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Hersh Shefrin, Meir Statman

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Behavioral Capital Asset Pricing Theory

Hersh Shefrin and Meir Statman*

Abstract

This paper develops a capital asset pricing theory in a market where noise traders interact with information traders. Noise traders are traders who commit cognitive errors while information traders are free of cognitive errors. The theory includes the determination of the mean-variance efficient frontier, the return on the market portfolio, the term structure, and option prices. The paper derives a necessary and sufficient condition for the existence of price efficiency in the presence of noise traders and analyzes the effects of noise traders on price efficiency, volatility, return anomalies, volume, and noise trader survival.

I. Introduction

We provide a behavioral theory of capital asset prices and the volume of trade. The theory centers on a market where both information traders and noise traders participate. Information traders use a proper Bayesian learning rule to form estimates of returns while noise traders commit errors as they employ non-Bayesian rules.

There is no need for a behavioral theory in a market composed entirely of information traders. This is a market where price efficiency and the CAPM hold. Risk premia are determined solely by beta and the distribution of the returns on the market portfolio, the term structure corresponds to the Cox, Ingersoll, and Ross (1985) model, and option prices follow the Black-Scholes formula.[1] However, the twin paradigms of price efficiency and the CAPM are under steady and forceful challenge, and there is a need for a comprehensive and structured theory that is not only consistent with the existing empirical evidence, but also links the components of the evidence. We argue that the paradigms of market efficiency and the CAPM fail because they do not incorporate the actions of noise traders.

Consider the challenge to price efficiency and the CAPM. Abnormal returns are associated with size (Banz (1981)), earnings to price ratios (Basu (1983)), past winners and losers (De Bondt and Thaler (1985), (1987)), and turn-of-the-year

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[1] Ancillary assumptions are required in order to arrive at these pricing structures.

(Roll (1983)). Volatility might be excessive (Shiller (1981)), LeRoy and Porter (1981)) and overreaction is reflected in option prices (Stein (1989) and Bates (1991)). We are also faced with the equity premium puzzle (Mehra and Prescott (1985)), the closed-end fund puzzle (Lee, Shleifer, and Thaler (1991)), and the failure of beta to reflect risk (Fama and French (1992)).

The empirical findings have sparked interest in the effect of noise traders in financial markets. Recent work includes Kyle (1985), Black (1986), De Long, Shleifer, Summers, and Waldmann (DSSW hereafter) (1990a), and Blume and Easley (1992). Earlier work includes Friedman (1953), Jaffe and Winkler (1974), Figlewski (1978), (1982), and Shefrin (1984).

What features distinguish markets in which prices are efficient from markets in which prices are inefficient? We argue that the key difference between the two markets is what we call the *single driver property*. In markets where prices are efficient, there is a single, specific variable that drives the mean-variance efficient frontier, the return distribution of the market portfolio, the premium for risk, the term structure, and the prices of options. This single driver is the minimal amount of new information necessary to infer changes to the return distribution of the market portfolio. Noise traders introduce a second driver into the market and drive prices away from efficiency. The actions of noise traders are manifested in many ways. They distort the mean-variance efficient frontier, thereby creating abnormal returns to particular securities. In this respect, they create a link between market beta and abnormal returns (Chopra, Lakonishok, and Ritter (1992)). They create excess volatility in the risk premium on the market portfolio and in long-term interest rates (Brown and Schaefer (1994)). They create a link between the slope of the yield curve and the risk premium on the market portfolio (Ferson and Harvey (1991)). They create a gap between the subjective volatility implied in option prices and its objective counterpart (Canina and Figlewski (1993)). We argue that the manifestations of the actions of noise traders are not a collection of unrelated phenomena. Rather, they are a joint manifestation of the failure of the single driver property.

The interest in noise trading within finance has proceeded simultaneously with the investigation of specific cognitive errors by psychologists. Some cognitive errors have already been incorporated into financial models, as in De Bondt and Thaler (1985), (1987) (overreaction in the stock market), Shefrin and Statman (1986) (the use of investment advisers and money managers), Solt and Statman (1988), (1989) (the belief that the bearish sentiment index contains information and the belief that good stocks are stocks of good companies), and De Bondt and Bange (1992) (expectations about the term structure). We focus on specific cognitive errors and show that the effect of noise traders in the market depends crucially on the type of errors they commit.

Our behavioral theory is a structured theory. We emphasize this point because of the prevailing concern that the abandonment of price efficiency and the CAPM might lead to a framework without structure that cannot be falsified by empirical evidence. Our theory provides specific functional forms for the term structure, option prices, and mean-variance frontier. Because we nest the case of price efficiency within our model, our results extend the standard pricing models to the case of price inefficiency.

The type of errors noise traders commit is important, especially to the link between noise trading and price efficiency. We demonstrate that price efficiency protects noise traders. Indeed, when prices are efficient, noise traders (as a group) are codominant with information traders, and their impact is restricted to an increase in volume.

The organization of the paper is as follows. Section II contains the basic model, and Section III addresses the condition under which noise traders interfere with efficient prices. Section IV describes the structure of noise trader errors, and Section V describes the volatility induced by noise trades. Section VI addresses mean-variance efficiency and the anatomy of abnormal returns. Section VII describes how volume is affected by noise traders. Section VIII contains conclusions.

II. The Model

Consider a financial market with H participants that we call traders. Time is discrete, with a set of dates indexed $t = 0, 1, 2, \ldots, T$. At the beginning of each date, new information s is revealed. We call s a state, and assume that it belongs to a finite set $S = \{s_i\}$. Let $s^t \epsilon S$ denote the state revealed at date t. The public information at the beginning of t is denoted by $x_t = (s^0, s^1, \ldots, s^t)$. That is, uncertainty unfolds according to a tree whose nodes are the date event pairs $\{x_t\}$. Let s_i occur at date t, and s_j occur at date $t+1$. Transitions from s_i to s_j are governed by an irreducible Markov chain with transition matrix $\Pi = [\Pi_{ij}]$.[2]

At the outset of date 0, trader h holds an initial portfolio ω_h. If h holds ω_h through date t, and date-event pair x_t materializes, then h receives dividend $\omega_h(x_t)$ during date t.[3] The symbol $\omega = \Sigma_h \omega_h$ denotes the market portfolio. The Markov state s, which occurs at date $t+1$, determines the dividend growth rate $\omega(x_{t+1})/\omega(x_t)$. For each s, partition S into $S_\omega(s)$ subsets $\{s_\omega\}$ according to the distinct transition distributions $\text{Prob}\{\omega(x_{t+1})/\omega(x_t) \mid s\}$. Call s_ω a *sufficient statistic* for the dividend growth rate, conditional on s.

A financial security is represented as a vector $Z = [Z(x_t)]$ where $Z(x_t)$ is the amount that one unit of the security pays its owner at x_t. We assume that financial markets offer a sufficiently rich set of securities to form complete markets (in the sense of Duffie and Huang (1985)). Therefore, there are state primitives that underlie security prices. Let $r(x_t)$ denote the price of an x_t-state contingent claim, and $r = [r(x_t)]$. We take x_0 as numeraire: that is, $r(x_0) = 1$. On the date 0 market, the price $q_Z(x_0)$ of security $Z = [Z(x_t)]$ is $r \cdot Z$. On the x_t market, the price $q_Z(x_t)$ of Z is the r-value of the Z-payoffs from date t on, divided by $r(x_t)$.[4]

We are interested in the impact of noise traders on a particular set of securities, and assume that these securities are available for trade at every date t. The securities set includes: i) zero coupon, risk-free bonds: these bonds underlie the term structure of interest rates. We assume that a zero coupon bond maturing at

[2]The Markov assumption is quite weak. It prevents the infinite past from affecting the distribution of the dividend growth rate, thereby allowing discovery of the stochastic process from the observational sequence by means of Bayes rule.

[3]Dividends are in real units of consumption.

[4]Define the vector $r'(x_t)$ as follows: the x_jth component of $r'(x_t)$ is $r(x_j)$ for all successor nodes x_j to x_t, and the x_jth component is zero otherwise. Then $q_z(x_t) = r'(x_t) \cdot Z/r(x_t)$.

any date t is available for trade at any date before t. ii) The market portfolio: this security is denoted by Z_ω, and is a scalar multiple of ω. iii) Put and call options on the market portfolio.[5] A call option issued at x_t has an exercise price of K, expires at date $t+j$, and pays $\max\{q_\omega(x_{t+j}) - K, 0\}$, where $q_\omega(x_{t+j})$ denotes the price of the market portfolio on the x_{t+j}-market. A put option is analogous to a call option, but returns $\max\{0, K - q_\omega(x_{t+j})\}$.

A trader's wealth at the beginning of t consists of the market value of his x_{t-1} portfolio, including dividends paid in x_t. The trader then divides his x_t-wealth into a portion to be consumed at t, and a portion to be saved. The saved portion is invested in the securities that comprise his x_t-portfolio. Denote trader h's net trade of the x_t-contingent commodity by $z_h(x_t)$. Then the consumption vector $c_h = [c_h(x_t)]$ is given by $c_h = \omega_h + z_h$.

Trader h's subjective beliefs are represented by a probability distribution P_h over the process $\{x_t\}$. Logarithmic utility plays a central role in the issue of long-run trader survival. Our model assigns all traders logarithmic utility with positive rates of discount. Every trader h's consumption plan c_h is obtained by maximizing expected utility EU_h subject to the lifetime budget constraint $r \cdot z_h \leq 0$. Formally, each trader h's intertemporal utility function is given by

$$U_h(x_t) = \sum_{t=1}^{T} \gamma_h^{t-1} \ell n\,(c_h(x_t)), \tag{1}$$

where $\gamma_h < 1$ is h's time discount factor. Assume that preferences over alternative states x_T are represented by the expected utility function EU_h, where the expectation is taken over the subjective probability distribution P_h.

To obtain the equilibrium state primitives r, consider the expected utility-maximizing solution for trader h. Define

$$\alpha_h(t) = \gamma_h^{t-1} \left\{1 + \gamma_h + \gamma_h^2 + \ldots + \gamma_h^{T-1}\right\}^{-1} P_h(x_t). \tag{2}$$

The demand function obtained by maximizing EU_h subject to $r \cdot z_h \leq 0$ is given by

$$c_h(x_t) = \alpha_h(x_t) W_h / r(x_t), \tag{3}$$

where W_h is h's market wealth $r \cdot \omega_h$: (3) is the well-known condition for maximizing logarithmic utility in which budget shares for contingent claims mirror probability beliefs.

The equilibrium state primitives r are defined by the condition $\Sigma_h z_h(r) = 0$ (with numeraire x_0). In our model, these prices serve to aggregate trader beliefs and discount functions into the beliefs and discount function of a *representative trader*.[6] This feature will enable us to analyze the effect of noise traders through their impact on the beliefs of the representative trader, which we now describe.

[5]We note that we can guarantee that markets will be complete by allowing enough variation in the option exercise prices.

[6]Benninga and Mayshar (1993) have also developed a representative trader model in which there is investor heterogeneity over the degree of risk aversion. In our model, the degree of risk aversion is constant across traders.

The representative trader is defined by probabilities $\{\Gamma(x_t)\}$ and discount factors $\{\gamma_R^t\}$. The probabilities are obtained as follows. Let w_h denote trader h's relative wealth share $W_h/\Sigma_j W_j$, where the summation is over $j = 1$ to H. The probability $\Gamma(x_t)$ is formed as a convex combination of trader probabilities $\Sigma_h \delta_h P_h(x_t)$, where δ_h is the ratio $w_h\gamma_h^t/\Sigma_j w_j\gamma_j^t$. The discount factor $\gamma_R(t)^t$, which is applied to date t payoffs, is defined by $\Sigma_h w_h \gamma_h^t$. It is straightforward to establish that[7]

$$r(x_t) = \gamma_R^t(t)\Gamma(x_t)\,\omega(x_0)/\omega(x_t). \tag{4}$$

III. Equilibrium Prices

In this section, we describe the structure of equilibrium prices and distribution of returns. The main results are summarized in three theorems, and enable us to analyze how volatility and abnormal returns depend on the participation of noise traders in the market.[8]

The first theorem describes the equilibrium rates of return[9] for the market portfolio and risk-free securities of all maturities, as well as options on the market portfolio. As we discussed in Section II, prices can be regarded as being set by the beliefs Γ and discount factor $\gamma_R(t)$ of a representative trader. The representative trader feature enables us to see how noise traders cause security prices to deviate from their efficient levels. When prices are efficient, the representative trader is an information trader. When prices are inefficient, the representative trader is a noise trader. Theorem 1 describes the structure of security prices in terms of the beliefs of the representative trader. To simplify exposition, most results are stated for the special case in which traders share the same discount function, i.e., $\gamma_h = \gamma$. The general case is described in footnotes.

Theorem 1.

i) (Return on market portfolio) The rate of return $\rho_{\omega,j}(x_{t+j})$ earned by purchasing the market portfolio Z_ω on the x_t-market, and holding it j periods is the product of the dividend growth rate and the inverse discount factor.[10] That is,[11]

[7] When all traders agree about the probability distribution (or discount factor), then the representative trader's probability distribution (or discount factor) takes on the common value. In (4), state primitives r are characterized in terms of the probabilities of a representative trader with probability beliefs Γ and discount factors $\gamma_R(t)$. However, the weights used to arrive at Γ and γ_R are themselves a function of r, since W_h is a function of r. To obtain an expression for r which is a function only of exogenous variables, observe that the equilibrium condition $\Sigma_h z_h(r) = 0$ (with numeraire x_0) has the linear form $Ar = b$. Hence, $r = A^{-1}b$.

[8] To save space, we have omitted the proofs of the theorems presented in the paper. However, all results are obtained through straightforward computation.

[9] Returns are always measured in gross terms, where gross equals 1+net.

[10] Hence, the state primitive $r(x_t)$ is the ratio of the probability $\Gamma(x_t)$ to the return $\rho_{\omega,t}(x_t)$. Interestingly, the return distribution on the market portfolio is not dependent on the probability beliefs of traders. This is an artifact of logarithmic utility in which the utility discount rates are constant across time, and common across traders. When discount rates are heterogeneous, probability beliefs enter indirectly through the W_h terms.

[11] To see the generalization to the case of heterogeneous discount factors, focus on $t = 0, 1$ and write (5) in the following form: $\omega(x_1)[1+\gamma+\ldots+\gamma^{T-1}]/\omega(x_0)[\gamma+\ldots+\gamma^t]$. This gives the single-period return $\rho_{\omega,1}$, which generates the multiperiod return $\rho_{\omega,j}(x_{t+j})$ through multiplication. In the heterogeneous case, γ is replaced by $\gamma_R(x_\tau, t)$. The numerator γ_R terms pertain to x_1, and the denominator γ_R terms

$$\rho_{\omega,j}\left(x_{t+j}\right) = \gamma^{-j}\omega\left(x_{t+j}\right)/\omega\left(x_t\right). \tag{5}$$

ii) (Price of market portfolio) The x_t-price $q_\omega(x_t)$ of Z_ω is

$$q_\omega\left(x_t\right) = \omega\left(x_t\right)\left[\gamma + \gamma^2 + \gamma^3 + \ldots + \gamma^{T+t-1}\right]. \tag{6}$$

The limiting value of (6) as T goes to infinity is $(\gamma/(1-\gamma))\omega(x_t)$.

iii) (Sufficient statistic) The variable s_ω is a sufficient statistic for the return distribution of $\rho_{\omega,j}(x_{t+j})$, conditional on x_t.[12]

iv) (Term structure) The equilibrium rate of return to a zero coupon risk-free security that is traded on the x_t-market and matures at date $t+j$ is given by[13]

$$\left(1 + i_j\left(x_t\right)\right)^j = \left[E_\Gamma\left(\rho_{\omega,j}\left(x_{t+j}\right)^{-1} \mid x_t\right\}\right]^{-1}, \tag{7}$$

where $E_\Gamma(\cdot)$ denotes conditional expectation under the representative trader's beliefs Γ.[14]

v) (Option price) Consider a European call option on security Z with exercise price K that expires at date $t+j$. Define Γ_E to be the Γ-probability that the option will be exercised at $t+j$. Define Γ_ρ to be the product of Γ_E and the Γ-expectation of $1/\rho_{\omega,j}$, conditional on exercise. Define Γ_q to be the product of Γ_E and the Γ-expectation of $((q_Z(x_{t+j})/q_Z(x_t))/\rho_{\omega,j})$, conditional on exercise. The expectation in Γ_q concerns the ratio of the growth rate in the security price q_Z relative to the return on the market portfolio. The x_t-price $q_c(x_t)$ of the call option is given by

$$q_c\left(x_t\right) = q_Z\left(x_t\right)\Gamma_q - K\Gamma_\rho. \tag{8}$$

If Z is the market portfolio Z_ω, then (8) takes the form,

$$q_c\left(x_t\right) = q_\omega\left(x_t\right)\gamma^j C\left(\gamma, t, j\right)\Gamma_E - K\Gamma_\rho,$$

pertain to x_0. Notably, γ_R is not time invariant. Rather it evolves stochastically and is a function of both x_t and j. It is obtained as follows. Trader h's wealth $W_h(x_t)$ at x_t is the present value of his consumption in all successor nodes to x_t, normalized by $r(x_t)$. That is, $W_h(x_t) = r'(x_t)\cdot c_h/r(x_t)$, where $r'(x_t)$ is defined in footnote 4. The variable $\gamma_R(x_t)^j$ is defined analagously to γ_R^j in Section II, with $W_h(x_t)$ used in place of W_h. Notice that the heterogeneity of γ_h injects an additional source of volatility into returns.

[12]Part iii of Theorem 1 depends crucially on the homogeneity of $\{\gamma_h\}$, and this feature will be significant for the results described in Section V. The key point is that the value of γ_R to be used on the x_t-market depends on the joint distribution of (w_h, γ_h) at x_t. Keep in mind that with the passage of time, the numeraire keeps shifting to x_t-consumption. Therefore, s_ω alone will not serve as a sufficient statistic for ρ_ω. This is because γ_R itself is a random variable that depends on x_t through the wealth distribution. Consider the question of what information is sufficient for the return distribution on the market portfolio, (and as we shall see in Section V the term structure, options, and mean-variance efficient frontier). The answer is that s_ω, the sufficient statistic for the dividend growth rate, must be augmented by the appropriate cross moment(s) of the (w_h, γ_h) joint distribution. This point should not be confused with the property that as t becomes large, the variable that dominates $\gamma_R(t)$ is $\gamma_{\max}$, where $\gamma_{\max} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_H\}$.

[13]Notice that the term structure is determined by the beliefs of a representative trader having logarithmic utility, just as in Cox, Ingersoll, and Ross (1985).

[14]The general expression for (7) in the heterogenous discount factor case involves $\omega(x_{t+1})/(\omega(x_t)\gamma_R)$ being used in place of $\rho_{\omega,j}$.

where $C(\gamma, t, j)$ is $[\gamma + \ldots + \gamma^{T-t-j-1}]/[\gamma + \ldots + \gamma^{T-t}]$.[15] A similar pricing equation holds for put options. □

A trader h whose subjective beliefs P_h coincide with the objective probabilities Π is an *information trader*. In an *efficient market*, security prices are determined as if all traders were information traders. Equation (4) provides us with a necessary and sufficient condition for market efficiency. Let $a_h(x_t)$ be the value of $\alpha_h(x_t)$ under the condition that $P_h(x_t)$ is the objectively correct probability $\Pi(x_t)$. Define h's discounted forecast error as $\epsilon_h(x_t) = \alpha_h(x_t) - a_h(x_t)$. Then (4) implies the following theorem.

Theorem 2. Prices are efficient if and only if[16]

$$(9) \qquad \sum_{h=1}^{H} W_h \epsilon_h = \operatorname{Cov}\{W_h, \epsilon_h\} + \left(\sum_{h=1}^{H} \epsilon_h / H\right)\left(\sum_{h=1}^{H} W_h\right) = 0. \quad \square$$

Equation (9) implies that price efficiency holds when trader errors are uncorrelated with wealth, and noise trading errors average to zero.[17] This is the condition under which the representative trader is an information trader. There is also the knife edge case of equal and opposite, when neither term in (9) is zero, yet the sum is zero.[18]

Our analysis of anomalies concerns the impact of noise traders on the premium for risk. On the x_t-market, one-period risk is priced in terms of a mean-variance efficient factor ρ_{MV}. Note that we suppress the unit subscript in $\rho_{\omega,1}$ in dealing with one-period returns. Theorem 3 below describes such a factor. Let Z be a typical security that is traded on the x_t-market. Its one-period return, $\rho(Z, x_t)$, from t to $t+1$ is $(Z(x_{t+1}) + q_Z(x_{t+1}))/q_Z(x_t)$. The expected one-period net return to Z, $E_\Pi \rho(Z) - 1$, on the x_t-market is given by the sum of the risk-free rate i_1 and the x_t-risk premium on Z; that is,

$$(10) \qquad E_\Pi \rho(Z) - 1 = i_1 + \beta(Z)\,[E_\Pi \rho_{\mathrm{MV}}(x_{t+1}) - 1 - i_1],$$

where $\beta(Z)$, the mean-variance beta of Z, is $\operatorname{Cov}(\rho(Z), \rho_{\mathrm{MV}})/\operatorname{Var}(\rho_{\mathrm{MV}})$. In (10), computations are made with respect to all successor nodes x_{t+1} of x_t. Therefore, $\beta(Z)$, like $E_\Pi \rho(Z)$, is a function of x_t. The difference $E_\Pi \rho_{\mathrm{MV}} - 1 - i_1$ is the risk premium per unit of beta risk.

In the binomial case ($S = 2$), market completeness is achieved when the risk-free security and the market portfolio are the only securities traded at each date. In this situation, ρ_ω serves as the risk factor ρ_{MV}. However, this need not be true when $S > 2$. Theorem 3 below presents the general case.

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[15] $C(\gamma, t, j)$ is a finite correction factor: $C \to 1$ as $T \to \infty$.

[16] The statement of the theorem is not conditioned on $\gamma_h = \gamma$.

[17] Varian (1985) considers risk-tolerance conditions under which an increase in the spread of trader beliefs reduces the value of the state primitive. He proves that logarithmic utility constitutes a borderline case for this problem.

[18] We interpret a *fad* as a situation in which the average noise trading error is nonzero. In this case, we can speak of noise trader *sentiment* being reflected in prices.

Theorem 3.

i) Consider the likelihood ratio $\Lambda_{ij} = \Pi_{ij}/\Gamma_{ij}$.[19] Let the date $t-1$ state be s_i and the date t state be s_j. The return $\rho_I(x_t)$, to the portfolio of an information trader h for whom $P_h = \Pi$, is given by $\rho_I(x_t) = \Lambda_{ij}\rho_\omega(x_t)$.

ii) The return ρ_{MV} to a mean-variance efficient portfolio has the form,

$$\rho_{MV}(x_t) = \left[1 - \left(\zeta/\rho_I(x_t)\right)\right]/\nu, \tag{11}$$

where $\zeta = [(1 + i_1(x_{t-1}))^{-1} - \nu][E_\Gamma\{\rho_\omega(x_t)^{-1}\rho_I(x_t)^{-1} \mid x_{t-1}\}]^{-1}$, and ν is a positive parameter whose variation generates the mean-variance efficient frontier. On the x_{t-1}-market, the expected return $E_\Pi(\rho_{MV})$ is given by

$$(1/\nu)\left[1 - \frac{\left[\left(1 + i_1\left(x_{t-1}\right)\right)^{-1} - \nu\right] E_\Pi\left\{1/\rho_I\left(x_t\right) \mid x_{t-1}\right\}}{E_\Pi\left\{1/\rho_I\left(x_t\right)^2 \mid x_{t-1}\right\}}\right]. \tag{12}$$

□

For (12), we have used the equivalence relationship $r = \Gamma/\rho_\omega = \Pi/\rho_I$ to convert the Γ-expectation in ζ into a Π-expectation. For small ν, ζ can be interpreted as a discounted risk measure, which enters (11) and (12) in a discounted risk-reward ratio.

IV. Probability Beliefs

In our model, the probability beliefs P_h of every trader reflect learning. We say that h is an *information trader* if his learning process is Bayesian, and the support of his prior includes all Markov transition matrices on S. As Blume and Easley (1992) indicate, Bayesian beliefs are "derivable from the postulates of expected utility maximization." Hence, information traders are rational processors of information. We define a *noise trader* as any trader who is not an information trader; that is, noise traders do not process information rationally.[20] Blume and Easley demonstrate that, under suitable conditions, Bayesians "will dominate relative to any trader" in the long run. We discuss these conditions in IV.D below.

This section provides a formal description of the learning process for both information traders and noise traders. In our model, noise traders are assumed to commit *particular* cognitive errors in the way that they process information and learn. These errors have been well studied within the behavioral decision literature. See Kahneman, Slovic, and Tversky (1982) for a general treatment.

A. Bayesian Beliefs

We begin the discussion of learning for the case where h is an information trader. Imagine that the information trader is aware that the stochastic evolution of states follows a Markov Chain with Π_{ij} denoting the "true" (objective) transition probability of moving to s_j from s_i. The Markov assumption is employed in order

[19]For a general trader h, $\Lambda_{ij}(h)$ is defined as the ratio of h's subjective transition probability to Γ_{ij}. See Section VII.

[20]The term noise trader has other definitions. It may mean traders who are not fully modeled, as opposed to being irrational.

to provide an environment in which it is possible for the true stochastic structure to be learned through observation. We assume that at date 0, no trader knows with certainty the identity of the true Π.

Let the information trader observe the first t transitions of states, and suppose that the state at date 0 is s_1. In a Bayesian framework, initial beliefs are represented by a "prior" density function $Q_0(\Psi)$ representing h's "degree of belief" that Ψ equals Π; see Shefrin (1983) for a detailed exposition of Markov learning. When h observes that state s_i, for instance, occurs at date 1 he revises his initial beliefs. Specifically, he uses Bayes' rule to form the posterior $Q_1(\Psi)$ as the conditional density attached to Q_0 given the transition s_1 to s_i. That is,

$$
\begin{aligned}
(13) \qquad Q_1(\Psi) &= \Pr\{\Psi \mid \text{transition } (s_1, s_i)\} \\
&= \Pr\{(s_1, s_i) \mid \Psi\} \Pr\{\Psi\} / \Pr\{(s_1, s_i)\} \\
&= K_1 Q_0(\Psi) \Psi_{1i}.
\end{aligned}
$$

Continuing in this way produces a Bayesian posterior Q_t at node x_t, with $Q_t = \Pr\{\Psi \mid x_t\}$ being dependent on the history of transitions encapsulated within x_t.

Consider the integral $E_t = \int \Psi dQ_t(\Psi)$. This integral represents h's "estimate" of the Markov transition matrix at x_t. The following assumption leads to a particularly convenient expression for E_t. Suppose that E_0 is given by a vector of positive integers $\{m_{ij}\}$ with the ijth transition probability having the form m_{ij}/m_i, where m_i is equal to $\Sigma_j m_{ij}$. Let t_{ij} be the number of transitions from i to j that have occured along the path leading to a particular node x_t. Also, define t_i to be the associated number of times that s_i has occurred along this path. Shefrin (1983) establishes that the ijth component of E_t is given by

$$
(14) \qquad [E_t]_{ij} = \left(t_{ij} + m_{ij}\right) / \left(t_i + m_i\right).
$$

With probability one, the relative frequency $t_{ij}/t_i \to \Pi_{ij}$ as $t \to \infty$. Therefore, Bayesian revision leads E_t to converge to the objective transition matrix Π in the limit.

Associated with each node x_t is a transition matrix (14) providing the one-step transition probabilities from x_t to x_{t+1}. A Bayesian trader h forms P_h by multiplying branch probabilities from the x_t-transition matrices.

B. Underweighting of Base Rate Information

We divide non-Bayesian noise trader beliefs into two categories that we call i) underweighting of base rate information, and ii) probability mismapping.[21] Base

[21] Our model can accommodate a wide variety of cognitive errors. To fix ideas, we focus on two of the errors that are particularly prominent in the behavioral decision literature and that pertain to issues in finance. We note that other cognitive errors, such as anchoring, hindsight bias, and even random errors, are formally similar to at least one of the two error forms we discuss here. In addition, noise traders implicitly act as if they commit the error of overconfidence, by not searching for evidence that would disconfirm their beliefs. See Einhorn and Hogarth (1978).

rate underweighters form forecasts by overweighting recent events and by underweighting more distant events (the base rate). In our model, base rate underweighters misapply Bayes rule when they use the observational sequence to update their current prior in order to obtain their next posterior. Specifically in (13), $Q_0(\Psi)$ is underweighted relative to the likelihood function $\Pr\{\text{transition} \mid \Psi\}$. Since the posterior Q_t derived at date t becomes the prior for the revision at date $t+1$, underweighting of the base rate occurs at each revision.

In our formulation, overweighting of the likelihood relative to the prior is accomplished as follows. Consider a sequence of positive numbers $k_0, k_2, \ldots, k_t, \ldots$, having the following properties: i) $k_0 = 1$; and ii) if s_i occurs at dates t and $t+j$, then $k_{t+j} \geq k_t$. Suppose that node x_{t+1} features the occurrence of s_i at date t and s_j at date $t+1$. Underweighting of the base rate can be accomplished by treating this single transition from s_i to s_j as if it had occurred $k_{t+1} > 1$ times, instead of once. For example, underweighting of the base rate at date 1 occurs because the information embodied within the transition from s_1 to s_i is overweighted by a factor of k_1. This means that at date 1, the ijth component of E_1 will be set equal to $(m_{1i}+k_1)/(m_i+k_1)$ rather than the correct Bayesian value of $(m_{1i}+1)/(m_i+1)$.[22,23]

Define $K_{t,i}$ as the partial sum Σk_j along the subsequence of transitions from s_i between dates 0 and t. Let s_i occur at date $t-1$. Consider the ratio $\theta_t = k_t/K_{t,i}$. A Bayesian sets $k_t = 1$ for all t, so that the value of θ_t is $1/t_i$. However, the θ_t sequence for a base rate underweighter will exceed $1/t_i$. Indeed, if k_t is chosen such that $k_t > \theta K_{t,i}$ where θ is a positive scalar, then $\theta_t > \theta$ for all t.[24]

Because the $\{k_t\}$ sequence is monotone increasing, the updating procedure will overweigh the contribution of the conditional probability relative to the preceding posterior. Notice that the updated posterior at $t+1$ will appear to feature $k_1 + \ldots + k_t$ transitions rather than t transitions. The ratio (14) (with the k_t transformation) is well defined for a noninteger valued θ as well as an integer valued θ.

C. Probability Mismapping

Probability mismapping describes a family of cognitive errors based on an incorrect mapping between probabilities and realized sequences. In order to motivate the formal definition of a probability mismapping error, consider an example. Imagine tossing a fair coin five times in succession. Suppose that four of the five coin tosses result in heads. Someone who feels that the probability of tails is higher than one-half, because a "tail is due by the law of averages," commits gambler's fallacy. Kahneman and Tversky facetiously refer to this error as the "law of small numbers." Whereas base rate underweighting leads to "positive feedback" forecasts in which recent events are expected to continue, gambler's fallacy leads to "negative feedback" forecasts in which recent events are expected to reverse.

[22]Noninteger values of k_t are permitted.

[23]The base rate underweighting counterpart to (14) is determined as follows. Define $\chi_{ij}(t) = 1$ if s_i occurs at date $t-1$, and the transition at date t is from s_i to s_j: define $\chi_{ij} = 0$ otherwise. Define $\chi_i(t)$ if the date t state is s_i, and zero otherwise. Consider the ratio $[m_{ij}+\Sigma\chi_{ij}(t')k(t')]/[m_i+\Sigma\chi_i(t'-1)k(t')]$, where t' varies from 1 to t. This ratio is the counterpart to (14).

[24]Let the next occurrence of s_j be at date $t+j$. The inequality $\theta_t > \theta$ implies that $K_{t+j,i}/K_{t,i} > (1+\theta)K_{t,i}$, implying that $K_{t+j,i}/K_{t,i}$ grows exponentially.

We formalize probability mismapping as follows. Imagine that trader h uses a Bayesian updating rule. Therefore, at x_t the mean of his posterior E_t is given by (14). Let state s_i occur at date t, and consider the history of the τ most recent transitions from s_i, $\tau < t$. Form the histogram of relative frequencies $F_{t,i}$ associated with these τ transitions. Compare this histogram $F_{t,i}$ with the E_t-transition probabilities, $E_{t,i}$, from s_i. Note that both $F_{t,i}$ and $E_{t,i}$ are S-dimensional vectors. If h believes in the law of small numbers, then he does not use the ith row, $E_{t,i}$, of E_t as his transition matrix at x_t. Instead, he forms his x_t-transition probabilities (from s_i) by assuming that it is likely that the next transition will be chosen to bring the $t+1$-histogram $F_{t+1,i}$ closer to $E_{t,i}$.

We say that trader h commits a probability mismapping error when his x_t-transition vector has the form,

$$E_{t,i} + \xi \left(E_{t,i} - F_{t,i}\right), \tag{15}$$

where ξ is a positive scalar measuring the magnitude of the error. Note that ξ is constrained by the fact that the resulting vector sum is nonnegative. Relative to $E_{t,i}$, this vector sum will increase the likelihood of a state whose $F_{t,i}$-relative frequency is less than the probability specified by $E_{t,i}$. In the fair coin tossing example featuring four out of five heads, (15) has the form $(0.5, 0.5) + \xi(0.5 - 0.8, 0.5 - 0.2)$. For $\xi = 1$, this becomes $(0.2, 0.8)$, meaning that h treats the probability of tails as being 0.8. The maximum value that ξ can take is 1.4, in which case (15) becomes $(0, 1)$.

D. The Persistence of Errors in the Long Run

Are noise traders able to survive in the long run? Blume and Easley (1992) address this question. Their Theorem 5.2 establishes that the relative wealth share of an information trader h,[25] having a discount factor γ_h that is at least as great as that of any other trader, will be bounded away from zero. It is in this sense that the Bayesian beliefs of information traders dominate. Blume and Easley's Theorem 5.5 provides the condition under which the relative wealth of all noise traders declines to zero in the long run.[26] It is important to be aware that the definition of information trader assumes logarithmic utility. The result cannot be guaranteed for a patient Bayesian whose utility function is not logarithmic, and this makes our logarithmic utility assumption special. However, one way for a noise trader to survive is to be sufficiently more patient than all information traders, and not commit errors that are too serious. In our framework, the seriousness of errors is captured by the parameters θ and ξ: θ reflects the extent of base rate underweighting and ξ the extent of probability mismapping. Notably, some noise traders can also survive in efficient markets, provided that their discount factors are at least as high as that of the most patient information trader. We discuss this last point below.

In our framework, noise traders do not all commit the same error. When noise traders survive in the long run, how can we tell which noise traders survive? As in the preceding paragraph, patience enhances the prospect of survival. But what

[25] Blume and Easley's condition involves logarithmic utility, as is the case in our model.

[26] Their analysis deals with i.i.d. processes, whereas we deal with the slightly more general irreducible Markov chain process. The key feature of both is that the strong law of large numbers holds.

about the type of error committed? Blume and Easley have developed an "entropy measure" $I_{\Pi}(c_h, x_t)$ that is designed to address this issue.[27] The entropy function $I_{\Pi}(c_h, x_t)$ measures the distance between h's x_t-budget share trading rule and the true transition probabilities at x_t. If we assume that every noise trader has the same discount factor γ, then the noise trader with the lower entropy dominates: Theorem 5.3, Blume and Easley.[28] Blume and Easley's argument implies that there are some circumstances in which base rate underweighting dominates, and other circumstances in which probability mismapping dominates. This follows because Bayesian beliefs constitute the limiting forms of noise trader beliefs (when parameters ξ and θ_t go to zero). Therefore, when all noise traders vanish in the presence of information traders, continuity of the entropy measure implies that small perturbations of every information trader's beliefs and discount factor continue to be dominant.

There is an important caveat that needs to be added about the survival of noise traders in efficient markets. *Efficient prices protect particular noise traders.* To see why, let (9) hold for all x_t, and let $\gamma_h = \gamma$ for every h. Imagine that every trader's initial portfolio consists exclusively of the market portfolio Z_{ω}. In this case, information traders whose Bayesian posteriors have converged to Π refrain from trade in equilibrium. They continue to hold the market portfolio. Since noise traders trade only among themselves, their wealth cycles only among themselves, and their sole impact on the market is to generate excess volume. That is,

Theorem 4. Let $\gamma_h = \gamma$ for all h. If prices are efficient, then noise traders are codominant with information traders, as a group. In particular, the fraction of market wealth which noise traders hold as a group remains invariant over time. □

Notice that noise traders survive without having higher discount factors than information traders. The preceding argument does not rest on the failure of information traders to trade. Begin with any price efficient equilibrium. Then the composition of initial portfolios can always be reshuffled, but its value preserved at W_h, so that every trader's initial portfolio consists of Z_{ω}. In other words, any price efficient equilibrium can be effectively recast in the form described within the preceding paragraph. Noise traders are not exploited by information traders in price efficient markets and, therefore, survive.

Blume and Easley's Theorem 5.5 provides the condition under which a noise trader vanishes almost surely. That condition requires a noise trader's budget shares to stay bounded away from the true probabilities over the long run. Formally, $\liminf(1/t)\Sigma I_{\Pi}(c_h, x_t)$ is positive for a vanishing noise trader h. We know from (9) that when prices are efficient, *aggregate* noise trader errors average to zero at every x_t. We learn from Blume and Easley that an *individual* noise trader who

[27]The basis of the entropy measure is as follows. Let $X1(x_t)$ be the set of one-period successor nodes x_{t+1} to x_t, and $X'(x_{t+1})$ be the set of *all* successor nodes to x_{t+1}. Let $V'_h(x_{t+1})$ be the value of h's consumption in the union of $X'(x_{t+1})$ and $\{x_{t+1}\}$. Let $V(x_{t+1})$ be the ratio $V'(x_{t+1})/\Sigma V'(y_{t+1})$ where the sum is over all $y_{t+1} \epsilon X1(x_t)$. V is the one-period portfolio budget share. If Π_t is the one period transition probability distribution at t, then the Blume-Easley entropy measure $I_{\Pi}(c_h, x_t)$ at x_t is $\Sigma \Pi_t(x_{t+1}) \cdot \mathrm{Log}(\Pi_t(x_{t+1})/V(x_{t+1}))$ where the summation is over $X1(x_t)$.

[28]Blume and Easley use the logarithm of wealth share to distinguish among traders who respectively dominate, survive, and vanish. Dominance occurs when lim inf log wealth share is positive. Survival occurs when lim sup log wealth share is positive. A trader vanishes when lim sup log wealth share is zero.

survives in the long run, in the presence of information traders, must have correct beliefs about transition probabilities beliefs *infinitely often*. In this respect, we note the absence of bias in the noise trader errors described in Blume and Easley's 3.2 and 3.3. Over time, the expected value of E_t for both noise trader groups is Π.[29] These noise trader beliefs reflect volatility over time, but not bias.

The following example illustrates the fact that a noise trader can survive in the presence of information traders when his beliefs are correct infinitely often. Consider an i.i.d. binomial example ($S = 2$) with the objective transition probabilities being equiprobable. Let there be three traders $h1$, $h2$, and $h3$ who have initial portfolios consisting exclusively of Z_ω, who share the same discount factor, and who have the same level of initial wealth. Take trader $h1$ to be an information trader. Trader $h2$ is a base rate underweighter. Hence, $h2$'s beliefs reflect excess volatility relative to a Bayesian, but not bias. Trader $h3$'s beliefs are defined such that

$$\epsilon_{h3}(x_t) = -\epsilon_{h2}(x_t) \quad \text{for all} \quad x_t.$$

This implies that (9) is satisfied and therefore prices are efficient. Of course, trader $h3$ is also a noise trader. Whenever $h2$ predicts continuation, $h3$ predicts reversal. Let $h2$ and $h3$ have flat priors. In this example, trade only takes place between $h2$ and $h3$. Trader $h1$ does not trade in equilibrium, but instead consumes the dividends from his endowment. Therefore, $h2$ and $h3$ are codominant as a group with $h1$. By symmetry, the entropy measure argument applies equally to both traders. Therefore, $h2$ and $h3$ are both dominant.

V. Volatility

At the conclusion of Section IV, we presented an example in which both the average noise trading error and the wealth-error covariance were zero. As a result, prices are efficient. However, in general, requiring that the wealth-error covariance be zero for every x_t is very demanding, even if the average noise trader error is zero. Indeed, if noise trader errors are restricted to base rate underweighting and probability mismapping, then for suitably long T, prices cannot be efficient on every x_t-market.[30] In this section, we study the effect of noise trader errors

[29] As was discussed in IV.A and IV.B, both information traders and base rate underweighters form their probability estimates as weighted sums of observed relative frequencies. While any convex combination of observed frequencies provides an *unbiased* estimate of the true probabilities, equal weights are required for a minimum variance estimator (Feller (1967)). However, since base rate underweighters weigh recent observations more heavily than earlier observations, the associated E_t will not provide a minimum variance estimator for Π: the weights attached to any past observation approach zero with the passage of time. As a result, the variance of the base rate underweighting estimates does not tend to zero. Therefore, traders who underweigh the base rate form estimates of Π_{ij} that take on the correct value Π_{ij} *on average*, but that meander around the correct value because the variance of their estimates stays bounded away from zero. For probability mismappers, the expected value of the histogram F_t is E_t by the strong law of large numbers. Therefore, the expected value of the probability mismapper's Π-estimate (15) is $E(E_t) = \Pi$.

[30] If one type of noise trader predominates, then the average noise trader error is nonzero. If both types of noise traders participate, then the zero covariance condition cannot be maintained. Along runs, noise trader errors converge to nonzero constants, but wealth is transferred from one group of noise traders to another. This disturbs the zero covariance condition.

part of the representative trader. Brown and Schaefer's observation leads them to propose a "simple market timing rule to exploit the market's underestimation..." (p. 37).

Consider the characteristics of the option pricing formula (8). This expression indicates the way that the return on the market portfolio affects option prices. The key variables in this respect are the option delta Γ_q, the representative trader's probability Γ_E that the option will be exercised, and Γ_ρ which is based upon the conditional expected inverse return, $1/\rho_w$, of the market portfolio. Notice that the option delta Γ_q reflects the conditional expected value of the growth rate in the security price relative to the return on the market portfolio.[36] Significantly, the values of these variables are *implied* by option prices.[37] The next theorem concerns the stability properties of option prices when prices are efficient. To avoid the specification of additional technical assumptions on Z, the theorem is stated for the case in which Z is the market portfolio Z_ω.

Theorem 7.

i) Consider the infinite horizon case ($T \to \infty$).[38] When prices are efficient, then s_ω is a sufficient statistic for option prices. Specifically, for given q_ω, K, and j, the value of (8) along the equilibrium only depends upon the current s_ω.

ii) When prices are efficient, the ergodic theorem for Markov chains implies that the magnitude of the implied values of Γ_E, Γ_q, and Γ_ρ are time invariant for large j. They are determined by the ergodic distribution π, and are therefore objectively correct as well. Hence, for given K, prices of long-term options along the equilibrium only vary asymptotically in response to changes in q_ω. □

Consider how noise trader errors affect option prices. The easiest way to investigate this issue is to focus on one-period European call options on Z_ω in the binomial case. Choose an exercise price K between $uq_\omega(x_t)$ and $dq_\omega(x_t)$. Suppose that the terms of both call and put options are structured such that the x_{t+1}-dividend on Z_ω accrues to the trader who holds Z_ω at the end of x_t. In this case, a call option pays $uq_\omega(x_t) - K$ if s_u occurs at x_{t+1}, and zero otherwise. Let s_i denote the Markov state which occurred at x_t. By part v of Theorem 1, $q_c(x_t)$, the price of the call option on the x_t-market, is given by

$$q_c(x_t) = q_\omega(x_t)\gamma C(\gamma, t, 1)\Gamma_{iu} - K\Gamma_{iu}\gamma/u. \tag{16}$$

Notice that Γ_{iu} is the probability that the call option will be exercised at date $t+1$, conditional on x_t.

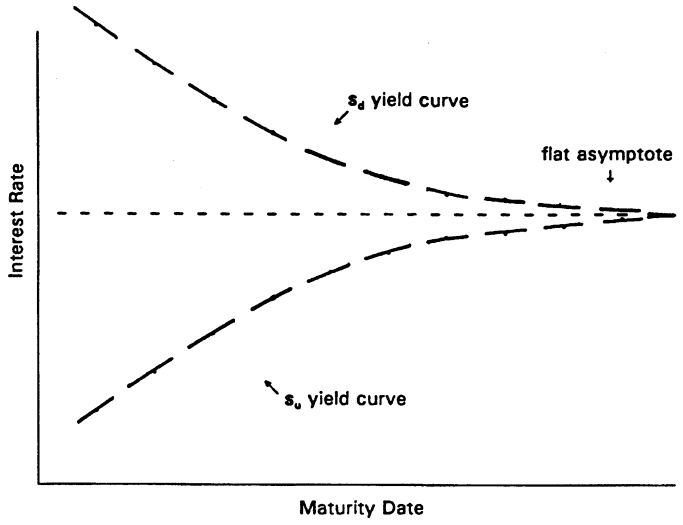
Imagine a run of s_u-states that is long enough to render base rate underweighters unduly optimistic. Because their (incorrect) beliefs will appear to be validated along the run, wealth will shift in their favor. If the run is sufficiently long, this will cause the beliefs of the representative trader to feature $\Gamma_{iu} > \Pi_{iu}$,

[36]If Z is a stock that pays zero dividends, then Γ_q is based upon the Γ-expectation of $\rho(Z)/\rho_\omega$, conditional on exercise. This ratio has the same units and interpretation as the market beta of Z, and can be regarded as an "up beta."

[37]The recovery of Γ_q and Γ_ρ is more complicated than the recovery of stock return variance in a Black-Scholes framework. Here the procedure involves recovering the state primitives from the prices of a collection of options having full span.

[38]By focusing on the infinite horizon case, we avoid the need to qualify statements to reflect the finite correction term $C(\gamma, t, j)$.

FIGURE 1



source of this noise is the oversensitivity of noise trader beliefs to the recent history. It is easy to see why the beliefs of probability mismappers are sensitive to recent events: Expression (15) has the form $(1 + \xi)E_{t,i} + \xi Ft, i$, where $F_{t,i}$ is the histogram based on the τ most recent transitions from s_i. The source of noise in base rate underweighters can be seen through a comparison with Bayesians. Consider the rate at which Bayesian expected transition probabilities change from one observation to another. This change is of the order $1/t$ (indeed, it is bounded above by $1/t_i$), which goes to zero with t. But for a base rate underweighter for whom $\theta_t = \theta$, the corresponding bound is $\theta/(1 + \theta)$, which is bounded away from zero as $t \to \infty$.[35]

Theorem 6 below enables us to compare the volatility characteristics of the term structure between the efficient and inefficient cases.

Theorem 6. Let prices be inefficient.

i) The variable s_ω is not a sufficient statistic for the term structure.

ii) There will not be a common asymptote attached to all equilibrium yield curve realizations. Instead of exhibiting zero volatility in the tail, the evolution of the yield curve exhibits nonzero volatility.

iii) For some choice of parameters, the yield curve at some x_t will slope in the wrong direction relative to its efficient counterpart. □

Evidence presented in Brown and Schaefer (1994) suggests that Theorem 6 describes the real term structure better than Theorem 5, (the Cox, Ingersoll, and Ross (CIR) case). Brown and Schaefer find positive volatility in long-term real rates and unstable parameters in their CIR estimates. Moreover, they find that the market underestimates "the speed at which short rates revert to their long-run mean" (p. 137). This finding is consistent with base rate underweighting on the

[35] Compute k_t/K_{t+1}, which is the ratio of the weight attached to the most recent observation divided by the weight total.

part of the representative trader. Brown and Schaefer's observation leads them to propose a "simple market timing rule to exploit the market's underestimation..." (p. 37).

Consider the characteristics of the option pricing formula (8). This expression indicates the way that the return on the market portfolio affects option prices. The key variables in this respect are the option delta Γ_q, the representative trader's probability Γ_E that the option will be exercised, and Γ_ρ which is based upon the conditional expected inverse return, $1/\rho_w$, of the market portfolio. Notice that the option delta Γ_q reflects the conditional expected value of the growth rate in the security price relative to the return on the market portfolio.[36] Significantly, the values of these variables are *implied* by option prices.[37] The next theorem concerns the stability properties of option prices when prices are efficient. To avoid the specification of additional technical assumptions on Z, the theorem is stated for the case in which Z is the market portfolio Z_ω.

Theorem 7.

i) Consider the infinite horizon case ($T \to \infty$).[38] When prices are efficient, then s_ω is a sufficient statistic for option prices. Specifically, for given q_ω, K, and j, the value of (8) along the equilibrium only depends upon the current s_ω.

ii) When prices are efficient, the ergodic theorem for Markov chains implies that the magnitude of the implied values of Γ_E, Γ_q, and Γ_ρ are time invariant for large j. They are determined by the ergodic distribution π, and are therefore objectively correct as well. Hence, for given K, prices of long-term options along the equilibrium only vary asymptotically in response to changes in q_ω. □

Consider how noise trader errors affect option prices. The easiest way to investigate this issue is to focus on one-period European call options on Z_ω in the binomial case. Choose an exercise price K between $uq_\omega(x_t)$ and $dq_\omega(x_t)$. Suppose that the terms of both call and put options are structured such that the x_{t+1}-dividend on Z_ω accrues to the trader who holds Z_ω at the end of x_t. In this case, a call option pays $uq_\omega(x_t) - K$ if s_u occurs at x_{t+1}, and zero otherwise. Let s_i denote the Markov state which occurred at x_t. By part v of Theorem 1, $q_c(x_t)$, the price of the call option on the x_t-market, is given by

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Notice that Γ_{iu} is the probability that the call option will be exercised at date $t+1$, conditional on x_t.

Imagine a run of s_u-states that is long enough to render base rate underweighters unduly optimistic. Because their (incorrect) beliefs will appear to be validated along the run, wealth will shift in their favor. If the run is sufficiently long, this will cause the beliefs of the representative trader to feature $\Gamma_{iu} > \Pi_{iu}$,

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[37]The recovery of Γ_q and Γ_ρ is more complicated than the recovery of stock return variance in a Black-Scholes framework. Here the procedure involves recovering the state primitives from the prices of a collection of options having full span.

[38]By focusing on the infinite horizon case, we avoid the need to qualify statements to reflect the finite correction term $C(\gamma, t, j)$.

for $i = u$. The resulting effect on option prices occurs through Γ_E and Γ_ρ. Observe that the representative trader's optimism imparts upward bias to the probability of exercise, Γ_E. However, the conditional expected value of $1/\rho_\omega$ remains constant at γ/u, although in the general multinomial case it declines. Equation (16) implies that the call option is overpriced, (and put option is underpriced). Hence, call options prices rise because traders become more optimistic about the return distribution of the asset underlying the option.

Theorem 8. Let prices be inefficient.

i) The variable s_ω is not a sufficient statistic for option prices.

ii) The prices of long-term options are not, asymptotically, a function of the price of the underlying asset alone.[39]

iii) If implied volatility is time varying, then the Black-Scholes formula cannot be the limiting form of the binomial option price formula. □

In our theory, the most important moment of the return distribution for the price of an option on Z_ω is the expected inverse return $E_\Pi(1/\rho_\omega)$, rather than the return standard deviation σ.[40] However, to the extent that volatility affects option prices, it is Γ-volatility, the return standard deviation with respect to Γ, that is relevant. If prices are inefficient, Γ-volatility will generally differ from its objective counterpart, Π-volatility.[41]

If option prices asymptotically conform with Black-Scholes, then Black-Scholes implied volatility will be Γ-volatility. In this case, implied volatility will be the same regardless of the exercise price of the option.[42] However, Theorem 8 suggests a difficulty in the applicability of Black-Scholes when prices are inefficient. This is because implied volatility in the binomial model underlying Black-Scholes is driven by the support of dividend growth rates $\{d, u\} = \{\omega(x_{t+1})/\omega(x_t) \mid s \epsilon S\}$, but not their probabilities. However, the support is fixed in our model. Hence, there is only *one* limiting value of implied volatility $\sigma_\omega(t+j \mid x_t)$ consistent with Black-Scholes. Of course, probabilities are the object of noise trader errors. Consequently, Black-Scholes implied volatility cannot be time varying.[43]

We turn next to mean-variance efficiency. Theorem 3 describes the structure of a mean-variance efficient factor ρ_{MV} that can be used to price risk on the x_t-market. It follows from (11) that there are three interdependent variables that determine the price of risk on the x_t-market, namely ρ_I, ρ_ω, and i_1.

---

[39] See Stein (1989) for an empirical discussion of inefficiency associated with the price of long-term options.

[40] The option price (8) in our model takes a different form from the standard binomial expressions described in Cox, Ross, and Rubinstein (1979), and its limiting form(s). Nevertheless, we can recast (8) into the standard form, although this will mask the role of the market portfolio in pricing options. Moreover, care needs to be taken with the impact of return volatility on option prices. Specifically, the increase in q_C cannot always be attributed to increased volatility. In the binomial example above, an increase in Γ_{iu} leads to a higher call option price. But the derivative $\partial\sigma/\partial\Gamma_{iu}$ can take on either sign depending upon whether Γ_{iu} lies above or below ½.

[41] See Canina and Figlewski (1993).

[42] If this condition fails, then option prices do not conform with Black-Scholes.

[43] It follows from Cox, Ross, and Rubinstein (1979), and the extension described in Madan, Milne, and Shefrin (1989) that if there is a limiting form for the option price formula, then it must be Poisson based, and feature jumps. In respect to implied volatility, we note that the multinomial case has more degrees of freedom in achieving a Black-Scholes limit than the binomial case. But it is still extremely restrictive and admits no limiting role for return moments, other than the variance.

Theorem 9. Let prices be efficient.

i) The variable s_ω is a sufficient statistic for ρ_{MV}, the interest rate i_1, and the risk premium $E_\Pi(\rho_{\mathrm{MV}}) - 1 - i_1$.[44] If the dividend growth rate of a traded security Z is solely a function of the current Markov state s, then s_ω is a sufficient statistic for $\beta(Z)$, the mean-variance beta of Z.

ii) If Π is i.i.d., then (11) implies that the mean-variance risk factor ρ_{MV} is also i.i.d. Thus, the following variables are time invariant: the expected return $E_\Pi(\rho_{\mathrm{MV}})$, the interest rate i_1, and the risk premium $E_\Pi(\rho_{\mathrm{MV}}) - 1 - i_1$. If the dividend growth rate of a security Z is solely a function of the Markov state s, then its mean-variance beta, $\beta(Z)$, is time invariant. In this case, risk is priced according to an intertemporal one-factor asset pricing model with time invariant betas.[45] □

Expression (11) describes exactly how the risk factor ρ_{MV} is modified by the presence of "noise" in prices. Recall that noise traders impact ρ_I and i_1, two of the three determinants of ρ_{MV}. This implies the next theorem.

Theorem 10. Let prices be inefficient. The variable s_ω is not a sufficient statistic for the factor ρ_{MV}, which prices risk. □

Our theory enables us to investigate the comovement in security prices. When prices are efficient, the sufficient statistic s_ω drives the return ρ_ω on the market portfolio Z_ω, the term structure, prices of options on Z_ω, and the mean-variance efficient frontier. We call this the *single driver property*. It is straightforward to describe the relationship between any two of these variables when the single driver property holds. For example, consider the relationship between the risk premium on the market portfolio and the change in the differential between long-term and short-term interest rates.[46] In the binomial case, the relationship can be described by four points that correspond to the transitions *du*, *dd*, *uu*, *ud*. Associated with each transition is a value for the change in the yield curve differential. Associated with the first state of each transition is a risk premium. It is readily shown that the relationship between the two variables is precise and can be plotted as a discrete "step function."[47]

When prices are efficient, the risk premium for any security is determined by beta and the return distribution of the market portfolio; see Theorems 3 and 9.

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[44]This is because when prices are efficient, ρ_ω determines both ρ_I and i_1.

[45]The strong law of large numbers for Markov Chains implies that the i.i.d. results generalize to long holding period returns. That is, when prices are efficient, the long-term interest rate, long-term risk premium, and long-term beta are time invariant. If prices are inefficient, this is not necessarily the case.

[46]The relationship between the change in yield curve differential and the return premia on securities has been discussed by Ferson and Harvey (1991).

[47]There are at most three possible values for the change in yield curve differential, with the value associated with *dd* and *uu* being zero. These transitions involve no change in the yield curve. There are at most two values for the conditional risk premium, one associated with s_u and the other with s_d. The conditional risk premium can be expressed as the difference of two functions, the conditional expectation $E_\Pi \rho_\omega$, which is a linear function of Π_{iu}, and $(1 + i_1)$, which is a nonlinear function of Γ_{iu}. The plot of these two functions reveals that the magnitude of each conditional risk premium is determined by how far Π_{iu} is from the $\{0, 1\}$ boundary, for $i = u, d$: a boundary probability produces a zero risk premium. The plot of the four yield curve-risk premium points can be described as a "discrete step function." The slope of this function indicates the sign of the relationship for the specific probabilities in question. But it also indicates that the correlation between the variables is imperfect.

When prices are inefficient, the contribution of the return on the market portfolio to the explanation of risk premia weakens. The failure of the single driver property enables some other variable, such as the change in yield curve differential, to provide some additional explanatory power for risk premia.[48]

When prices are inefficient, noise trader beliefs contribute a second driver. The second driver makes the relationship between the risk premium on the market portfolio and changes in the differential between the long-term and short-term interest rates more complex. To get a sense of the effect of the second driver, consider a lengthy s_u-run in the binomial case. The run produces a noise trader bubble. Along the run, the wealth of base rate underweighters grows relative to the wealth of probability mismappers and information traders, and as a result prices are inefficient. Specifically, relative to the efficient price benchmark, index call options are overpriced and index put options are underpriced; hence, the put-call price difference is excessive. Implied volatility declines. The entire yield curve rises, short-term rates rise above long-term rates, but the differential narrows. The return that the representative trader expects on the market portfolio rises, but the associated risk premium ($E_\Gamma \rho_\omega - 1 - i_1$) declines. Of course, the true premium ($E_\Pi \rho_\omega - 1 - i_1$) also declines, but, in addition, it turns *negative* for a sufficiently long run. As a result of mispricing, the premium ($E_\Pi \rho_I - 1 - i_1$) attached to the portfolio of an information trader rises. Therefore, the premium ($E_\Pi \rho_{\mathrm{MV}} - 1 - i_1$) attached to a mean-variance efficient portfolio rises. As we shall see in Section VII, volume declines along the run.

VI. The Anatomy of Return Anomalies

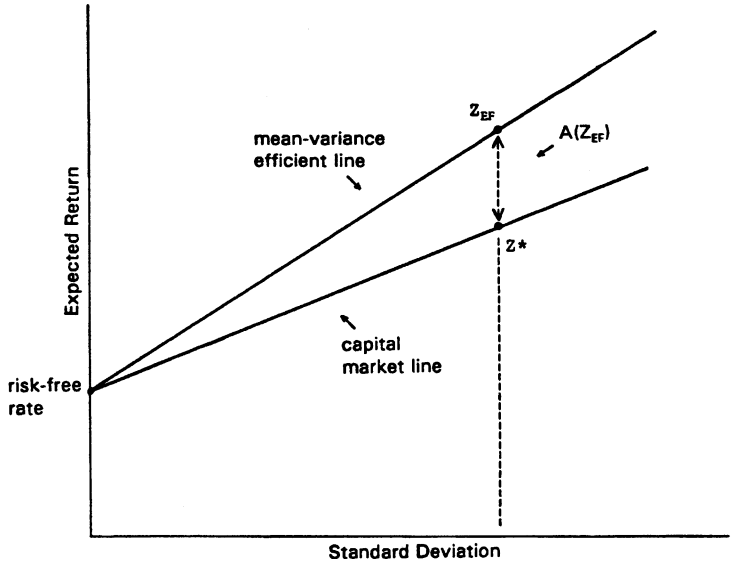
Let ρ^* be a mean-variance factor (ρ_{MV}) for the case in which prices are efficient ($\Gamma = \Pi$). Theorem 3 indicates that ρ^* is a function of ρ_ω, the return on the market portfolio. We call ρ^* the market factor. Let $\beta^*(Z)$ be the beta of security Z measured relative to the market factor ρ^*. Call β^* the market beta of Z. The expected return $E_\Pi \rho(Z) - 1$ to security Z is given by the sum $E^* = i_1 + \beta^*(Z)(E_\Pi \rho^* - 1 - i_1) + A(Z)$. We call $A(Z)$ the expected abnormal return. If prices are efficient and we price the risk of any security in terms of ρ^*, then $A(Z)$ is zero. However, suppose we use ρ^* as the sole factor to price risk when prices are inefficient. In this case, we should expect to find nonzero abnormal returns. This is because risk is priced in terms of ρ_ω only when prices are efficient. Recall that Theorem 3 indicates that risk is priced in terms of ρ_I, the return to the portfolio of an information trader, and ρ_I is the product of the two functions Λ and ρ_ω. Notably, $\Lambda \neq 1$ when prices are inefficient.

Theorem 11 below characterizes the abnormal return function. To develop the notation for this theorem, choose a risk factor ρ_{MV} that conforms with (11) and has the same standard deviation as the market factor ρ^*. If prices are inefficient, then ρ_{MV} is a true risk factor, whereas ρ^* is not. In Figure 2, we depict the mean-variance efficient risk-return line from the risk-free rate through the risk-return point for

[48] As to the failure of the single driver property, Ferson and Harvey (1991) state that "the stock return premium is the most important for capturing predictable variation of the stock portfolios, while premiums associated with interest rate risk capture predictability of the bond returns" (p. 385).

ρ_{MV}. We also depict the capital market line from the risk-free rate through the risk-return point for the market factor ρ^*.

FIGURE 2



Let $\beta(Z)$ be the mean-variance beta of any portfolio Z relative to ρ_{MV}: $\beta(Z)$ is the "true" beta. The beta $\beta(\rho^*)$ of ρ^*, relative to the mean-variance efficient return ρ_{MV}, is $\text{Cov}(\rho^*, \rho_{MV})/\text{Var}(\rho_{MV})$. Effectively, $\beta(\rho^*)$ measures the degree to which ρ^* is efficient. Because ρ_{MV} has been selected to have the same standard deviation as ρ^*, $\beta(\rho^*) \leq 1$. If $\beta(\rho^*) = 1$, ρ^* is mean-variance efficient. If $\beta(\rho^*) = 0$, all risk in ρ^* is unpriced.

Consider the ratio $\beta(Z)/\beta(\rho^*)$. Observe that this ratio has the same units as the market beta $\beta^*(Z)$, namely the return on Z divided by the return ρ^*. Both $\beta^*(Z)$ and $\beta(Z)/\beta(\rho^*)$ relate the premium on Z to the premium on ρ^*. However, keep in mind that not all risk in ρ^* need be priced. What the market beta $\beta^*(Z)$ measures is the amount of ρ^*-risk in $\rho(Z)$, both priced and unpriced. But $\beta(Z)/\beta(\rho^*)$ reflects all priced risk in $\rho(Z)$ relative to the premium in ρ^*. Therefore, we can interpret the difference $\beta(Z)/\beta(\rho^*) - \beta^*(Z)$ as a correction to the market beta. We refer to it as the "beta correction."

Theorem 11.

i) The expected abnormal return $A(Z)$ to Z is given by

$$A(Z) = \left(\left[\beta(Z)/\beta\left(\rho^*\right)\right] - \beta^*(Z)\right)\left(E_{\Pi}\left(\rho^*\right) - 1 - i_1\right). \tag{17}$$

ii) If prices are efficient, then $A(Z) = 0$.

iii) If the return $\rho(Z)$ is perfectly correlated with ρ^*, then $A(Z) = 0$. □

Theorem 11 tells us that abnormal returns are proportional to the risk premium on the mismeasured risk factor ρ^*. Notably, the factor of proportionality is the

"beta correction," which provides a link between market beta and abnormal returns. At the same time, keep in mind that part iii of Theorem 11 tells us that the beta correction will be small for securities whose returns are closely correlated with the mismeasured benchmark ρ^*.

Equation (17) indicates that the expected abnormal return to a security is i) an increasing function of its mean-variance efficient beta; and ii) a decreasing function of its market beta. Hypothetically, suppose that we were to sort all securities according to the value of the ratio $\beta(Z)/\beta^*(Z)$. Consider a group of securities that share the same value for $\beta(Z)/\beta^*(Z)$. As can be seen from Figure 2, within this group, the expected abnormal return is increasing in market beta $\beta^*(Z)$. In other words, the highest abnormal returns within each group will be associated with the highest market betas. In this regard, consider the empirical evidence linking abnormal returns, market beta, and cognitive errors.

De Bondt and Thaler (1985) attribute the overreaction effect to base rate underweighting. De Bondt and Thaler (1989) describe how the betas of losers are lower than the betas of winners during the formation period when the winners are outperforming the losers. However, the direction of the inequality is reversed during the test period, when the previous losers earn positive abnormal returns and the previous winners earn negative abnormal returns.[49] This is to be expected if market beta serves as a proxy for our "beta correction" term in Theorem 11. To take the argument one step further, we suspect that the winner betas and loser betas will move into equality at the conclusion of the test period when the abnormal returns disappear. Chopra, Lakonishok, and Ritter (1992) extend the analysis by regressing realized returns on beta, past returns, and the market value of equity. They report that extreme losers, small firms, and high betas occur in conjunction, thereby reinforcing the link between abnormal returns and beta.

Theorem 11 suggests that securities that feature high abnormal returns should also have high market betas. However, the theorem also makes clear that not every high beta security will feature high abnormal returns. Indeed, the theorem provides the reason why market beta, $\beta^*(Z)$, may lack power in explaining returns.[50] Fama and French (1992) argue that beta offers no residual explanatory power for returns after market value of equity and book to market are taken into account. Indeed, market beta will also lack power in explaining abnormal returns if it does not serve as a proxy for beta correction.

VII. Volume

This section discusses how volume emerges as a byproduct of the reaction of noise traders to new information. Notably, trading volume is not determined by differences in opinion, but by the *rate of change* of those differences (see Karpoff (1986)). Below we discuss why, in a Markov setting, noise trader errors prevent these differences from stabilizing. For purposes of illustration, we use the

[49] Ball and Kothari (1989) argue that the difference in returns between winners and losers stems from the change in financial structure. The stock of losers (winners) becomes more (less) risky since their debt ratios increase (decrease) at the beginning of the test period.

[50] If the average high beta stock contains low levels of priced risk, then the relation between returns and market beta may well be negative.

binomial case in which all portfolios consist of the one-period risk-free security and the market portfolio.

Consider the x_t-market, and let the date t state be s_i. Define the transition likelihood ratio $\Lambda_{ij}(h) = P_{ij}(h)/\Gamma_{ij}$.[51] If the date $t+1$ state is j, it follows from (3) and (4) that the single-period return to h's portfolio in x_{t+1} is $\Lambda_{ij}(h)\rho_\omega(x_{t+1})$. Straightforward calculation based on (3) and (4) indicates that h's x_t-portfolio contains the following proportion of the risk-free security,

$$N_h = [\Lambda_{id}(h) - \Lambda_{iu}(h)]\left[ud/(u-d)i_1\right].$$

Similarly, the portfolio h selects on the x_t-market contains the following proportion (hedge ratio) of units of Z_ω,

$$M_h = [u\Lambda_{iu}(h) - d\Lambda_{id}(h)]/(u-d).$$

Observe that when h's beliefs agree with those of the representative trader, meaning $\Lambda_{ij}(h) = 1$, then h invests his wealth exclusively in the market portfolio. That is, $N_h = 0$ and $M_h = 1$.

When $\gamma_h = \gamma$, every trader's consumption is completely financed from the dividends and interest generated by his portfolio.[52] Therefore, after date 0, the only reason to trade, apart from rolling over the risk-free security, is a change in the transition likelihood difference $\Lambda_{id}(h) - \Lambda_{iu}(h)$. In the binomial model, trading volume occurs when traders shift between the risk-free security and market portfolio. Therefore, we can measure trading volume by focusing on the risk-free security. Since N_h refers to the fraction of h's portfolio allocated to the risk-free security, trading volume is obtained by applying $|\partial N_h|$, the absolute value of the change in the risk-free allocation from x_{t-1} to x_t, to the x_t-value of h's portfolio. Theorem 12 follows.

Theorem 12.

i) Volume, expressed in units of date t consumption, is proportional to[53]

$$\sum_{h=1}^{H} W_h(x_t)\left|\partial\left[\Lambda_{id}(h) - \Lambda_{iu}(h)\right]\right|/i_1. \tag{18}$$

ii) $\Lambda_h = 1$ for all x_t if and only if h is an information trader. If $\Lambda \neq 1$, the maintenance of a constant Λ in the face of changing Γ is a knife edge case. In the presence of noise traders, it is impossible to maintain the likelihood ratio Λ for all x_t as the one-step transition probabilities change. This generates noise trader volume. □

Expression (18) indicates how trading volume depends upon the degree of dispersion in beliefs. Notably, it emphasizes that it is the *change in dispersion*

[51] $\Lambda_{ij}(h)$ is a function of x_t, although we suppress this dependence in the notation.

[52] When γ_h varies across h, impatient traders also trade to finance consumption. In models with varying rates of risk aversion, such as Benninga and Mayshar (1993), volume also reflects risk shifting. In setting aside these issues, this section concentrates entirely on trading volume associated with noise trading.

[53] A zero coupon bond is normalized so that its x_t-price is always one unit of x_t-consumption.

from period to period that is the fundamental determinant of trading volume. Indeed, zero volume is compatible with belief dispersion. This is easily seen when all traders believe that Π is i.i.d., but disagree about the value of the transition probabilities. Suppose that we hold constant the transition probability beliefs of every trader h. Then $\Lambda(h)$ is time invariant, and (18) implies that trading volume is zero, even though the dispersion of beliefs is nonzero.

Outside the i.i.d. case, the condition for a zero *change* in dispersion, when there is already nonzero dispersion, is quite restrictive. In our model, positive dispersion cannot involve a zero change in dispersion at all dates in which there is a transition from one Markov state to another. For instance, outside the knife edge cases, including $\Lambda = 1$, it is impossible to maintain the likelihood ratio Λ as the one-step transition probabilities change. That is, we cannot maintain a constant Λ in the face of changing Γ.[54] What is more important is the condition necessary for a noise trader to maintain the same portfolio as the one-step transition probabilities Γ evolve.[55] Therefore, noise traders must generate nonzero volume in the transition from one state s_i to another s_j.[56]

VIII. Conclusion

We develop a capital asset pricing theory in a market where noise traders interact with information traders. Noise traders are traders who commit cognitive errors while information traders are free of cognitive errors. We provide a behavioral mean-variance efficiency theory, a behavioral option pricing theory, and a behavioral term structure theory. The framework is a comprehensive one, not a collection of separate theories, and we describe the links between the various components of the overall structure. We derive a necessary and sufficient condition for the existence of price efficiency in the presence of noise traders and analyze the effects of noise traders on price efficiency, volatility, return anomalies, volume, and noise trader survival.

When prices are efficient, security prices and, hence, volatility are determined through a single driver, a sufficient statistic consisting only of new information. That single driver drives the mean-variance efficient frontier, the return distribution of the market portfolio, the premium for risk, the term structure, and the price of options. One manifestation of the single driver property is that the volatility of long-term interest rates is zero. Another is that the return moments implied in the prices of options are also stable functions of the sufficient statistic. Indeed,

[54]Let $P = \lambda\Gamma$, so that we maintain the ratio $P/\Gamma = \lambda$ as Γ varies. Then we cannot maintain as constant the ratio $(1 - P)/(1 - \Gamma)$, unless $\lambda = 1$.

[55]To maintain a constant M_h, P must have the form $\Gamma + K\Gamma(1 - \Gamma)[(u - (u - d)\Gamma)]^{-1}$, where K is the difference $u\Lambda_{iu} - d\Lambda_{id}$. Neither base rate underweighting nor probability mismapping generate subjective transition probabilities that adhere to this restriction. Moreover, if noise trader errors were to have this form, then their wealth-weighted convex combination would have to produce Γ at every date. This is even more restrictive.

[56]Significantly, the noise trader errors described in Section IV feature nonconstant likelihood ratios in general. The magnitude of noise trading errors associated with the most recent transitions from s_u need not be the same as those associated with the most recent transitions from s_d. For instance, along a long s_u-run, trading volume actually shrinks along the run, but will increase dramatically once the run concludes, as long as the most recent transitions from s_d were not exclusively to s_u. See Gallant, Rossi, and Tauchen (1992) for empirical evidence on the volume effects following dramatic price changes.

the return moments implied in the prices of long-term options are time invariant. Similarly, the premium for risk is a stable function of beta and the return distribution of the market portfolio. No other variable, such as the market value of equity or the term structure affects the premium for risk. However, conditions for price efficiency are not compelling. Noise traders act as a second driver and they steer the market away from price efficiency.

When prices are inefficient, new information is no longer a sufficient statistic. Old information continues to affect prices, volatility, the premium for risk, the term structure, and option prices. The effect of noise traders is uniform neither across securities nor across time. For example, the impact of noise traders on the return to the market portfolio is less pronounced than the impact on the term structure. For the term structure, volatility is especially pronounced in long-term rates. Moreover, noise traders can cause inefficient inversions in the term structure. Noise traders also distort option prices by affecting the relevant implied return moments, including volatility. The Black-Scholes model is not flexible enough to accommodate fluctuations in volatility induced by noise traders. In addition, our theory suggests that the distribution moment relevant to option prices is the expected inverse return rather than the return variance.

The shock that noise traders generate is common to all security markets, and that shock is reflected in correlations among the term structure, the premium for risk, and option prices. As a result, the term structure and/or option prices may appear to contribute to the explanation of the risk premium. Noise traders also affect the volume of trade, and we analyze the effect of the rate of change in trader beliefs on volume.

The analysis of the premium for risk offers insights into the relationship between abnormal returns and beta. The actions of noise traders weaken the relationship between security returns and beta, but they create a positive conditional correlation between abnormal returns and beta. As to trader survival, noise traders are not eliminated in markets where prices are efficient. Rather, price efficiency protects particular noise traders. In such a world, the only effect of noise traders is an increase in trading volume. However, not every noise trading error is protected by market efficiency, and we discuss the fitness properties of particular cognitive errors.

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