

In this last three-lecture unit, we are going to consider Markov chains, which are random processes that satisfy the Markov property. Named after Andrey Markov, perhaps the most well known member of a family of famous Russian mathematicians from the late 19th to early 20th centuries.

A random process which satisfies the Markov property has the following interesting feature. One can make predictions on its future based only on its current state independently of what happened in the past up to that current state. In other words, conditional on the current state of the process, its future and past evolutions are independent. As such, Markov chains are quite general and enjoy a wide range of applications.

For example, in physics there are used extensively in statistical mechanics. In information sciences, they are used in signal processing, coding, data compression, and pattern recognition. In queuing theory, they provide the analytical backbone for the analysis of queues. We will expand later on this example.

In internet applications, they have been used to rank web pages. And we will see that as well. In statistics, they are often used to make Bayesian inference more practical under the name of Markov chain Monte Carlo. In finance, they are used to describe asset prices evolution. They are also used in games, music, genetics, baseball, history and so on.

In this unit we will only consider discrete time Markov chains evolving within finite state of spaces. This allows us to concentrate on the main concepts without being overwhelmed by technical details needed to address in a rigorous fashion continuous time Markov processes in general state spaces.

We will then concentrate on the behavior of a Markov chain in the long run. That is, after we let it evolve for a long time. And study under what conditions a Markov chain exhibits some sort of steady state behavior, and under what conditions that behavior is independent of the initial starting state.

Finally, we will look at a classical application of Markov chains associated with the design of a phone system. We will then conclude by making use of all what we have learned so far in order to calculate some interesting quantities associated with short term behaviors of Markov chains.