavy tails, and skewness, which is not the case for other models in the literature. The new method is adopted to portfolio returns generated by historical simulation under time varying minimum variance portfolio weights. The predictive performance of the new method is compared with the several widely used existing methods. The empirical findings reveal that the new method outperforms all other benchmarked methods according to three out of four backtesting criteria.

The promising results obtained in this study show that the new method can be utilized by financial institutions and investors for value-at-risk estimation of portfolios containing highly volatile assets such as cryptocurrencies. A possible future research topic is the application of the new method to portfolios of assets from different markets such as commodity, foreign exchange, and derivative markets.

Authors' contribution

Baykar Silahli: Conceptualization, Writing – original draft, undertook data preparation, methodology, empirical applications.
Kemal Dincer Dingec: Conceptualization, Writing – original draft, undertook data preparation, methodology, empirical applications.
Atilla Cifter: Conceptualization, Writing – original draft, undertook data preparation, methodology, empirical applications.
Nezir Aydin: Conceptualization, Writing – original draft, designed the study.

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