Some exam-type exercises

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The following are some problems that you may find on a final exam session of Inferential Statistics. The solution provided here are kept short for the sake of brevity. During the exam session you must be as detailed as possible, by justifying and explaining the reasoning you are applying in orderd to solve the problem.

Exercise 1

Let $X \sim F_\theta$ be a discrete r.v. with $\theta \in \Theta$, where the parameter space is $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$. The four possible distributions are as follows.

X	1	2	3
$f(x;\theta_1)$	.4	.1	.5
$f(x;\theta_2)$	.2	.1	.7
$f(x;\theta_3)$	.2	.4	.4
$f(x;\theta_4)$	.6	.3	.1

Let (X_1, X_2) be an i.i.d. random sample from F_θ.

(a) Compute the maximum likelihood estimator

(b) Compute the most powerful test of level $\alpha = .09$ for testing the hypthesis $H_0 : \theta = \theta_2$ against $H_1 : \theta = \theta_4$.

(c) Compute the power of the test under the hypothesis in (b).

(d) Perform a test of level $\alpha = .09$ for testing the hypthesis $H_0 : \theta = \theta_2$ against $H_1 : \theta \neq \theta_2$.

Solution

Exercise 2

Let $X_i \overset{iid}{\sim} \text{Wei}(\alpha, 1/\lambda)$, for $i = 1, \ldots, n$, $\alpha > 0, \beta > 0$. Note that $\text{Wei}(\alpha, 1/\lambda)$ has p.d.f.

$$f(x;\alpha,\lambda) = \alpha\lambda x^{\alpha-1} e^{-\lambda x^\alpha},\ x > 0.$$

Note:

$$E(X) = \frac{\lambda^{-1/\alpha}}{\alpha}\Gamma(1/\alpha),\ E(X^2) = \lambda^{-2/\alpha}\Gamma[(2+a)/a].$$

Assume $\alpha = 1$.

(a) Find $\hat{\lambda}_{MM}$, the method of moments estimator for λ.

(b) Compute the bias and the variance of $\hat{\lambda}_{MM}$.

(c) Is $\hat{\lambda}_{MM}$ consistent?

(d) Is $\hat{\lambda}_{MM}$ efficient ?

(e) Compute $\hat{\lambda}$, the maximum likelihood estimator for λ.

(f) Compute the exact and an approximate distribution of $\hat{\lambda}$.

(g) If possible find a UMP test for $H_0 : \lambda \leq \lambda_0$ aginst $H_1 : \lambda > \lambda_0$ with size α.

(h) Does there exists an UMP test for $H_0 : \lambda = \lambda_0$ vs $H_0 : \lambda \neq \lambda_0$? Why ?

(i) Compute an approximate confidence interval for λ.

(j) In a study about the lifetime of washing machines (measured in years), the observed sample of size $n = 20$ led to $\sum_{i=1}^{20} x_i = 9.849$. Get the p-value of the hypothesis $H_0 : \lambda = 1$ vs $H_0 : \lambda \neq \lambda_0$ using an exact test of size $\alpha = .05$ and compare it by the p-value obtained by an approximate test of the same size.

Solution