

# FE522 homework 1

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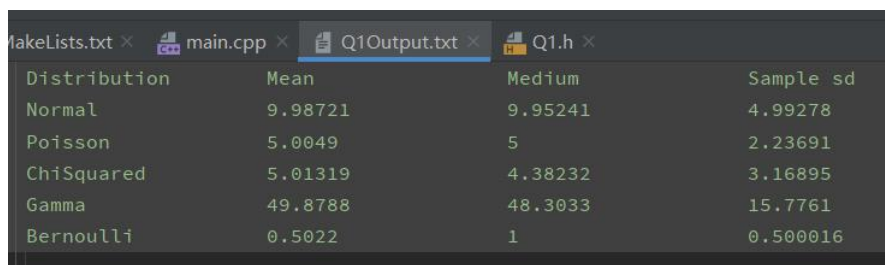
## 1 Problem 1

For this problem, the 5 distributions I choose are: Normal distribution; Poisson distribution; Chi-squared distribution; Gamma distribution and Bernoulli distribution. Here is the parameters I set for each distribution to get random numbers.

```
// Problem 1
// Generate 5 random number distributions
vector<double> normal = normalDist( mean: 10, sd: 5, length: 10000);
vector<double> poisson = poissonDist( k: 5, length: 10000);
vector<double> chi_squared = chisquaredDist( k: 5, length: 10000);
vector<double> gamma = gammaDist( mean: 10, sd: 5, length: 10000);
vector<double> bernoulli = bernoulliDist( p: 0.5, length: 10000);
```

Figure 1: Parameter setting for Problem 1

And for output path, I'm not sure if it's better to put outputs of all problems into a single file, or separately. Finally I create folders for both paths but use only one in my code. Just to make sure I know the usage of "../". Each "../" means jumping back to once to the parent folder. I use "../.." at the beginning of my output path so I finally put all outputs together.



Distribution	Mean	Medium	Sample sd
Normal	9.98721	9.95241	4.99278
Poisson	5.0049	5	2.23691
ChiSquared	5.01319	4.38232	3.16895
Gamma	49.8788	48.3033	15.7761
Bernoulli	0.5022	1	0.500016

Figure 2: Output of Problem 1

## 2 Problem 2

For this problem, the polynomial function I use was  $f(x) = -4 + x^2$  because the root for this one is easy to check.

Then, my C++ function will take input of an index vector and two different initial guesses. The index vector, for example, while solving  $a_0 + a_1x + a_2x^2 + \dots + a_nx^n = 0$ , the index vector should be  $a_0, a_1, a_2, \dots, a_n$ . And I made my functions print a report instead of only a number. Here is my function and output:

```

4 // Problem 2-1 — Bisection method
5 // vector<double> indexVec = {-4, 0, 1}; // stands for the function: f(x) = -4 + x^2
6 double solution1 = BisectionMethod(indexVec, -6.0, 0.0);
7 cout << "Bisection method solution is around: " << solution1 << " according to Bisection method." << endl;
8 double solution1_2 = BisectionMethod(indexVec, 0.0, 5.0);
9 cout << "Bisection method solution is around: " << solution1_2 << " according to Bisection method." << endl;
10
11 // Problem 2-2 — Secant method
12 double solution2 = SecantMethod(indexVec, -6.0, 0.0);
13 cout << "Secant method solution is around: " << solution2 << " according to Secant method." << endl;
14 double solution2_2 = SecantMethod(indexVec, 0.0, 5.0);
15 cout << "Secant method solution is around: " << solution2_2 << " according to Secant method." << endl;
16
17 }

```

Q2

```

E:\Stevens\2020-SPRING\FE-522\homework1\Q2\cmake-build-debug\Q2.exe
Bisection method solution is around: -1.99805 according to Bisection method.
Bisection method solution is around: 2.00195 according to Bisection method.
Secant method solution is around: -1.99951 according to Secant method.
Secant method solution is around: 2.00049 according to Secant method.
Process finished with exit code 0

```

Figure 3: Output of Problem 2

Root of my polynomial function which falls between interval  $[-6, 0]$  is -2, and that falls between interval  $[0, 5]$  is 2. The results I got from functions of both methods are very close to the exact values. Therefore, they work well.

### 3 Problem 3

Based on the requirements, I created a class named Money which takes string as input and output. When creating a Money variable, the default value will be "\$0.0", and I can also set values using similar format. For example, "\$1234.567".

In test cases below, in order to make it more convenient for graders to check my work, I make output more detailed. Here, all the "money1" and "money2" four decimal places are printed out because this is the initial value I set and professor didn't ask us to make any change on it. The calculation results follow 4/5 rounding rules.

```

8 // Problem 3
9 Money money1( moneyAmount: "$1.3221"), money2( moneyAmount: "$22.0029");
10 Money money3 = money2 + money1;
11 cout << money2 << " + " << money1 << " = " << money3 << endl;
12 Money money4 = money2 - money1;
13 cout << money2 << " - " << money1 << " = " << money4 << endl;
14 Money money5_1 = money2 * 2;
15 cout << money2 << " times " << 2 << " = " << money5_1 << endl;
16 Money money5_2 = money2 * 0.25;
17 cout << money2 << " times " << 0.25 << " = " << money5_2 << endl;
18 Money money6_1 = money2 / 4;
19 cout << money2 << " divided by " << 4 << " = " << money6_1 << endl;
20 Money money6_2 = money2 / 0.5;
21 cout << money2 << " divided by " << 0.5 << " = " << money6_2 << endl;
22
23 main

```

Q3

```

E:\Stevens\2020-SPRING\FE-522\homework1\Q3\cmake-build-debug\Q3.exe
$22.0029 + $1.3221 = $23.33
$22.0029 + $1.3221 = $20.68
$22.0029 times 2 = $44.01
$22.0029 times 0.25 = $5.50
$22.0029 divided by 4 = $5.50
$22.0029 divided by 0.5 = $44.01
Process finished with exit code 0

```

Figure 4: Some test cases for Problem 3

This is how I make my bool value "visible". And variable "money7" is used to test the "&" operator.

```

24 Money money7;
25 Money money8 = money1;
26 if (money1 == money2) cout << "money1 = money2" << endl;
27 else cout << "money1 not equal to money2" << endl;
28 if (money1 == money8) cout << "money1 = money8" << endl;
29 else cout << "money1 not equal to money8" << endl;
30 if (money1 == "$1.3221") cout << "money1 = $1.3221" << endl;
31 else cout << "money1 not equal to $1.3221" << endl;
32
33 cin >> money7;
34 cout << money7;
35
main
Q3 x
E:\Stevens\2020-SPRING\FE-522\homework1\Q3\cmake-build-debug\Q3.exe
money1 not equal to money2
money1 = money8
money1 = $1.3221
Please enter your money amount as a string:
$987.65
$987.65
Process finished with exit code 0

```

Figure 5: Some other test cases for Problem 3

## 4 Problem 4

Here is what my class contains. Two grey functions will be used in Problem 5.

```

// For problem 4:
class EuropeanOption{
private:
    string Type;
    double call();
    double put();
    double St, K, r, vol, d1, d2;
    int T;

public:
    EuropeanOption(string type, double spotPrice, double strikePrice,
        double interestRate, double volatility, int timeToMature);
    double getPrice();
    double getBisectionVol(double price);
    double getSecantVol(double price);
};

```

Figure 6: Test cases and result for Problem 4

Here, input values are option type, spot price, strike price, interest rate, volatility and time to mature(year). As required, I didn't define an  $N()$  function myself.

```

double EuropeanOption::getPrice() {
    if(Type == "call") return call();
    else return put();
}
double EuropeanOption::call() {
    return (0.5 * erfc(-d1 * M_SQRT1_2))*St-(0.5 * erfc(-d2 * M_SQRT1_2))*K*exp(-r*T);
}
double EuropeanOption::put() {
    return (0.5 * erfc(d2 * M_SQRT1_2))*K*exp(-r*T)-(0.5 * erfc(d1 * M_SQRT1_2))*St;
}

```

Figure 7: Test cases and result for Problem 4

This is the test cases I use and price I get. The exit code is not 0 because professor said we should not accept

illegal input and I tested wrong type setting, wrong volatility input and wrong time to mature input.

```

8 // Problem 4:
9 EuropeanOption option1( type: "call", spotPrice: 100, strikePrice: 105, interestRate: 0.05, volatility: 0.2, timeToMature: 1);
10 cout << option1.getPrice() << endl;
11
12 EuropeanOption option2( type: "put", spotPrice: 100, strikePrice: 105, interestRate: 0.05, volatility: 0.2, timeToMature: 1);
13 cout << option2.getPrice() << endl;
14
15 EuropeanOption option3( type: "test", spotPrice: 100, strikePrice: 105, interestRate: 0.05, volatility: 0.2, timeToMature: 1);
16
17 return 0;
18 }
19
20 main

```

Q4Q5 x

E:\Stevens\2020-SPRING\FE-522\homework1\Q4\cmake-build-debug\Q4Q5.exe

8.02135

7.90044

Wrong input of type!

Process finished with exit code 1

Figure 8: Test cases and results for Problem 4

```

14
15 EuropeanOption option3( type: "call", spotPrice: 100, strikePrice: 105, interestRate: 0.05, volatility: -2, timeToMature: 1);
16
17 return 0;
18 }
19
20 main

```

Q4Q5 x

E:\Stevens\2020-SPRING\FE-522\homework1\Q4\cmake-build-debug\Q4Q5.exe

8.02135

7.90044

Volatility out of range!

Process finished with exit code 2

Figure 9: Other test case and result for Problem 4

```

15 EuropeanOption option3( type: "call", spotPrice: 100, strikePrice: 105, interestRate: 0.05, volatility: 0.2, timeToMature: -1);
16
17 return 0;
18 }

```

Q4Q5 x

E:\Stevens\2020-SPRING\FE-522\homework1\Q4\cmake-build-debug\Q4Q5.exe

8.02135

7.90044

Time to mature cannot be negative!

Process finished with exit code 3

Figure 10: Other test case and result for Problem 4

## 5 Problem 5

For this problem, there isn't too much to say. This time the input value "timeToMature" in the constructor of EuropeanOption class should be mature day. I did change "time to mature" from day to year while calculating. Below is screenshot of my output file. All results from both methods are close to given volatility.

	Type	Bid	Ask	Spot	Strike	Rate	DaysToMaturity	ImpliedVolatility	BisectionVol	SecantVol
1	Call	5.45	5.6	158.28	155	0.0099	29	0.2084	0.202148	0.20274
2	Call	2.73	2.82	158.28	160	0.0099	29	0.1986	0.190289	0.190508
3	Call	1.18	1.25	158.28	165	0.0099	29	0.1987	0.198242	0.197508
4	Call	0.78	0.84	158.28	167.5	0.0099	29	0.2037	0.202148	0.202874
5	Call	0.35	0.38	158.28	172.5	0.0099	29	0.2153	0.21582	0.215493
6	Put	0.37	0.41	158.28	145	0.0099	29	0.2275	0.229492	0.230062
7	Put	0.61	0.65	158.28	148	0.0099	29	0.2139	0.217773	0.217072
8	Put	0.86	0.92	158.28	150	0.0099	29	0.208	0.208961	0.210825
9	Put	1.34	1.4	158.28	152.5	0.0099	29	0.2009	0.204102	0.204507
10	Put	2.04	2.09	158.28	155	0.0099	29	0.1947	0.199219	0.199184
11	Put	3	3.05	158.28	157.5	0.0099	29	0.1898	0.194336	0.194831
12	Put	5.8	6.05	158.28	162.5	0.0099	29	0.1935	0.194336	0.19447

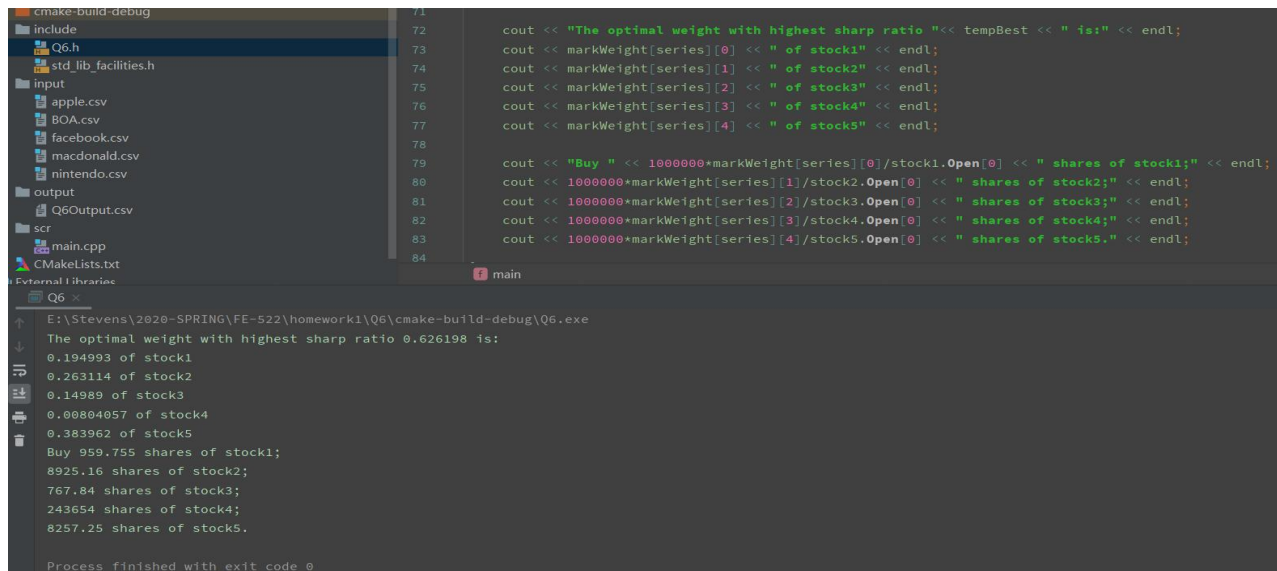
Figure 11: Output of Problem 5

Because  $d_1$  and  $d_2$  will change when volatility changes, I built a new class "dInBS" (stands for d in BS model), in order to update  $d_1$  and  $d_2$  when do iteration to vol.

## 6 Problem 6

For this problem, I first use R to download stocks data from June 30<sup>th</sup>, 2019 to Dec. 31<sup>st</sup>, 2019, and use C++ to do later work. I choose 5 stocks from different industries, generate random weight and see whether current sharp ratio I get is larger than all before. If so, I mark down the weight as temp optimal. Finally I got a best one for that simulation. Also, in order to get sharp ratio, I find risk free rate on [a website](#). It turns out that if I start from June 6<sup>th</sup>, risk free rate is 2.19%.

In CLion, I print the optimal weights I got from that simulation, as well as the shares of stocks I should buy in the beginning. Then, I output the change of my portfolio value into a csv file and use R to visualize it.



```
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72
73     cout << "The optimal weight with highest sharp ratio "<< tempBest << " is:" << endl;
74     cout << markWeight[series][0] << " of stock1" << endl;
75     cout << markWeight[series][1] << " of stock2" << endl;
76     cout << markWeight[series][2] << " of stock3" << endl;
77     cout << markWeight[series][3] << " of stock4" << endl;
78     cout << markWeight[series][4] << " of stock5" << endl;
79
80     cout << "Buy " << 1000000*markWeight[series][0]/stock1.Open[0] << " shares of stock1;" << endl;
81     cout << 1000000*markWeight[series][1]/stock2.Open[0] << " shares of stock2;" << endl;
82     cout << 1000000*markWeight[series][2]/stock3.Open[0] << " shares of stock3;" << endl;
83     cout << 1000000*markWeight[series][3]/stock4.Open[0] << " shares of stock4;" << endl;
84     cout << 1000000*markWeight[series][4]/stock5.Open[0] << " shares of stock5;" << endl;
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