# Investment Homework I

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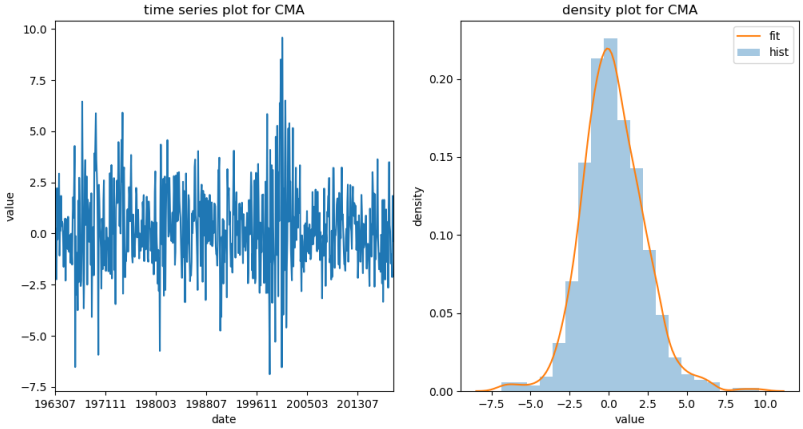
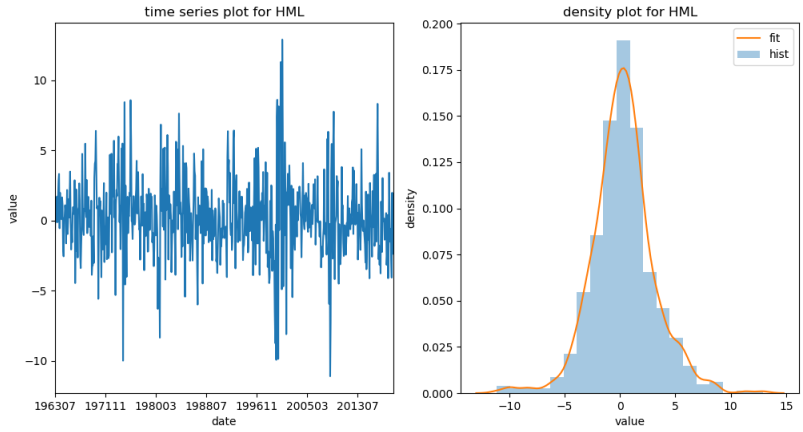
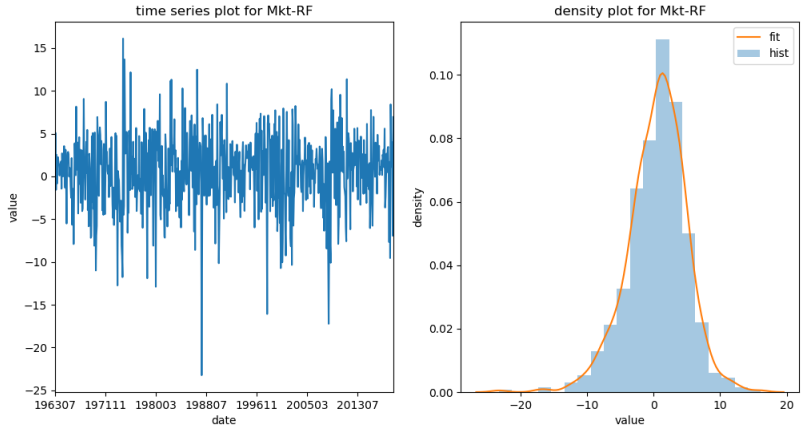
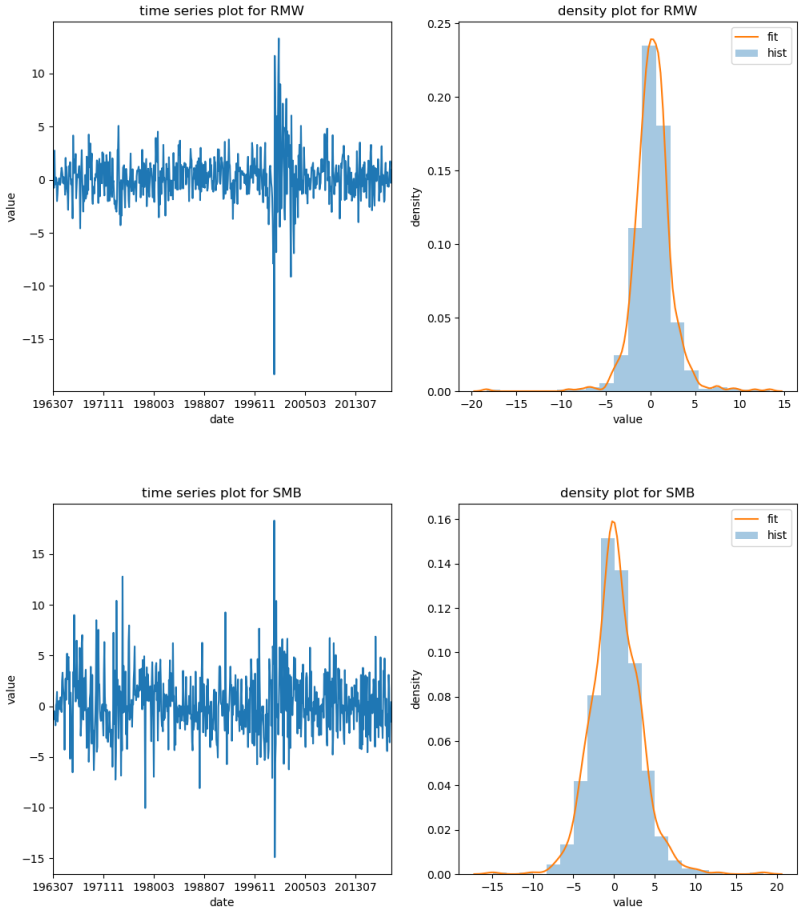
## 1. (a)

The following five figures show the time series plot and density plot for the five factors






From the time series plots, we find that there is no significant trend or seasonality in these factors. Meanwhile, the factors do not have constant variance, which suggests that they may not be stationary.

From the density plots, we find that all the factors are nearly normal distributed. The SMB, RMW, HML and CMA factors distribute more normally, while the Mkt-RF factor is most left skewed.

| | **Mkt-RF** | **SMB** | **HML** | **RMW** | **CMA** |
|---|---|---|---|---|---|
| **Mkt-RF** | 1.0000 | 0.2763 | -0.2554 | -0.2273 | -0.3866 |
| **SMB** | 0.2763 | 1.0000 | -0.0681 | -0.3475 | -0.1053 |
| **HML** | -0.2554 | -0.0681 | 1.0000 | 0.0610 | 0.6930 |
| **RMW** | -0.2273 | -0.3475 | 0.0610 | 1.0000 | -0.0374 |
| **CMA** | -0.3866 | -0.1053 | 0.6930 | -0.0374 | 1.0000 |

The table above is the correlation matrix of the five factors. We find that the CMA and HML factors have highest correlation coefficient, which is nearly 0.7. The correlation between other pairs of factors is relatively small.

**1. (b).**

Let PT denotes contractions, TP denotes expansions. The averages and SDs of factors conditional on economic cycles are shown below

| | Mkt-RF | SMB | HML | RMW | CMA |
|---|---|---|---|---|---|
| **average PT** | -1.24310 | -0.06170 | 0.64520 | 0.39690 | 0.86010 |
| **average TP** | 0.78283 | 0.28093 | 0.25578 | 0.23761 | 0.18902 |
| **SD PT** | 6.57720 | 3.69280 | 3.78050 | 2.20170 | 2.62200 |
| **SD TP** | 3.94112 | 2.90752 | 2.63231 | 2.15722 | 1.87754 |

From the table, we find that during contractions, Mkt-RF and SMB have lower average value than during expansions. Besides, they have negative average values during contractions and positive average values during expansions. However, HML, RMW and CMA are in the opposite situation and the average values are always positive. As for the SDs, all the factors except RMW have lower volatilities during expansions.

**2.**

Intuitively for equation (1), variance of daily return is approximately the combining effect of trading hour news, overnight news variance, plus the stock's inherent variance during trading hour/overnight.

To justify Equation (1) simply sum these factors up on a probability space for each factor. I.E. sum the products of each factor's effect on variance and the probability of such factor to happen. ("the expected variance")

For equation (2), daily return consists of idiosyncratic return (alpha), variance (beta) and random noise. And variance can be broken down into news/no news during day and overnight news. Intuitively it makes sense because stock performance is generally affected by news during trading hours or overnight, also market movements (if no news).

**3.**

Since it is a 3-date stock, we have

$$r_1 = \frac{c_1 + E_1(c_2)MDF_1}{P_0} = \frac{c_1 + P_1}{P_0}$$

$$r_2 = \frac{c_2}{E_1(c_2)MDF_1} = \frac{c_2}{P_1}$$

where $c_1, c_2, P_1$ are random variables, $P_0$ is a constant number.

Then we have

$$cov(r_1, r_2) = cov\left(\frac{c_1 + P_1}{P_0}, \frac{c_2}{P_1}\right)$$

$$= \frac{1}{P_0}\left(cov\left(c_1, \frac{c_2}{P_1}\right) + cov\left(P_1, \frac{c_2}{P_1}\right)\right)$$

$$= \frac{1}{P_0}\left(cov\left(c_1, \frac{c_2}{E_1(c_2)MDF_1}\right) + cov\left(E_1(c_2)MDF_1, \frac{c_2}{E_1(c_2)MDF_1}\right)\right)$$

This formula suggests that the autocorrelation in return depend not only on the autocorrelation of dividends but also on the correlation between dividend and expected dividend. Hence, autocorrelation in the dividends alone cannot cause autocorrelation in stock returns between times 1 and 2.