Functions: A special type of relation is called function. It is one of the most important concept in mathematics. We can visualize function as a rule, which produces new elements out of some given elements. There are many terms such as 'map' or 'mapping' used to denote a function.

Definition: A relation f from a set A to a set B is called a function if every element of set A has one and only one image in set B.

If f is a function from A to B and $(a, b) \in f$ then $f(a) = b$, where 'b' is called the image of 'a' under f and 'a' is called the preimage of 'b' under f.

Notation: The function f from A to B is denoted by

$$f : A \longrightarrow B.$$

Example: Let $A = \{1, 2, 3, 4, 5, 6\}$. Define the relation R from A to A by

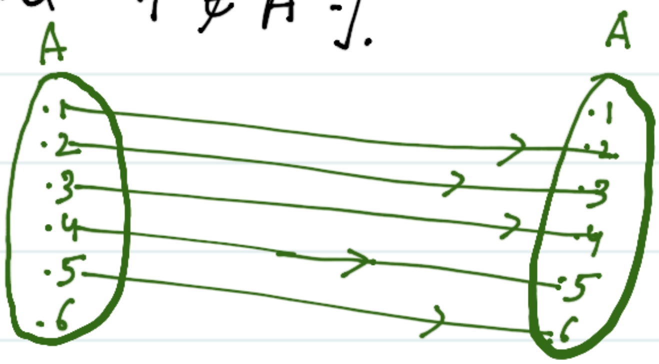
$$R = \{(x, y) : y = x + 1\}.$$

Then check whether this relation is a function or not?

Solution: Here,

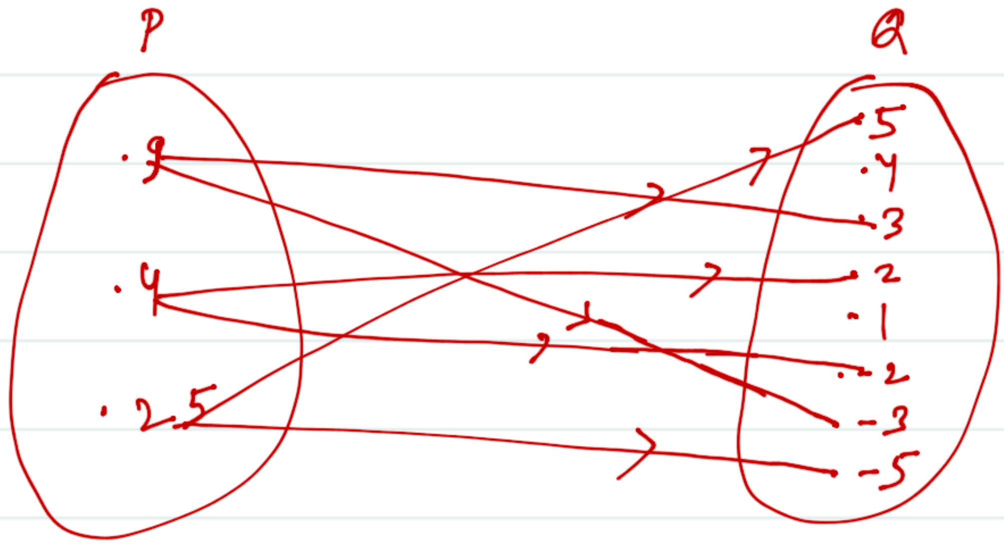
$$R = \{(1,2), (2,3), (3,4), (4,5), (5,6)\}.$$

$[(6,7) \notin R$ since $7 \notin A.]$



Here, we observe that the element 6 has no image in A. So, the relation R is not a function here.

Example: Consider the relation R between the sets P and Q.



Check whether the relation R between P and Q is a function or not?

Solution: Here, $R = \{(9, 3), (9, -3), (4, 2), (4, -2), (25, 5), (25, -5)\}$.

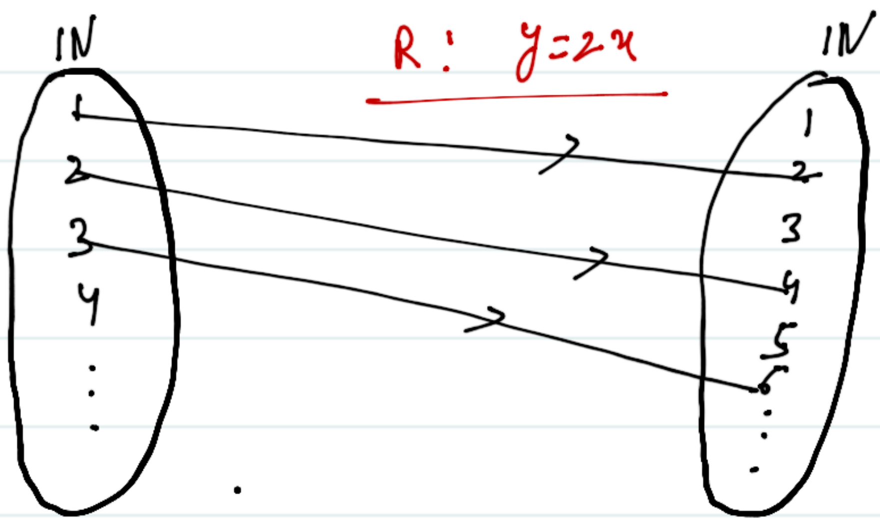
Here elements of domain, 9, 4 and 25 are connected to more than one images. Thus the relation R between P and Q is not a function.

Example: Let N be the set of natural numbers and the relation R be defined on $\mathbb{N}$ such that

$$R = \{(x,y) : y = 2x,\ x,y \in \mathbb{N}\}.$$

What is the domain, codomain and range of R? Is this relation a function?

Solution: Here the domain of R is the set of natural numbers $\mathbb{N}$. The codomain of R is also $\mathbb{N}$.



The range is the set of even natural numbers.

Since every natural number n has one and only one image, this relation is a function.

Imp| Let $f: A \longrightarrow B$ be a function.

Then, domain $(f) = A$

Codomain $(f) = B$

Range (f) = set of images in B

Clearly Range $(f) \subseteq B$.

Ex: Let $f: \mathbb{N} \longrightarrow \mathbb{N}$ be a function defined by

$$f(x) = 2x.$$

Then domain $(f) = \mathbb{N} \leftarrow$ set of all natural numbers

Codomain $(f) = \mathbb{N} \leftarrow$ "

Range $(f) = \{2, 4, 6, 8, \ldots\} \leftarrow$ set of all even natural numbers.

Example: Let $\mathbb{N}$ be the set of natural numbers. Define a function, $f: \mathbb{N} \to \mathbb{N}$ by $f(x) = 2x+1$.

Using this definition, complete the table given below.

x	1	2	3	4	5	6
y	$f(1)=$	$f(2)=$	$f(3)=$	$f(4)=$	$f(5)=$	$f(6)=$

Solution: Given function is

$$f(x) = 2x+1.$$

The completed table is given by

x	1	2	3	4	5	6
y	$f(1)=3$	$f(2)=5$	$f(3)=7$	$f(4)=9$	$f(5)=11$	$f(6)=13$

Some functions and their graphs:

① Identity function: Let $\mathbb{R}$ be the set of real numbers. Define the real function $f: \mathbb{R} \to \mathbb{R}$ by

$y = f(x) = x$ for each $x \in \mathbb{R}$.

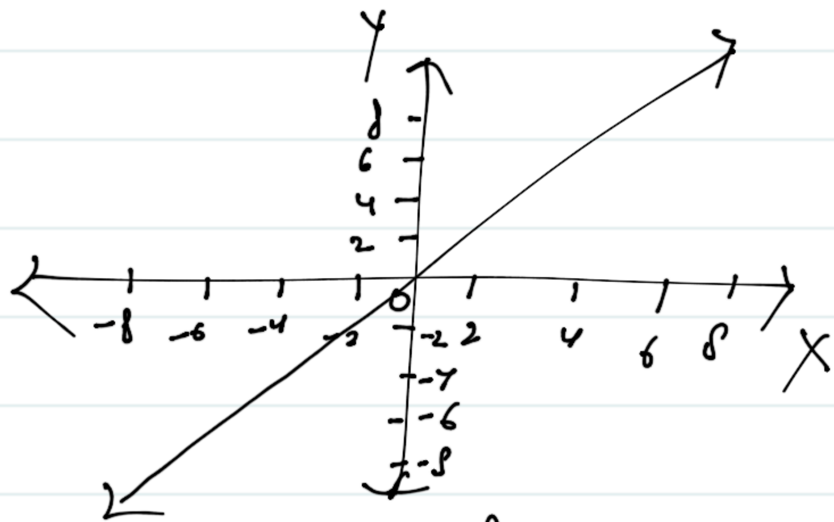
Such a function is called the identity function.

Under the given function:

$$f(1) = 1$$
$$f(2) = 2$$
$$f(3) = 3$$
$$f(4) = 4$$
$$f(0) = 0$$
$$f(-4) = -4$$
$$f(-3) = -3$$
$$f(-2) = -2$$
$$\vdots$$

Here the domain, codomain and range of f are $\mathbb{R}$.

The graph is a straight line as shown in the following figure.



$y = f(x) = x$

(2) Constant Function: Define the function

$f : \mathbb{R} \longrightarrow \mathbb{R}$ by

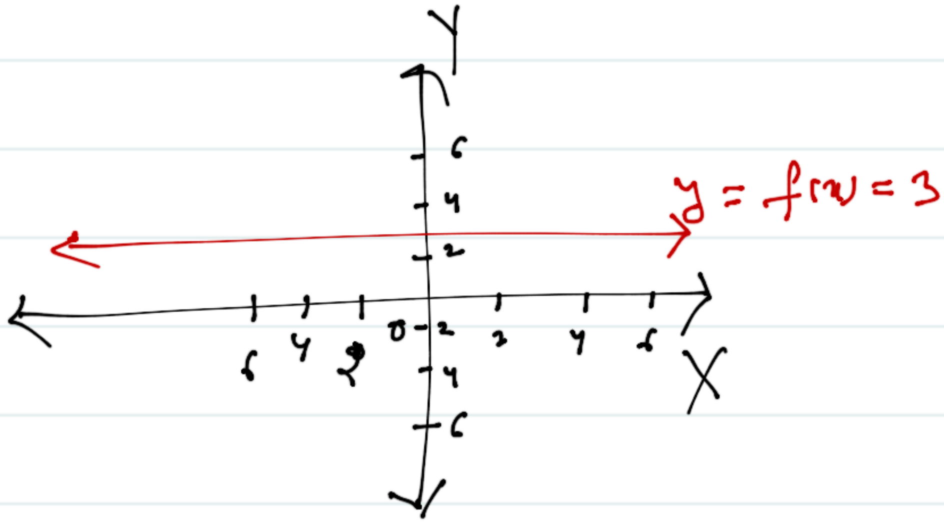
$y = f(x) = c, \quad x \in \mathbb{R}$, where

c is a constant and $x \in \mathbb{R}$.

Here the domain of f is $\mathbb{R}$.

Codomain $(f) = \mathbb{R}$, Range $(f) = \{c\}$.

Let $c = 3$ then $f(x) = 3$.



$y = f(x) = 3$

The graph is a line parallel to x-axis. For example, if $f(x) = 3$ for each $x \in R$, then its graph will be a line as shown in the above figure.

Example: Let f be function defined by $f(x) = \frac{1}{x}$. Find largest possible domain, codomain and range of f.

Solution: Given function is

$f: A \longrightarrow B$ such that

$$f(x) = \frac{1}{x}.$$

Our aim is to find set A, B and set of all images.

Clearly $1/x$ is not defined at $x=0$, thus 0 can not be a part of domain i.e. $x=0$ can not be input for f.

Thus

$$\text{Domain}(f) = R - \{0\}.$$

$$\text{Codomain}(f) = R$$

$$\text{Range}(f) = R - \{0\}.$$

Graph of $f(x) = \frac{1}{x}$: