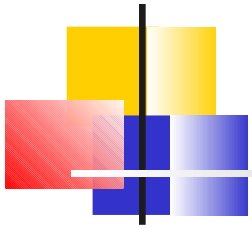
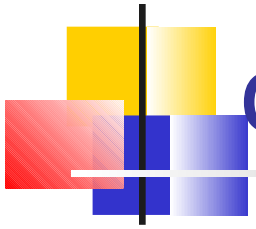


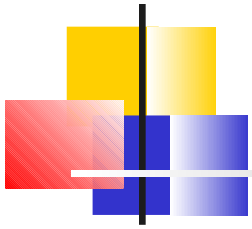
Properties and Relationships of Set Theory

Properties and Relationships of Set Theory



How are Venn Diagrams used to show relationships among sets?

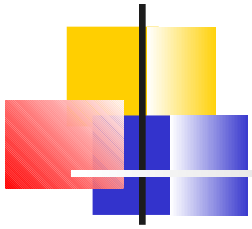
How are sets, subsets, unions, intersections, and complements identified?



Sets and Venn Diagrams

- A set is a collection of objects called members or elements.
- There are three ways to describe a set:

1. We can use words.
2. We can make a list.
3. We can use *set-builder notation*.



Examples of Sets

1. Words:

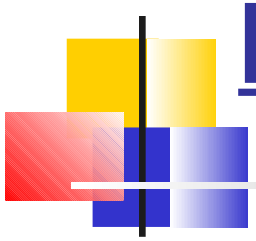
   N is the set of natural numbers or counting numbers.

2. List:

   $N = \{1, 2, 3, \ldots\}$

3. Set-builder notation:

   $N = \{x \mid x \in N\}$



Example

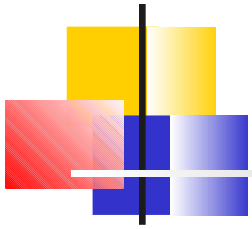
Write the set B of whole numbers greater than 5 using (a) roster notation and (b) set-builder notation.

a) roster notation:

$$B = \{6, 7, 8, \ldots\}$$

b) set-builder notation:

$$B = \{x \mid x \text{ is a whole number and } x > 5\}$$



Kinds of Sets

- A finite set has a limited number of members.

Example: The set of students in our Math class.

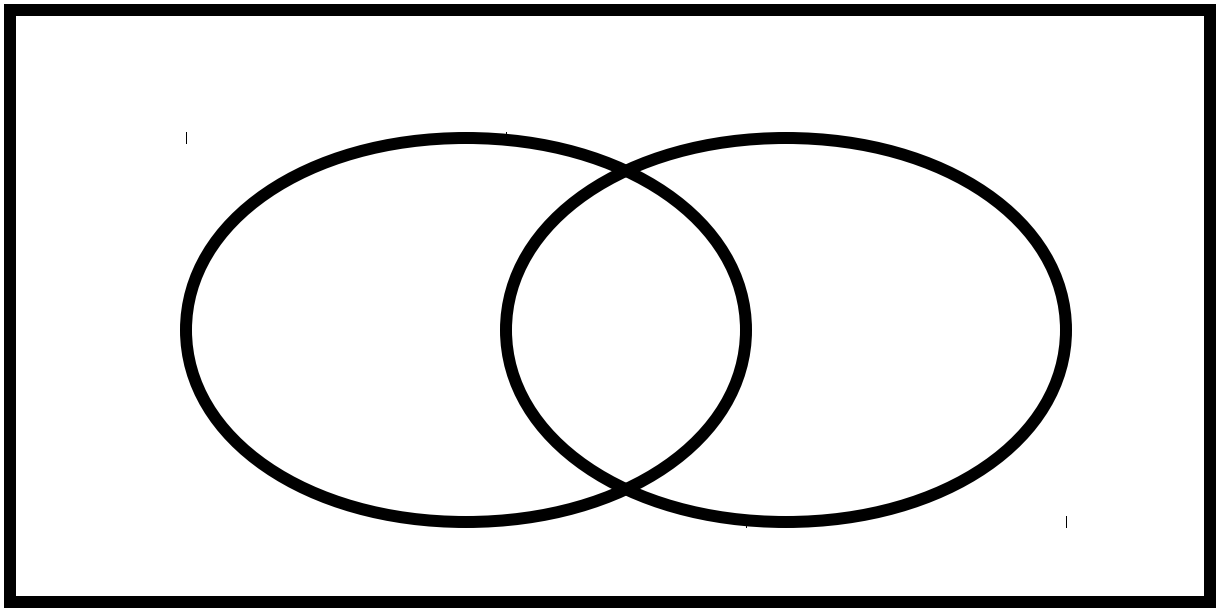
- An infinite set has an unlimited number of members.

Example: The set of integers.

- A well-defined set has a universe of objects which are allowed into consideration and any object in the universe is either an element of the set or it is not.

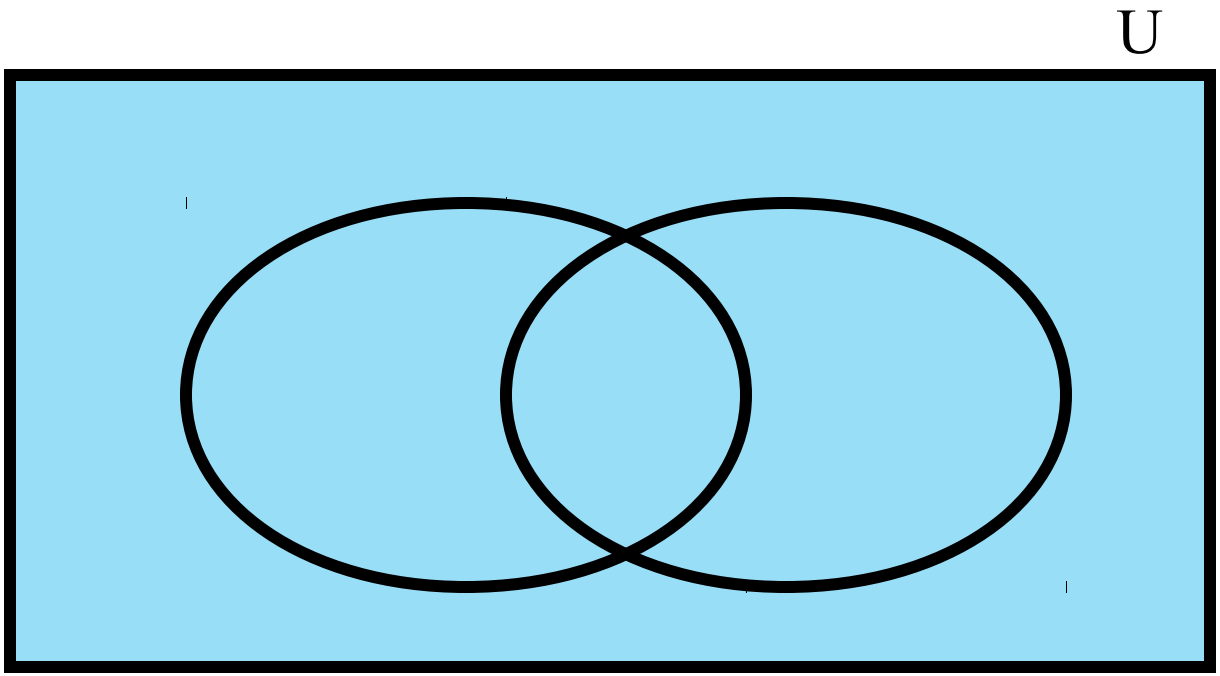
Venn Diagrams

One way to represent or visualize sets is to use Venn diagrams:



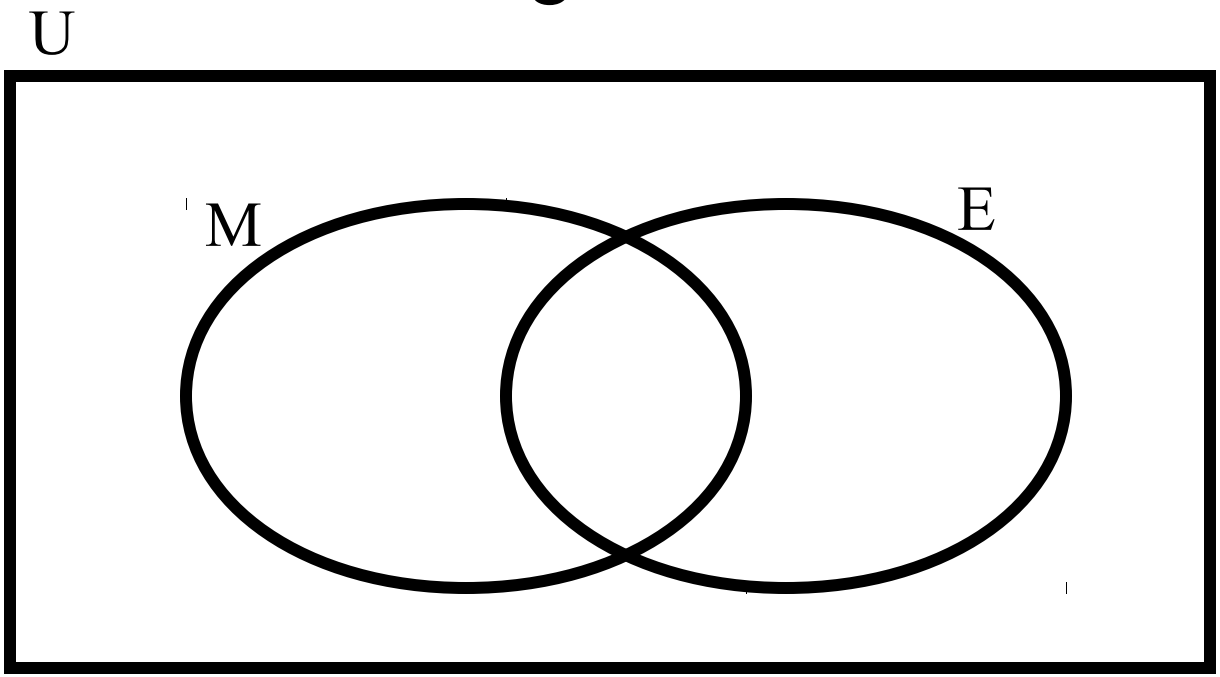
Universe or Universal Set

Let U be the set of all students enrolled in classes this semester.



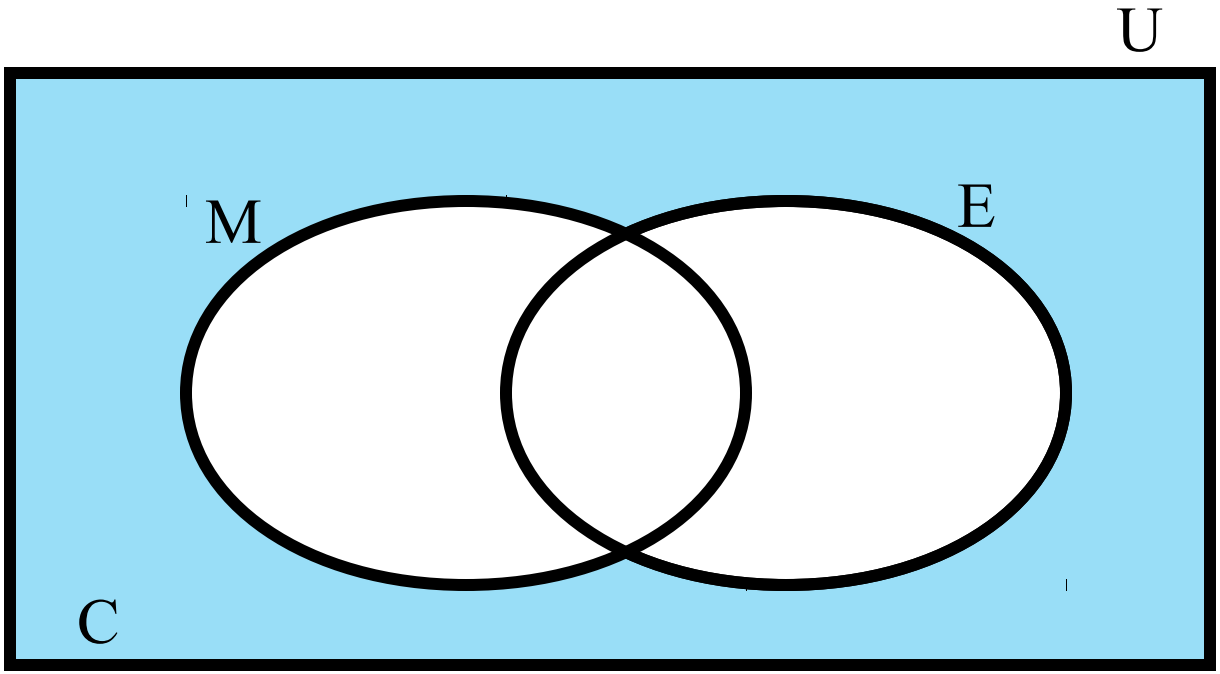
Let M be the set of all students enrolled in Math this semester.

Let E be the set of all students enrolled in English this semester.



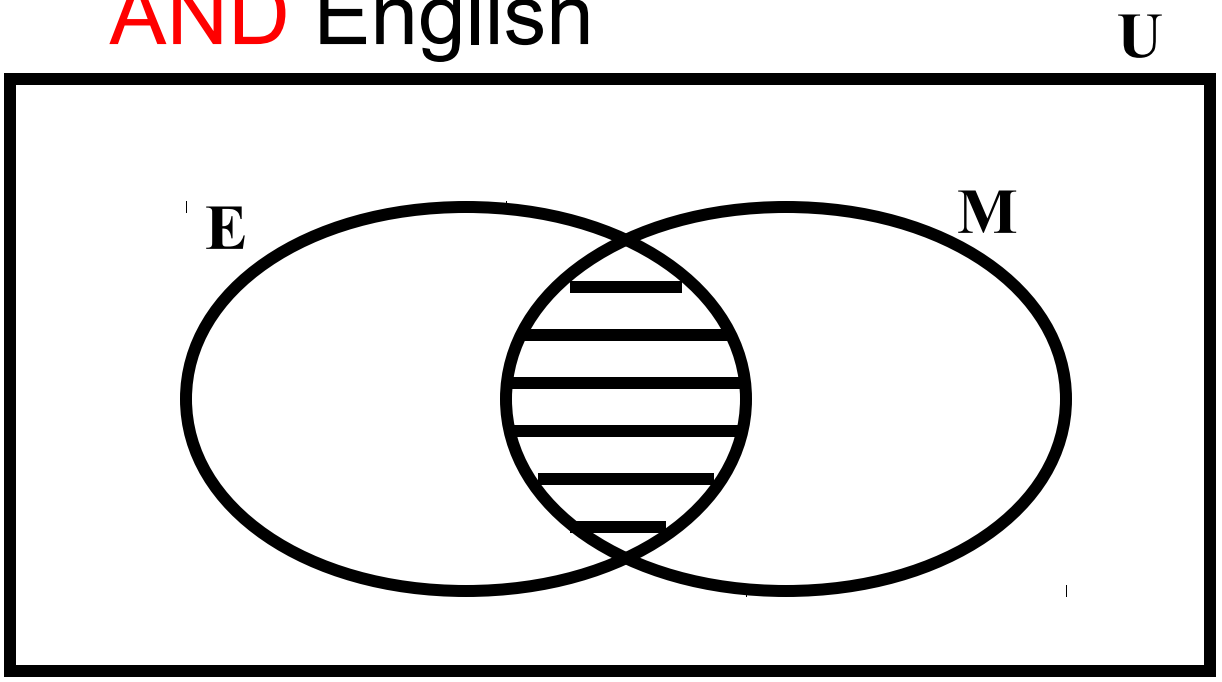
Complement of a set

Let C be the set of all students enrolled in classes this semester, but who are not enrolled in Math or English



Intersection (∩)

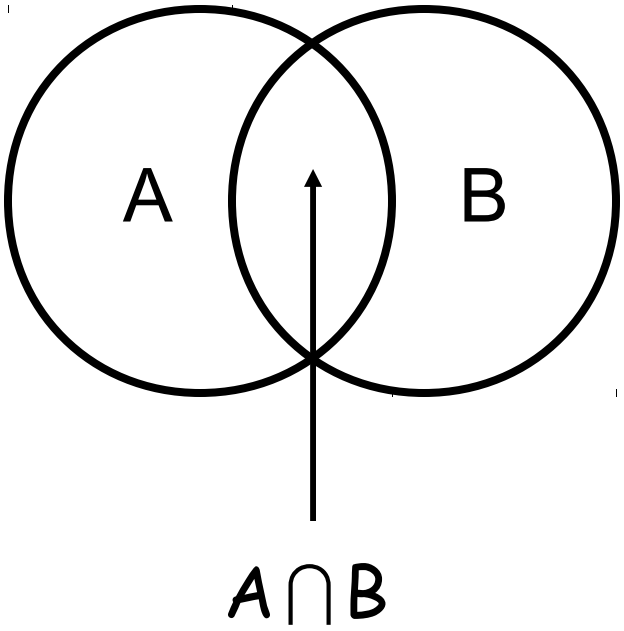
E ∩ M = the set of students in Math AND English

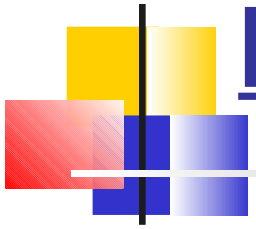


Intersection of Sets

The **intersection** of two sets A and B, written $A \cap B$, is the set of all members that are common to both sets.

$A \cap B$ is read "A intersection B"





Example

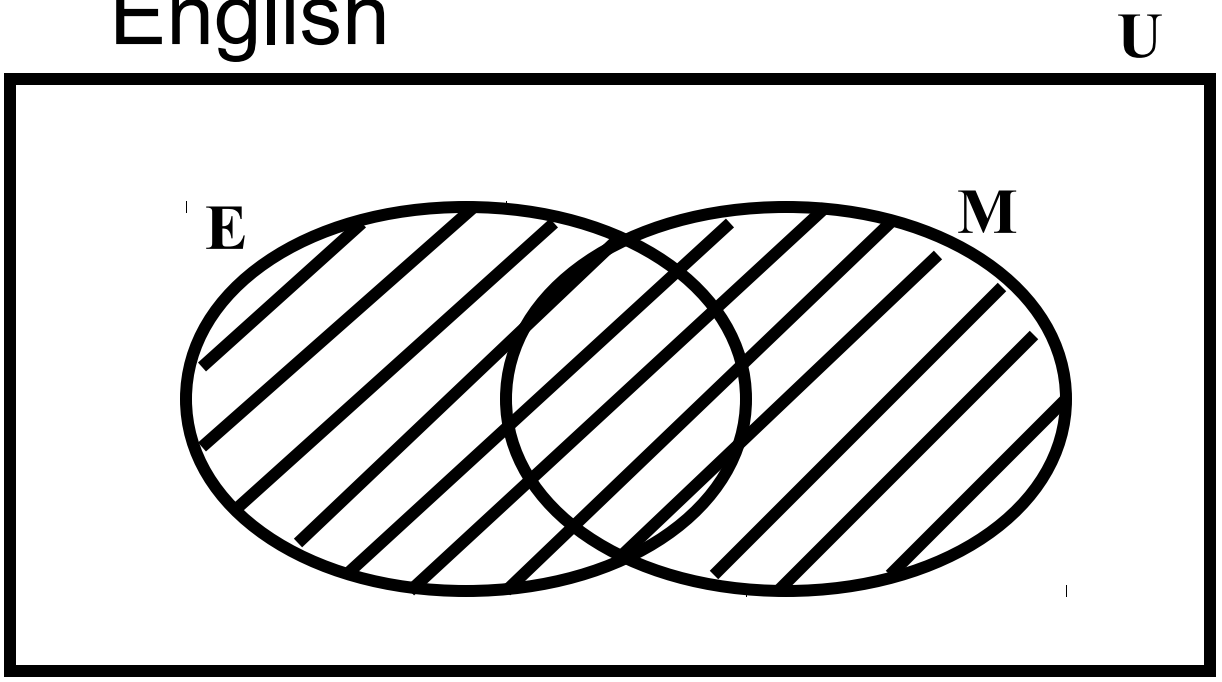
Let U = $\{1,3,5,7\}$ and V = $\{7,6,5,4\}$.

Find $U \cap V$.

$U \cap V = \{5,7\}$

Union (∪)

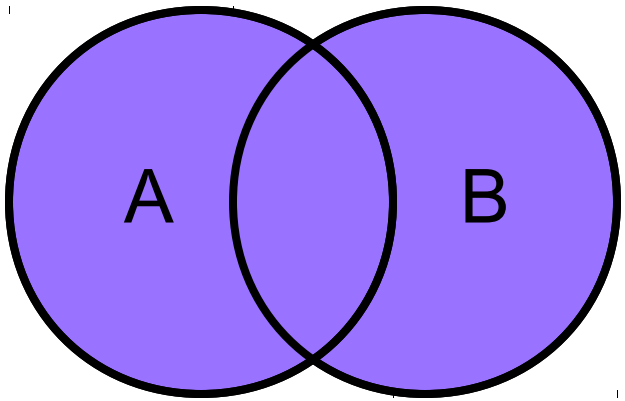
E ∪ M = the set of students in Math OR English



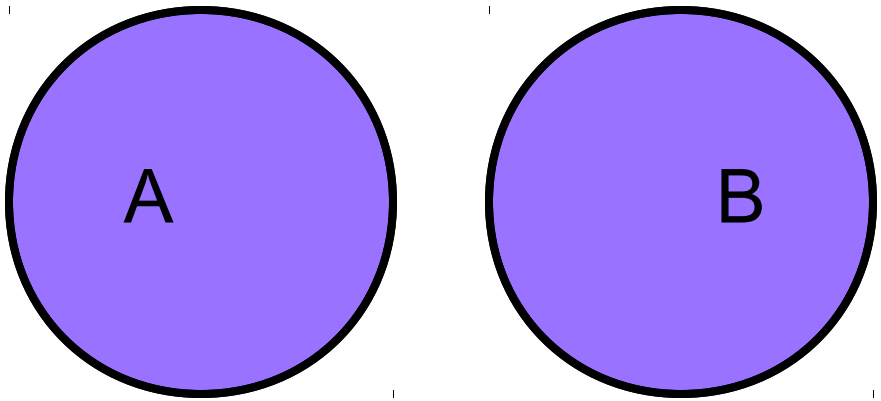
Union of Sets

The **union** of two sets A and B, written $A \cup B$, is the set of all members that are common to both sets.

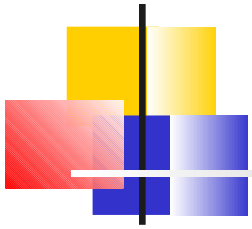
$A \cup B$ is read "A union B"



$A \cup B$



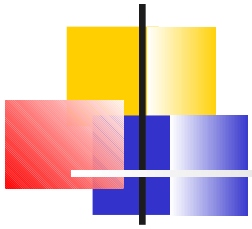
$A \cup B$



Example

Let C = {0,1,2,3} and D = {1,3,5}. Find $C \cup D$.

$C \cup D = \{0,1,2,3,5\}$



Example

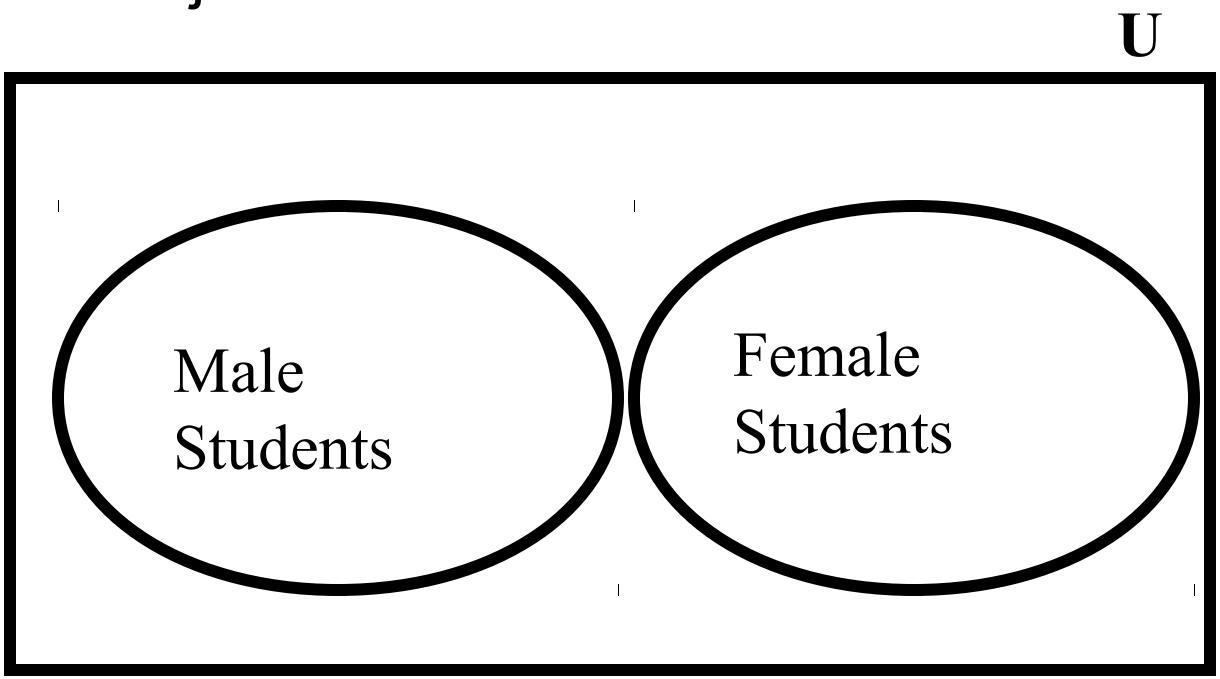
Let S be the set of positive divisors of 4 and let T = {1,5,10}. Find $S \cup T$.

S = {1,2,4}

$S \cup T = \{1, 2, 4, 5, 10\}$

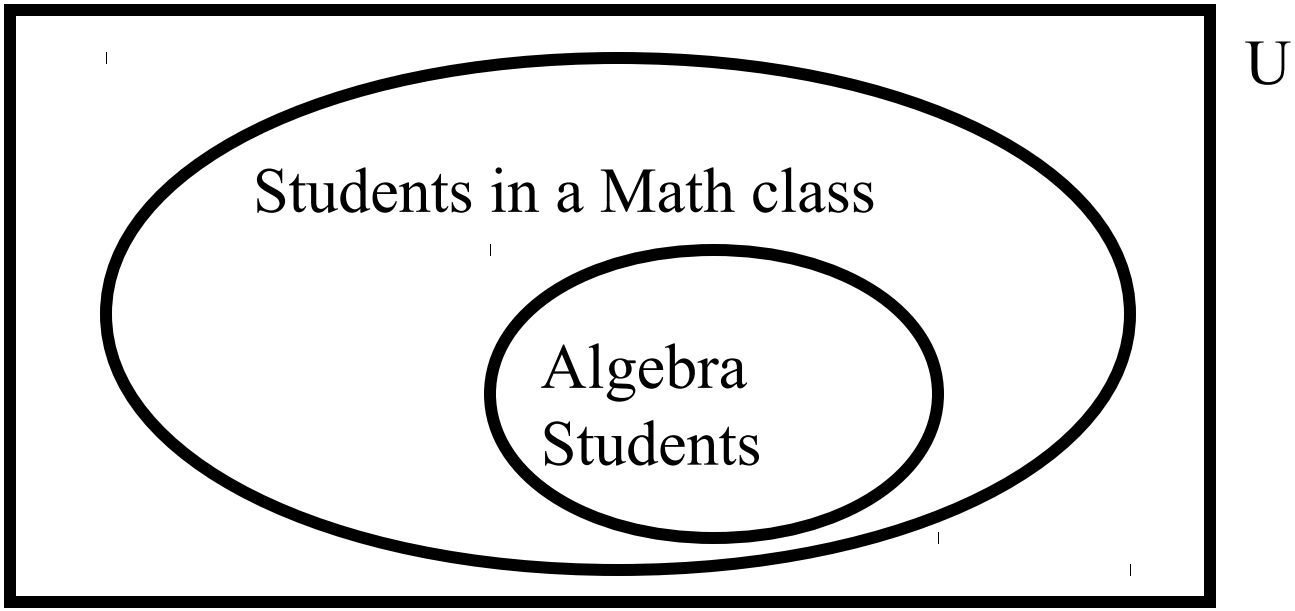
Disjoint Sets

Two sets with no elements in common are called disjoint sets.



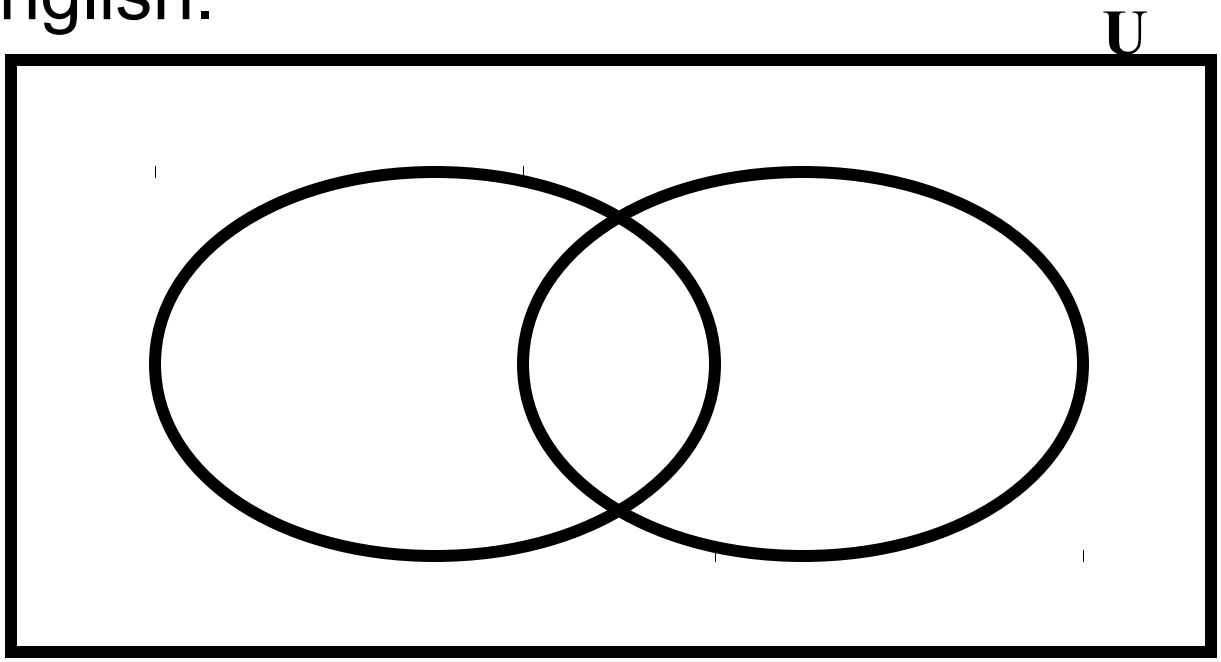
Subset ($\subseteq$)

X is a subset of Y if and only if every member of X is also a member of Y.



Example

A survey of 100 students revealed that 82 were in Math and 65 were in English. How many students are taking both Math and English? All 100 students are either in Math or English.



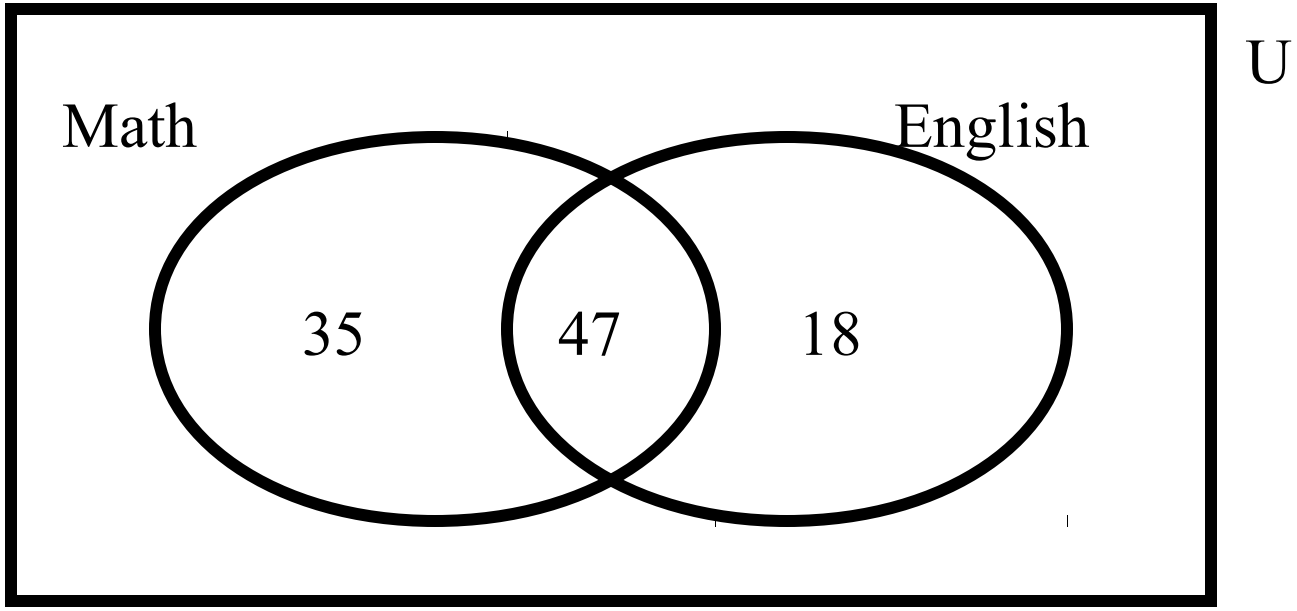
Solution

82 + 65 = 147

82 – 47 = 35

147 – 100 = 47

65 – 47 = 18



Example

The manager at a local Country -Western station reviewed the songs played during one 3-hour program on her station.

12 songs were about a truck driver who is in love while in prison.

13 songs talked about a prisoner in love.

28 songs talked about a person in love.

18 songs were about a truck driver in love.

3 songs were about truck drivers in prison who are not in love.

8 songs talked about people who are not in prison, are not in love and don't drive a truck.

16 songs were about truck drivers who are not in prison.

2 songs were about people in prison who are not in love and are not truck drivers.

Draw a Venn diagram.

Use your Venn Diagram to answer the following questions.

a) How many songs were about truck drivers?

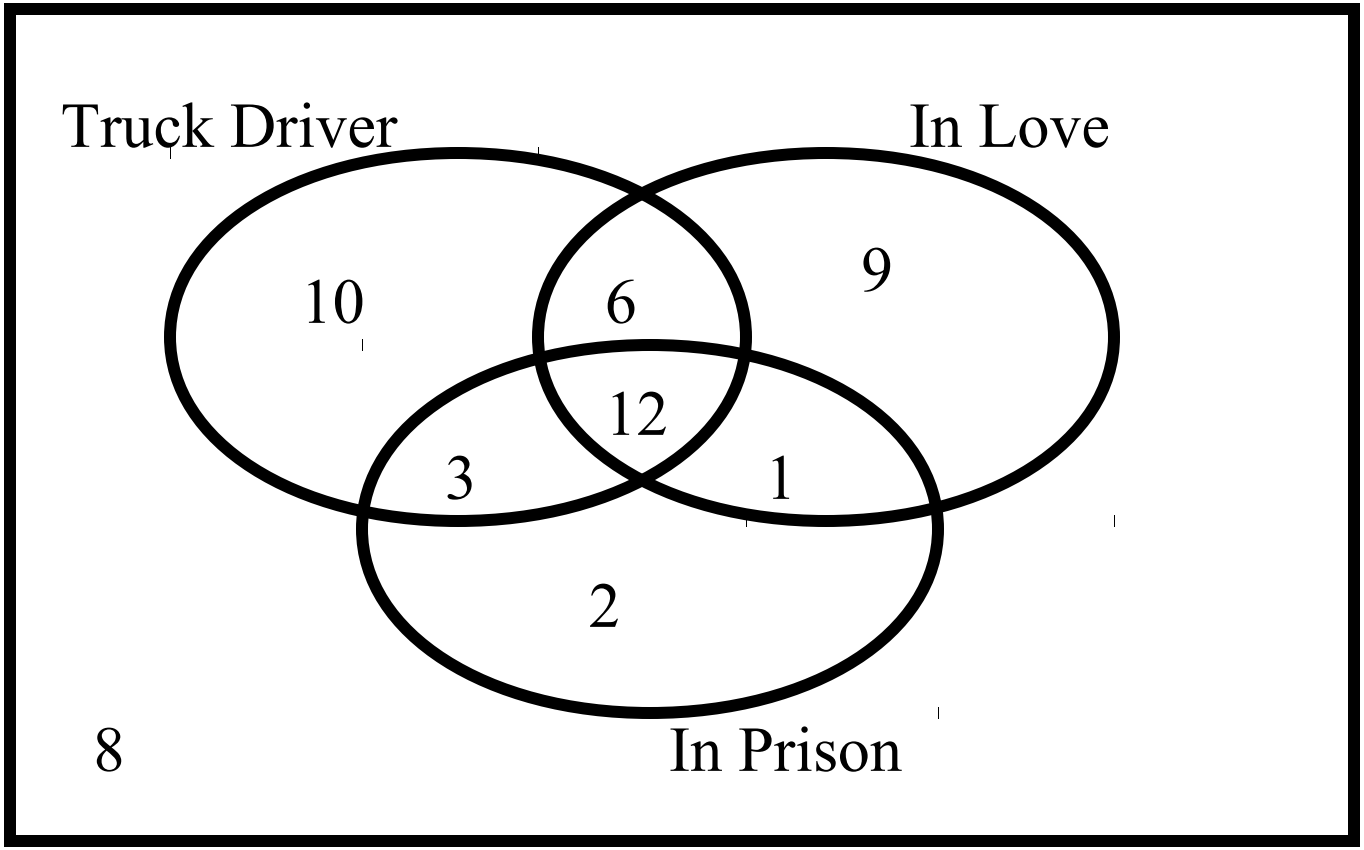
b) How many songs were about prisoners?

c) How many songs were about truck drivers in prison?

d) How many songs are about people in love who are not truck drivers and not in prison?

e) How many songs did the station manager review?

Venn Diagram:



Answers:

a. How many songs were about truck drivers? 31

b. How many songs were about prisoners? 18

c. How many songs were about truck drivers in prison? 15

d. How many songs are about people in love who are not truck drivers and not in prison? 9

e. How many songs did the station manager review? 51