Capital Asset Pricing Model Theory & Evidence

Fama & French

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Introduction

- The CAPM builds on the model of portfolio choice developed by Harry Markowitz (1959).
- The birth of asset pricing theory by William Sharpe (1964) and John Lintner (1965): CAPM:(Sharpe had Nobel Prize in 1990)
- CAPM still widely used in applications (more than four decades)
- The attraction of the CAPM:
  - Measure risk
  - Relation between expected return and risk
- CAPM weakness: The empirical record of the model is poor.
- The failure of the CAPM in empirical test implies that most applications of the model are invalid.

Outlining

- The Logic of CAPM
- The History of the Empirical Work
- Shortcomings of the CAPM that pose challenges to be explained by alternative models.

Logic of the CAPM

+ Minimize the variance of portfolio return, given expected return.
+ Maximize expected return, given variance

The Markowitz approach called a "mean variance model"

Assumes:

- Investors are risk averse
- Invertors care only about mean and variance of their one-period investment return.

The portfolio model provides an algebraic condition on asset weights in ***mean-variance-efficient*** *portfolio.*

The Logic of the CAPM

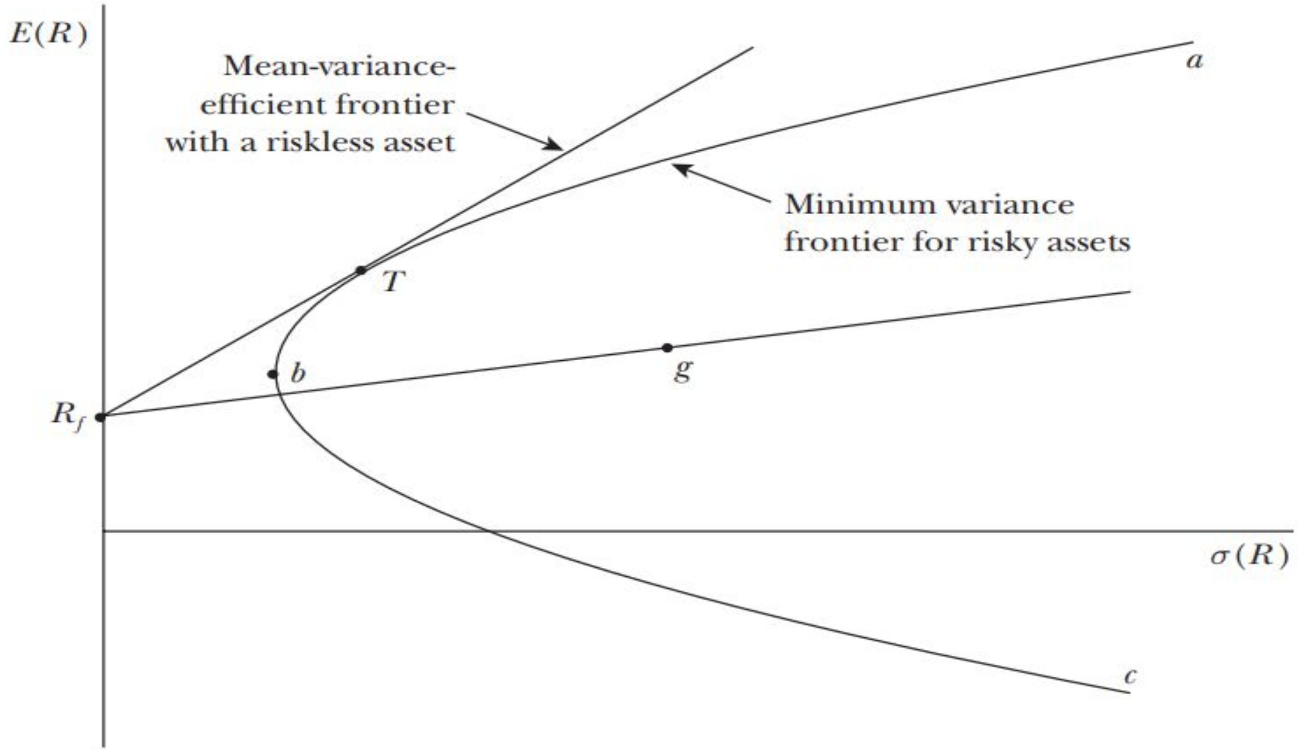
Sharpe (1964) and Lintner (1965) add two key assumptions to the *Markowitz model* to identify a portfolio:

- Complete agreement: given market clearing asset prices at t-1.
- Borrowing and lending at a risk-free rate, which is the same for all investors and does not depend on the amount borrowed or lent.

Describe The Portfolio Opportunities and Tells the CAPM Story

Figure 1

Investment Opportunities



Risk Free Borrowings and Lending

[2] Formally, the return, expected return and standard deviation of return on portfolios of the risk-free asset f and a risky portfolio g vary with x, the proportion of portfolio funds invested in f, as

$$R_p = xR_f + (1 - x)R_g,$$

$$E(R_p) = xR_f + (1 - x)E(R_g),$$

$$\sigma(R_p) = (1 - x)\sigma(R_g),\ x \leq 1.0,$$

which together imply that the portfolios plot along the line from R_f through g in Figure 1.

Market Portfolio

$$\text{(Minimum Variance Condition for } M) \quad E(R_i) = E(R_{ZM}) + [E(R_M) - E(R_{ZM})]\beta_{iM}, \; i = 1, \ldots, N.$$

In this equation, $E(R_i)$ is the expected return on asset i, and β_{iM}, the market beta of asset i, is the covariance of its return with the market return divided by the variance of the market return,

$$\text{(Market Beta)} \quad \beta_{iM} = \frac{\text{cov}(R_i, R_M)}{\sigma^2(R_M)}.$$

[3] Formally, if x_{iM} is the weight of asset i in the market portfolio, then the variance of the portfolio's return is

$$\sigma^2(R_M) = Cov(R_M, R_M) = Cov\left(\sum_{i=1}^{N} x_{iM}R_i, R_M\right) = \sum_{i=1}^{N} x_{iM}Cov(R_i, R_M).$$

Sharpe-Lintner Model

When there is risk-free borrowing and lending, the expected return on assets that are uncorrelated with the market return, $E(R_{ZM})$, must equal the risk-free rate, R_f. The relation between expected return and beta then becomes the familiar Sharpe-Lintner CAPM equation,

$$\text{(Sharpe-Lintner CAPM)} \quad E(R_i) = R_f + [E(R_M) - R_f)]\beta_{iM}, \; i = 1, \ldots, N.$$

In words, the expected return on any asset i is the risk-free interest rate, R_f, plus a risk premium, which is the asset's market beta, β_{iM}, times the premium per unit of beta risk, $E(R_M) - R_f$.

Early Empirical Tests

- Tests of the CAPM are based on:
  - Expected return on all assets are linearly related to their betas, and no other variable has marginal explanatory power.
  - The beta premium is positive.
  - Asset uncorrelated with the market have expected market returns equal to the risk-free interest rate (in the Sharpe-Lintner version of the model).

Most tests of these predictions use either *cross section or time-series regression*.

Test On Risk Premium

- Cross-Sectional Regression
  - Estimate of beta for individual assets are imprecise
  - The regression residuals have common sources of variation

Positive correlation in the residuals produces downward bias in the usual OLS estimates of the standard errors of the cross-section regression slopes.

Test On Risk Premium

Further research to improve the precision of estimated betas.

- Blume (1970)
- Friend and Blume (1970)
- Black, Jensen and Scholes (1972)
- Fama and MacBeth (1973)

[4] Formally, if x_{ip}, $i = 1, \ldots, N$, are the weights for assets in some portfolio p, the expected return and market beta for the portfolio are related to the expected returns and betas of assets as

$$E(R_p) = \sum_{i=1}^{N} x_{ip} E(R_i), \text{ and } \beta_{pM} = \sum_{i=1}^{N} x_{ip} \beta_{pM}.$$

Thus, the CAPM relation between expected return and beta,

$$E(R_i) = E(R_f) + [E(R_M) - E(R_f)]\beta_{iM},$$

holds when asset i is a portfolio, as well as when i is an individual security.

Test On Risk Premium

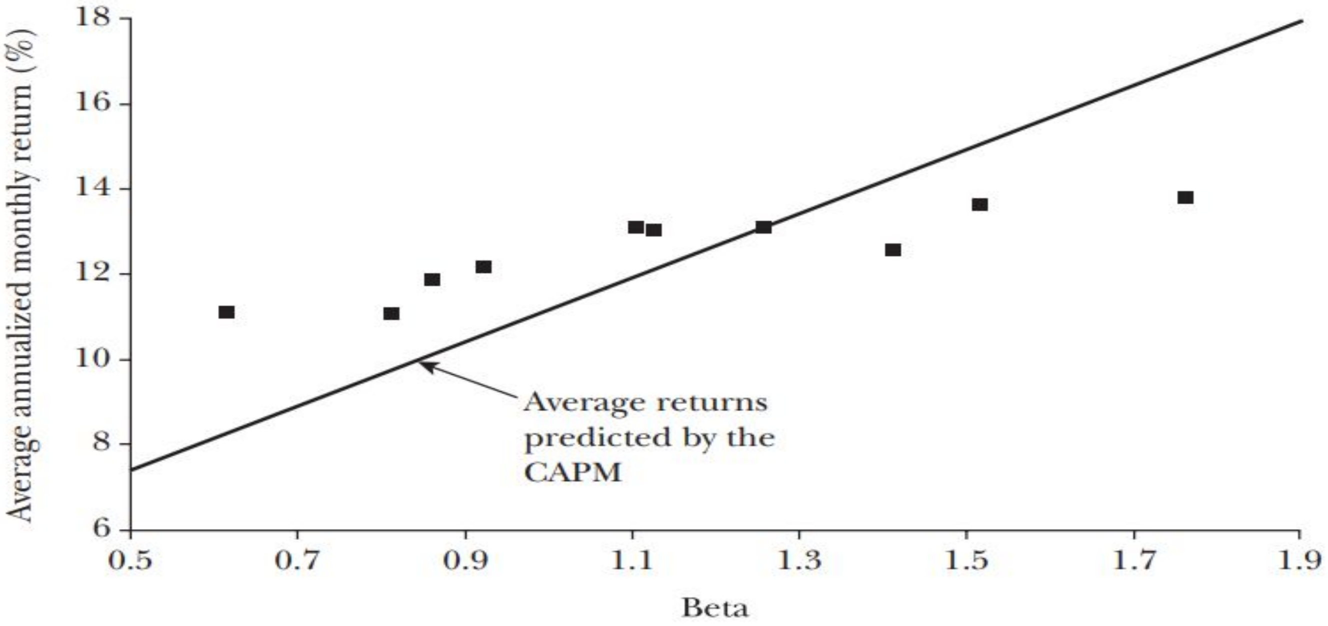
- Time-Series Regression
  - Jensen(1968)
    - S-L version also implies a time series regression test.

$$\text{(Time-Series Regression)} \quad R_{it} - R_{ft} = \alpha_i + \beta_{iM}(R_{Mt} - R_{ft}) + \varepsilon_{it},$$

Time Series Regression

Figure 2

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



Testing Whether Market Betas Explain Expected Return

- Fama and MacBeth (1973)

    - Add new variable: square market betas (to test the prediction that the relation between expected return and beta is linear)
    - Residual variances from regression of return on the market return (to test the prediction that market beta is the only measure of risk needed to explain expected return)

- The result:
    - Consistent with the hypothesis that their market proxy is on the minimum variance frontier.

Recent Tests

- Fama and French (1992)
  - Using cross-sectional regression approach Variable:
  - Size
  - Earning-price
  - Debt equity
  - Book to market ratios

- Expected stock returns provided by market beta.
- Results
  - Contradictions of the CAPM associated with price ratios are not sample specific (data problem)

Explanations: Irrational Pricing or Risk

- **The empirical failures of the CAPM:**
  - The Behavioralists side
    - Stocks with high ratios of book value to market price are typically firms that have fallen on bad time.
  - The Model side (unrealistic assumptions)
    - Ex. The assumption that investors care only about the mean and variance of one-period portfolio returns is extreme
- **Merton's (1973)**
  - Intertemporal CAPM
    - Different assumption about investor objective.

Explanations: Irrational Pricing or Risk

- Three Factor Model
  - Based on Implementation of Intertemporal CAPM(ICAPM), Fama & French(1993, 1996) propose a three factor model for expected returns:

Based on this evidence, Fama and French (1993, 1996) propose a three-factor model for expected returns,

(Three-Factor Model) $$E(R_{it}) - R_{ft} = \beta_{iM}[E(R_{Mt}) - R_{ft}] + \beta_{is}E(SMB_t) + \beta_{ih}E(HML_t).$$

In this equation, SMB_t (small minus big) is the difference between the returns on diversified portfolios of small and big stocks, HML_t (high minus low) is the difference between the returns on diversified portfolios of high and low B/M stocks, and the betas are slopes in the multiple regression of $R_{it} - R_{ft}$ on $R_{Mt} - R_{ft}$, SMB_t and HML_t.

The Market Proxy Problem

- Roll (1977): CAPM has never been tested and probably never will be.

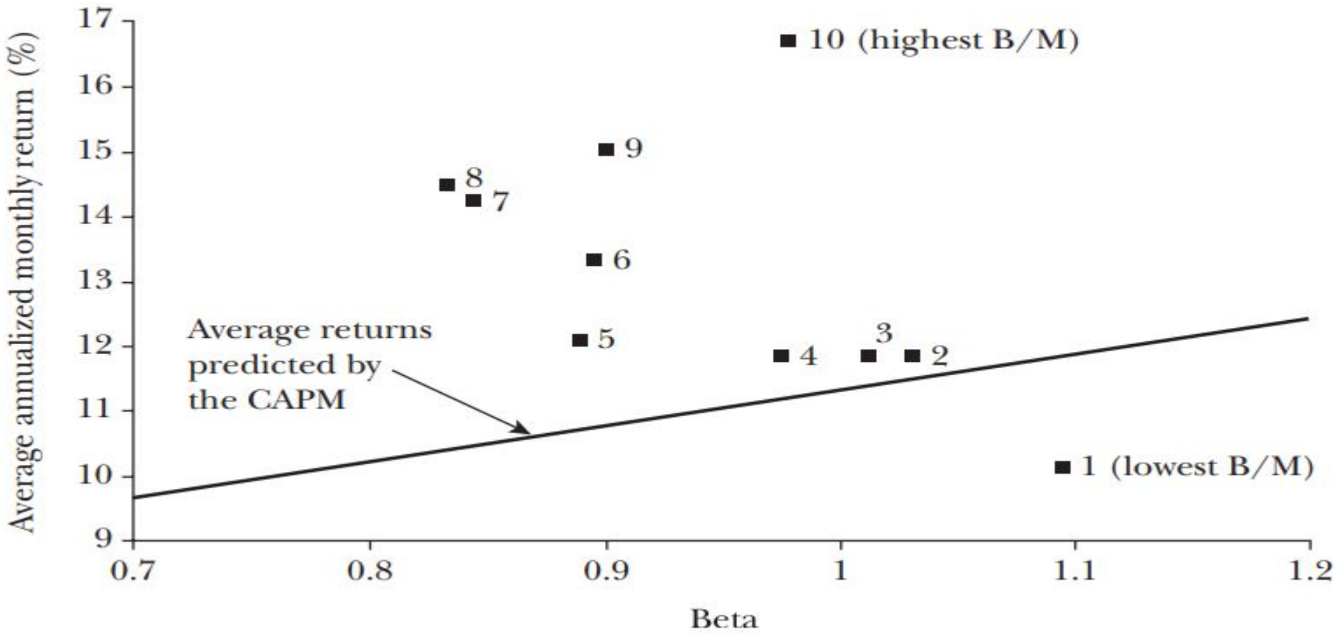
- A major problem for the CAPM
    - Portfolio formed by sorting on price ratios produce a wide range of average returns, but average returns ARE NOT positively related to market betas.

- If a market proxy does not work in tests of the CAPM, it does not work in applications.

The Market Proxy Problem

Figure 3

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on B/M, 1963–2003



Conclusion

- The version of the CAPM developed by Sharpe (1964) and Lintner (1965) has never been empirical success.

- Black (1972), has some success (accommodate a flatter tradeoff of average return for market data.

- The CAPM's empirical problems probably invalidate its use in applications
- Three factor model