Operations on Sets:

Union of Sets: Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.

The symbol '$\cup$' is being used to denote the union. Symbolically, we write $A \cup B$ and is read as 'A Union B'.

In symbols, we write

$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$

The union of two sets can be represented by a Venn diagram in fig. 3.

The shaded portion in fig. 3 represents $A \cup B$.

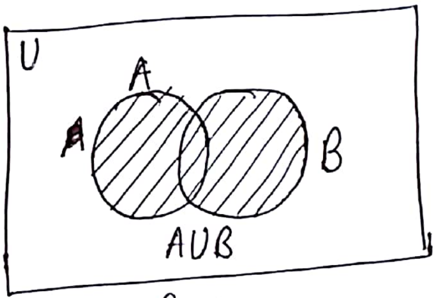


fig. 3

Example 1. Let $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$.

find $A \cup B$.

Solution: We have $A \cup B = \{2, 4, 6, 8, 10, 12\}$.

Example 2: Let $A = \{a, e, i, o, u\}$, $B = \{a, i, u\}$. Show that $A \cup B = A$.

Solution: We have $A \cup B = \{a, e, i, o, u\} = A$

This example shows that if $B \subset A$, then $A \cup B = A$.

Properties of the Operation of Union:

(i) $A \cup B = B \cup A$ (Commutative Law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative Law).

(iii) $A \cup \phi = A$ (ϕ is the identity of $\cup$).

(iv) $A \cup A = A$ (Idempotent Law)

(v) $U \cup A = U$, where U is the universal set.

Intersection of Sets: The intersection of sets A and B is the set of all elements which are common to both A and B. The symbol '$\cap$' is used to denote the intersection.

Symbolically, we write

$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

Example: Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

and $B = \{2, 3, 5, 7\}$.

Then $A \cap B = \{2, 3, 5, 7\}$.

Note: If $B \subset A$, then $A \cap B = B$.

(Symbolically)

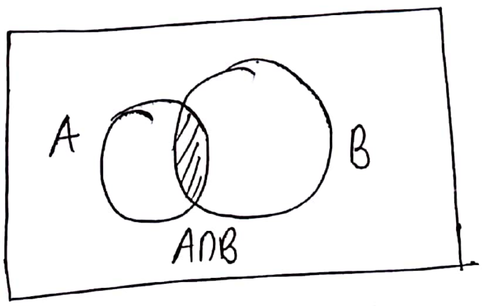


fig. 4

The shaded portion in fig. 4 indicates the intersection of A and B.

Disjoint Sets: If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

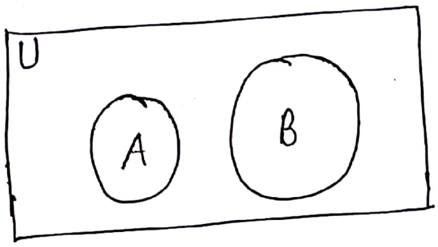
for example, let $A = \{2, 4, 6, 8\}$

and $B = \{1, 3, 5, 7\}$.

Then $A \cap B = \phi$.

$\Rightarrow$ A and B are disjoint sets.

The disjoint sets can be represented by means of Venn diagram as shown in the following figure.



Some Properties of Operations of Intersection:

(i) $A \cap B = B \cap A$ (Commutative Law)

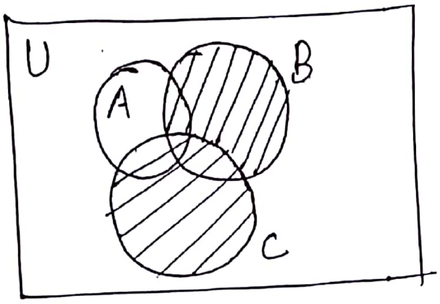
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U).

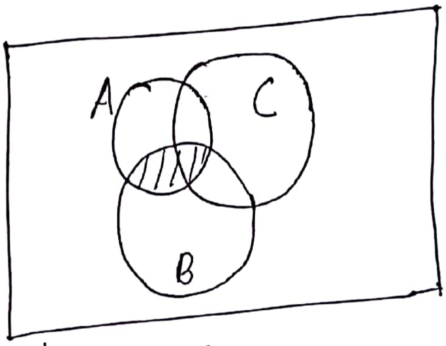
(iv) $A \cap A = A$ (Idempotent Law)

(V) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive Law)

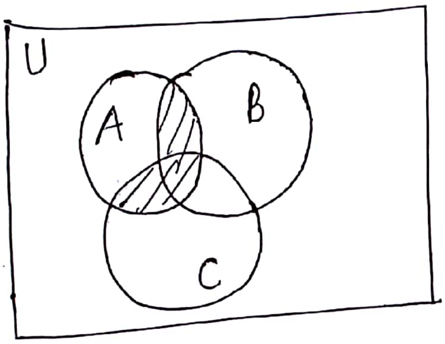
($\cap$ distributes over U).



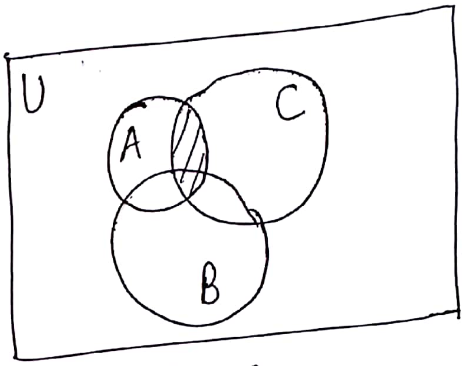
(i) $(B \cup C)$



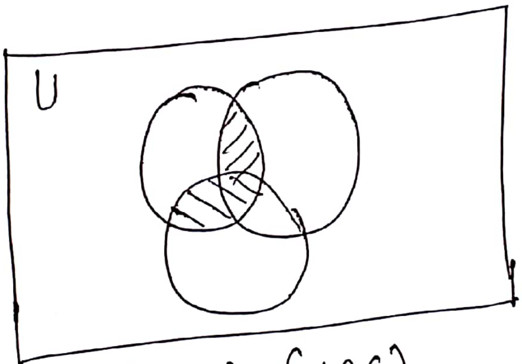
(iii) $A \cap B$



(ii) $A \cap (B \cup C)$



(iv) $(A \cap C)$



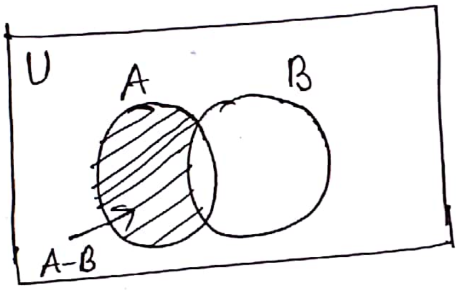
(v) $(A \cap B) \cup (A \cap C)$

Difference of Sets: The difference of sets A and B in this order is the set of elements which belong to A but not to B.

Symbolically, we write $A - B$ and read as "A minus B".

$$A - B = \{x : x \in A \text{ and } x \notin B\}.$$

The difference of two sets A and B can be represented by Venn diagram as shown in the following figure.



The shaded portion represents the difference of the two sets A and B.

Example: Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$. Find $A - B$ and $B - A$.

Solution:

$$A - B = \{1, 3, 5\}.$$

$$B - A = \{8\}.$$

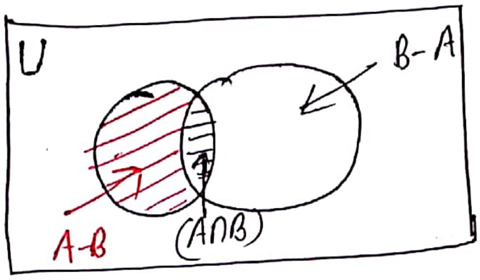
Thus, we note that $\boxed{A - B \neq B - A.}$

Example: Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$. Find $V - B$ and $B - V$.

Solution: $V - B = \{e, o\}$ and $B - V = \{k\}$.

Thus $V - B \neq B - V$.

Remark: The sets $A-B$, $A\cap B$ and $B-A$ are mutually disjoint sets, i.e intersection of any of these two sets is the null set.



Complement of a Set: Let U be the universal set and A be a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. Symbolically, we write A' to denote the complement of A with respect to U.

Thus

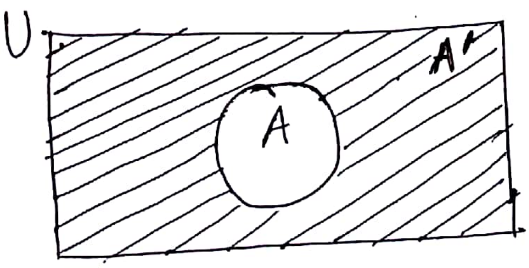
$$A' = \{x : x \in U \text{ and } x \notin A\}.$$
$$= U - A.$$

Example: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,

$$A = \{1, 3, 5, 7, 9\}.$$

Then $A' = \{2, 4, 6, 8, 10\}$.

The complement A' of a set A can be represented by a Venn diagram as shown in the following figure.



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Some Properties of Complement Sets:

1. Complement Laws : (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$.

2. De Morgan's Law : (i) $(A \cup B)' = A' \cap B'$

   (ii) $(A \cap B)' = A' \cup B'$

3. Laws of double complementation :

$$(A')' = A.$$

4. Law of empty set and universal set.

$$\phi' = U \text{ and } U' = \phi.$$

Examples: Let $U = \{1, 2, 3, 4, 5, 6\}$,

$A = \{2, 3\}$ and $B = \{3, 4, 5\}$.

Find A', B', $A' \cap B'$, $A \cup B$ and hence show that

$$(A \cup B)' = A' \cap B'.$$

Solution: $A' = \{1, 4, 5, 6\}$, $B' = \{1, 2, 6\}$.

Hence $A' \cap B' = \{1, 6\}$.

Also, $A \cup B = \{2, 3, 4, 5\}$,

$(A \cup B)' = \{1, 6\}$.

$(A \cup B)' = \{1, 6\} = A' \cap B'$.

Thus $(A \cup B)' = A' \cap B'$, Similarly, $(A \cap B)' = A' \cup B'$.

Practical Problems on Union and Intersection of Two Sets:

Let A and B be finite sets. If $A \cap B = \phi$, then

(i) $$\boxed{n(A \cup B) = n(A) + n(B)}$$

In general, if A and B are finite sets, then

(ii) $$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

(iii) If A, B and C are finite sets, then

$$\boxed{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)}.$$

Example: If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have?

Solution: Given that

$$n(X \cup Y) = 50, \quad n(X) = 28, \quad n(Y) = 32.$$

$$n(X \cap Y) = ?.$$

By using the formula,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$\Rightarrow n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$$

$$= 28 + 32 - 50$$

$$= 10$$

$$\Rightarrow \boxed{n(X \cap Y) = 10.}$$ Ans.

Example: In a school, there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics?

Solution: Let M = Set of teachers who teach mathematics.

P = Set of teachers who teach physics.

According to the statement,

$$n(M \cup P) = 20, \quad n(M) = 12, \quad n(M \cap P) = 4$$

Using the formula,

$$n(M \cup P) = n(M) + n(P) - n(M \cap P)$$

$$\Rightarrow \quad 20 = 12 + n(P) - 4$$

$$\Rightarrow \quad \boxed{n(P) = 12}$$

Thus 12 teachers teach physics.

Example: In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of two games. How many students like to play both cricket and football?

Solution: Let X be the set of students who like to play cricket.

and Y be the set of students who like to play football.

$X \cup Y$ = Set of students who like to play atleast one game.

$X \cap Y$ = Set of students who like to play both games.

Given $n(X) = 24$, $n(Y) = 16$, $n(X \cup Y) = 35$, $n(X \cap Y) = ?$

Using the formula,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y), \text{ we get}$$

$$35 = 24 + 16 - n(X \cap Y)$$

$$\Rightarrow \boxed{n(X \cap Y) = 5}$$

ie 5 students like to play both the games.

Example: In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Solution: Let U = set of surveyed students.

A = set of students taking apple juice.

B = set of students taking orange juice.

Then $n(U) = 400$, $n(A) = 100$, $n(B) = 150$ and

$n(A \cap B) = 75$.

Now, $n(A' \cap B') = n(A \cup B)'$ $\quad [\because A' \cap B' = (A \cup B)']$

$= n(U) - n(A \cup B)$ $\quad [\because (A \cup B)' = U - A \cup B]$

$= n(U) - n(A) - n(B) + n(A \cap B)$

$[\because n(A \cup B) = n(A) + n(B) - n(A\cap$

$= 400 - 100 - 150 + 75 = 225$

$\Rightarrow \boxed{n(A' \cap B') = 225}$.

Hence 225 students were taking neither apple juice nor orange juice.

<u>Example</u>: There are 200 individuals with a skin disorder, 120 had been ~~ex~~ exposed to the chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of ~~individuals~~ individuals exposed to

(i) Chemical C_1 but not chemical C_2 (ii) Chemical C_2 but not chemical C_1

(iii) Chemical C_1 or chemical C_2.

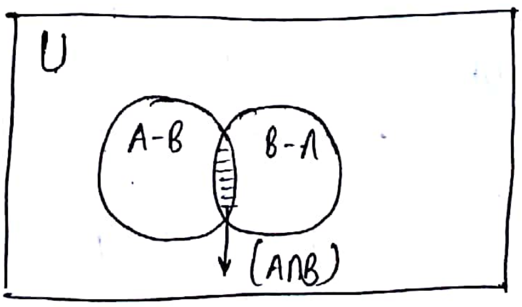
<u>Solution</u>: Let U denote the universal set consisting of individuals suffering from the skin disorder.

A = Set of individuals exposed to the chemical C_1.
B = Set of individuals exposed to the chemical C_2.

Here $n(U) = 200$, $n(A) = 120$, $n(B) = 50$ and

$n(A \cap B) = 30$.

(i) From the Venn diagram, we observe



$$A = (A-B) \cup (A \cap B)$$

$$\Rightarrow \quad n(A) = n(A-B) + n(A \cap B) \quad [\because A-B \text{ and } A \cap B \text{ are disjoint}]$$

$$\Rightarrow \quad n(A-B) = n(A) - n(A \cap B)$$

$$= 120 - 30 = 90$$

$$\Rightarrow \quad \boxed{n(A-B) = 90}$$

Hence the number of individuals exposed to chemical C_1 but not to chemical C_2 is 90.

(ii) Again, from the Venn diagram, we observe that

$$B = (B-A) \cup (A \cap B).$$

$$\Rightarrow \quad n(B) = n(B-A) + n(A \cap B).$$

$[\because$ Since $B-A$ and $A \cap B$ are disjoint$]$

$$\Rightarrow \quad n(B-A) = n(B) - n(A \cap B)$$

$$= 50 - 30 = 20$$

$$\Rightarrow \quad \boxed{n(B-A) = 20}$$

Thus the number of individuals exposed to chemical C_2 but not to chemical C_1 is 20.

(iii) The number of individuals exposed either to chemical C_1 or to chemical C_2, i.e.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$= 120 + 50 - 30 = 140.$$

$$\Rightarrow \boxed{n(A \cup B) = 140}.$$

Example: A market research group conducted a ~~geo~~ survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both the products?

Solution: Let U be the set of consumers questioned.

S = Set of consumers who liked the product A.

T = Set of consumers who liked the product B.

Given that,

$$n(U) = 1000, \quad n(S) = 720, \quad n(T) = 450.$$

So, $$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$

$$= 720 + 450 - n(S \cap T)$$

$$\Rightarrow n(S \cup T) = 1170 - n(S \cap T)$$

Therefore, $n(S \cup T)$ is maximum when $n(S \cap T)$ is least.

But $S \cup T \subset U$ implies that

$$n(S \cup T) \leq n(U) = 1000.$$

So, maximum values of $n(S \cup T)$ is 1000.

Thus, the least value of $n(S \cap T)$ is 170.

Hence, the least number of consumers who liked both products is 170.

Example : A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of three sports?

Solution: Let F = Set of men who received medals in football.

B = Set of men who received medals in basketball.

C = Set of men who received medals in cricket.

Then $n(F) = 38$, $n(B) = 15$, $n(C) = 20$.

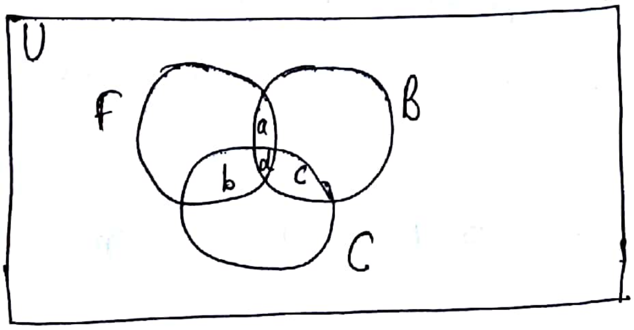
$n(F \cup B \cup C) = 58$ and $n(F \cap B \cap C) = 3$.

Therefore,

$$n(F \cup B \cup C) = n(F) + n(B) + n(C) - n(F \cap B) - n(F \cap C) - n(B \cap C) + n(F \cap B \cap C).$$

$$\Rightarrow \quad n(F \cap B) + n(F \cap C) + n(B \cap C) = 18.$$

Consider the Venn diagram as given in figure



Here, a denotes the number of men who got medals in football and basketball only,

b denotes the number of men who got medals in football and cricket only.

c denotes the number of men who got medals in basket ball and cricket only.

d denotes the number of men who got medal in all the three.

Thus $d = n(F \cap B \cap C) = 3$.

and $a + d + b + d + c + d = 18$

$\Rightarrow \boxed{a + b + c = 9}$

Which is the number of people who got medals in exactly two of the three sports.

Example: Out of 500 cars owners investigated, 400 owned car A and 200 owned car B, 50 owned both A and B cars. Is this data correct?

Solution: Let U be the set of car owners investigated.

M = Set of persons who owned car A.

S = Set of persons who owned car B.

Given that

$n(U) = 500$, $n(M) = 400$, $n(S) = 200$, $n(S \cap M) = 50$.

Then $n(S \cup M) = n(S) + n(M) - n(S \cap M) = 200 + 400 - 50$

$= 550$

$\Rightarrow$

$\Rightarrow n(S \cup M) = 550.$

But $S \cup M \subset U$

$\Rightarrow n(S \cup M) \leq n(U) = 500.$

Which is a contradiction. So, the given data is incorrect.