Inferential Statistics
Teacher in charge: Erlis Ruli (ruli@stat.unipd.it).
Academic Year: 2020/2021

November 29, 2020

The following are exercises to be performed with a computer using `R`. Choose two the following as assignment for evaluation. Place the `R` code, and relevant comments, tables and figures on a pdf file and upload it to the Moodle page within the deadline. Fix the random seed by using `set.seed(29)`. Uploads after the deadline won't be considered.

(1) For samples of size $n = \{10, 50\}$ from $\text{NegBin}(\theta_0, 10)$, with $\theta_0 = 2/3$, compute the distribution of the MLE of $\tau = \log[\theta/(1-\theta)]$ by simulation and compare it with its asymptotic distribution. (Hint: use the Delta method). For the same sample sizes and by means of simulations study the coverage probability of the 95% Wald confidence interval for τ and compare with the 95% Wald confidence interval for θ. Conclusions?

(2) For samples $Y_1, \ldots, Y_n$ of size $n = \{5, 15\}$ from the $\text{N}(\mu_1, \sigma_1^2)$ distribution and samples $X_1, \ldots, X_m$ of size $m = \{5, 15\}$ from the $\text{N}(\mu_2, \sigma_2^2)$ distribution with $\sigma_1^2 = 1, \sigma_2^2 = 2$, study by means of simulations the coverage of the 95% t-Student confidence interval. With the same parameters study also the coverage of the t-Student confidence interval which does not assume equal variances, given in Sect. 5.11. Conclusions?

(3) For the samples $Y_{1j}, \ldots, Y_{n_j k}$ for $j = 1, \ldots, k$ with $k = 4$, set $n_1 = n_3 = 10$, $n_2 = n_4 = 15$, and let $X_{ij} \overset{iid}{\sim} \text{N}(\mu_j, \sigma_j^2)$, with $\mu_j = 0$ and $\sigma_j^2 = 1$ for $j = 1, 2, 3, 4$. Compute the distribution of the F statistic for the ANOVA test by simulation and compare it with its exact distribution. Now let $\sigma_1^2 = \sigma_3^2 = 1/2$ and $\sigma_2^2 = \sigma_4^2 = 2$. Compute again by simulation the distribution of the F statistic. Does it follow its exact distribution?

(4) For samples of size $n = \{5, 10, 20\}$ from the $\text{Exp}(\lambda_0)$ distribution with $\lambda_0 = 1$, study the uniformity of the p-values when testing $H_0 : \lambda = 1$ vs $H_1 : \lambda \neq 1$ with the Wald test and with the likelihood ratio test. What can you conclude as n increases? Compute the coverage probability of the 95% likelihood confidence interval.