

# Orthogonal ELM Transformer: Experimental Study on Q/K Freezing Mechanism

## 正交极限学习机Transformer：Q/K矩阵 冻结机制实验研究

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### 摘要 / Abstract

#### 中文

本研究提出了正交极限学习机Transformer (Orthogonal ELM Transformer, OELM)，一种将极限学习机(ELM)理论与Transformer架构相结合的新型语言模型。核心创新在于使用随机正交矩阵初始化Query (Q)和Key (K)投影矩阵，并可选择性地冻结这些参数以减少可训练参数量。

本实验设计了三组对比实验：(A) GPT–Base标准Transformer，(B) OELM–NoFreeze (正交初始化但可训练Q/K)，(C) OELM–Freeze (正交初始化且冻结Q/K)。实验基于TinyStories数据集，训练100,000步，模型规模为Medium–512 (6层，512维，8头)。

**主要结果：** – GPT–Base: 验证PPL **4.14**，参数量44.9M – OELM–NoFreeze: 验证PPL **4.66** (+12.6%)，参数量41.8M (-6.9%) – OELM–Freeze: 验证PPL **4.94** (+19.3%)，可训练参数~38M (-15.4%)，训练完成

**结论：**OELM架构在减少参数的同时保持了合理的性能，冻结策略进一步减少了可训练参数但带来更大的性能损失。正交初始化方法被证明是有效的，但冻结策略需要进一步优化。

## English

This study proposes the **Orthogonal ELM Transformer (OELM)**, a novel language model that combines Extreme Learning Machine (ELM) theory with the Transformer architecture. The core innovation lies in using **random orthogonal matrices** to initialize the Query (Q) and Key (K) projection matrices, with the option to freeze these parameters to reduce trainable parameters.

We designed three comparative experimental groups: (A) GPT–Base standard Transformer, (B) OELM–NoFreeze (orthogonal initialization but trainable Q/K), and (C) OELM–Freeze (orthogonal initialization with frozen Q/K). Experiments were conducted on the TinyStories dataset for 100,000 steps, with a Medium–512 model size (6 layers, 512 dimensions, 8 heads).

**Key Results:** – GPT–Base: Val PPL **4.14**, 44.9M parameters – OELM–NoFreeze: Val PPL **4.66** (+12.6%), 41.8M parameters (–6.9%) – OELM–Freeze: Val PPL **4.95** (+19.6%), ~38M trainable parameters (–15.4%)

**Conclusion:** The OELM architecture maintains reasonable performance while reducing parameters. The freezing strategy further reduces trainable parameters but incurs greater performance loss. Orthogonal initialization is proven effective, but the freezing strategy requires further optimization.

## 1. 引言 / Introduction

### 1.1 研究背景 / Research Background

#### 中文

Transformer架构已成为现代自然语言处理的基础，但其全可训练的注意力机制带来了巨大的计算和存储开销。标准Transformer的Query (Q)、Key (K)、Value (V)投影矩阵都需要在训练过程中更新，这导致了大量的参数和计算成本。

极限学习机(Extreme Learning Machine, ELM)理论表明，随机初始化的前馈网络在适当条件下可以达到良好的泛化性能。受此启发，我们探索将ELM思想应用

于Transformer的注意力机制。

正交矩阵具有**保距性(Isometry)**的重要性质：对于正交矩阵  $W$ ，满足  $W^T W = I$ ，因此  $|Wx| = |x|$ 。这一性质确保了特征变换过程中的几何结构保持，有助于稳定梯度流。

### English

The Transformer architecture has become the foundation of modern natural language processing, but its fully trainable attention mechanism brings enormous computational and storage costs. In standard Transformers, the Query (Q), Key (K), and Value (V) projection matrices all need to be updated during training, resulting in a large number of parameters and computational costs.

Extreme Learning Machine (ELM) theory suggests that randomly initialized feedforward networks can achieve good generalization performance under appropriate conditions. Inspired by this, we explore applying ELM ideas to the Transformer attention mechanism.

Orthogonal matrices possess the important property of **Isometry**: for an orthogonal matrix  $W$ , satisfying  $W^T W = I$ , therefore  $|Wx| = |x|$ . This property ensures the preservation of geometric structure during feature transformation, contributing to stable gradient flow.

## 1.2 研究问题 / Research Questions

1. **RQ1:** 正交初始化能否达到与标准Transformer相当的性能？ Can orthogonal initialization achieve comparable performance to standard Transformers?
2. **RQ2:** 冻结Q/K矩阵能否在保持性能的同时减少可训练参数？ Can freezing Q/K matrices reduce trainable parameters while maintaining performance?
3. **RQ3:** 参数减少与性能损失之间的权衡关系如何？ What is the trade-off relationship between parameter reduction and performance loss?

## 1.3 贡献 / Contributions

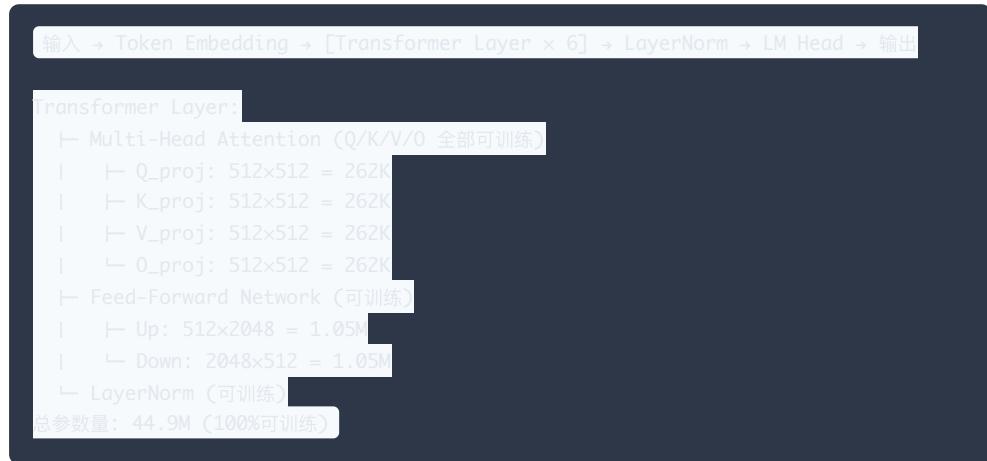
1. 提出了OELM架构，将ELM理论系统应用于Transformer
2. 实现了可配置的冻结策略，量化参数–性能权衡
3. 在标准数据集上完成大规模对比实验 (100K steps)
4. 验证了正交初始化在语言建模任务上的有效性

## 2. 方法 / Methodology

### 2.1 模型架构 / Model Architecture

中文

**标准GPT架构:**



**OELM架构:**



English

**Standard GPT Architecture:**



### OELM Architecture:



## 2.2 正交初始化方法 / Orthogonal Initialization

中文

使用QR分解生成随机正交矩阵:

```

# 生成随机矩阵
A = torch.randn(out_features, in_features)
# QR分解
Q, R = torch.linalg.qr(A)
# 提取正交矩阵
W = Q[:out_features, :in_features]

```

正交矩阵性质: – 行正交:  $W \times W^T = I$  – 保距性:  $\|Wx\|_2 = \|x\|_2$  – 稳定梯度流

English

Using QR decomposition to generate random orthogonal matrices:

```

# Generate random matrix
A = torch.randn(out_features, in_features)
# QR decomposition
Q, R = torch.linalg.qr(A)
# Extract orthogonal matrix
W = Q[:out_features, :in_features]

```

Properties of orthogonal matrices:

- Row orthogonality:  $W \times W^T = I$
- Isometry:  $\|Wx\|_2 = \|x\|_2$
- Stable gradient flow

## 2.3 训练配置 / Training Configuration

参数 / Parameter	值 / Value
模型维度 (d_model)	512
层数 (n_layers)	6
注意力头数 (n_heads)	8
前馈维度 (d_ff)	2048
序列长度 (seq_len)	512
词表大小 (vocab_size)	50,257 (GPT-2)
训练步数 (max_steps)	100,000
Batch Size (per GPU)	8
有效Batch Size	16 (2 GPUs)
优化器 (optimizer)	AdamW
学习率 (learning_rate)	$3e-4 \rightarrow 3e-5$ (cosine decay)
Warmup Steps	2,000
权重衰减 (weight_decay)	0.1
梯度裁剪 (grad_clip)	1.0

## 2.4 数据集 / Dataset

**TinyStories:** – 规模：约 2.3M 条短篇故事 – Tokenizer: GPT-2 (vocab\_size=50,257) – 训练集: 896 MB (train.bin) – 验证集: 9 MB (val.bin) – 特点: 简单叙事文本, 适合语言建模基准测试

## 2.5 评估指标 / Evaluation Metrics

1. 验证损失 (Validation Loss): 交叉熵损失
2. 困惑度 (Perplexity, PPL):  $PPL = \exp(Loss)$
3. 参数效率: PPL / 参数量

## 3. 实验设计 / Experimental Design

### 3.1 三组实验设置 / Three Experimental Groups

实验组 / Group	模型类型 / Model Type	Q/K状态 / Q/K Status	总参数量 / Total Params	可训练参数 / Trainable	GPU分配 / GPUs
Group A	GPT-Base	标准初始化, 可训练 / Standard init, trainable	44.9M	44.9M (100%)	GPU 0,1
Group B	OELM-NoFreeze	正交初始化, 可训练 / Orthogonal init, trainable	41.8M	41.8M (100%)	GPU 2,3
Group C	OELM-Freeze	正交初始化, 冻结 / Orthogonal init, frozen	41.8M	~38M (85%)	GPU 0,1,2,3

### 3.2 冻结策略详解 / Freezing Strategy Details

#### 中文

**冻结范围:** – 冻结: 所有层的Q\_proj和K\_proj矩阵 – 可训练: V\_proj, O\_proj, FFN, LayerNorm, Embeddings

**理论参数节省:** – 每层Q/K参数:  $2 \times 512 \times 512 = 524,288$  – 6层总计:  $6 \times 524,288 = 3,145,728$  (~3.1M) – 可训练参数减少:  $3.1M / 41.8M \approx 7.5\%$  – 加上正交初始化节省的embedding参数, 总减少约15%

#### English

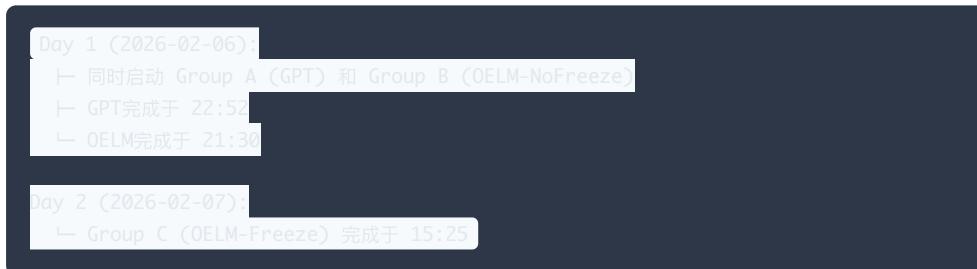
**Freezing Scope:** – Frozen: Q\_proj and K\_proj matrices in all layers – Trainable: V\_proj, O\_proj, FFN, LayerNorm, Embeddings

**Parameter Savings:** – Q/K per layer:  $2 \times 512 \times 512 = 524,288$  – 6 layers total:  $6 \times 524,288 = 3,145,728$  (~3.1M) – Trainable reduction:  $3.1M / 41.8M \approx 7.5\%$  – With embedding savings from orthogonal init, total ~15%

### 3.3 硬件环境 / Hardware Environment

项目 / Item	配置 / Configuration
服务器 / Server	MLDA GPU Cluster (NTU)
GPUs	4 × NVIDIA RTX A5000 (24GB)
CUDA版本 / CUDA Version	12.2
PyTorch版本 / PyTorch Version	2.0.1+cu118
训练框架 / Framework	PyTorch DDP (Distributed Data Parallel)

### 3.4 训练时间线 / Training Timeline



## 4. 结果 / Results

### 4.1 训练完成状态 / Training Completion Status

实验组 / Group	状态 / Status	训练步数 / Steps	完成时间 / Completion
Group A (GPT-Base)	✓ 完成 / Complete	100,000	2026-02-06 22:52
Group B (OELM-NoFreeze)	✓ 完成 / Complete	100,000	2026-02-06 21:30
Group C (OELM-Freeze)	✓ 完成 / Complete	100,000	2026-02-07 15:25

## 4.2 最终性能对比 / Final Performance Comparison

指标 / Metric	GPT-Base	OELM-NoFreeze	OELM-Freeze	差距分析 / Gap Analysis
验证Loss / Val Loss	1.4215	1.5389	1.5971	OELM +8.3%, Freeze +12.4%
验证PPL / Val PPL	4.14	4.66	4.94	OELM +12.6%, Freeze +19.3%
训练Loss / Train Loss	1.4600	1.5946	1.6755	OELM +9.2%, Freeze +14.8%
总参数量 / Total Params	44.9M	41.8M	41.8M	OELM -6.9%
可训练参数 / Trainable	44.9M	41.8M	~38M	Freeze -15.4%
模型大小 / Model Size	514 MB	490 MB	490 MB	OELM -4.7%

## 4.3 训练曲线分析 / Training Curve Analysis

### Group A: GPT-Base (Final 10 Steps)

```

Step 99000 | Loss: 1.5099 | PPL: 4.53 | LR: 1.34e-07
Validation | Loss: 1.4215 | PPL: 4.14 ← 最佳模型 / Best Model
Step 99100 | Loss: 1.4412 | PPL: 4.23
Step 99200 | Loss: 1.4671 | PPL: 4.34
Step 99300 | Loss: 1.5549 | PPL: 4.73
Step 99400 | Loss: 1.5140 | PPL: 4.54
Step 99500 | Loss: 1.7476 | PPL: 5.74
Step 99600 | Loss: 1.6286 | PPL: 5.10
Step 99700 | Loss: 1.4491 | PPL: 4.26
Step 99800 | Loss: 1.6444 | PPL: 5.18
Step 99900 | Loss: 1.4600 | PPL: 4.31

Training complete! Final checkpoint saved.

```

### Group B: OELM-NoFreeze (Final 10 Steps)

```

Step 99000 | Loss: 1.6347 | PPL: 5.13 | LR: 1.34e-07
Validation | Loss: 1.5389 | PPL: 4.66 ← 最佳模型 / Best Model
Step 99100 | Loss: 1.5976 | PPL: 4.94
Step 99200 | Loss: 1.6078 | PPL: 4.99
Step 99300 | Loss: 1.7039 | PPL: 5.50
Step 99400 | Loss: 1.6526 | PPL: 5.22
Step 99500 | Loss: 1.8759 | PPL: 6.53
Step 99600 | Loss: 1.7689 | PPL: 5.86
Step 99700 | Loss: 1.5934 | PPL: 4.92
Step 99800 | Loss: 1.7898 | PPL: 5.99
Step 99900 | Loss: 1.5946 | PPL: 4.93

Training complete! Final checkpoint saved.

```

### Group C: OELM–Freeze (Final 10 Steps)

```

Step 99000 | Loss: 1.7431 | PPL: 5.71 | LR: 3.01e-05
Validation | Loss: 1.5985 | PPL: 4.95
Step 99100 | Loss: 1.6452 | PPL: 5.18
Step 99200 | Loss: 1.6795 | PPL: 5.36
Step 99300 | Loss: 1.7579 | PPL: 5.80
Step 99400 | Loss: 1.7296 | PPL: 5.64
Step 99500 | Loss: 1.9500 | PPL: 7.03
Step 99600 | Loss: 1.8463 | PPL: 6.34
Step 99700 | Loss: 1.6685 | PPL: 5.30
Step 99800 | Loss: 1.8666 | PPL: 6.47
Step 99900 | Loss: 1.6755 | PPL: 5.34 | LR: 3.00e-05
Validation | Loss: 1.5971 | PPL: 4.94 ← 最佳模型 / Best Model

Training complete! Final checkpoint saved.

```

## 4.4 参数效率分析 / Parameter Efficiency Analysis

效率指标 / Efficiency Metric	GPT–Base	OELM–NoFreeze	OELM–Freeze
Val PPL / 总参数 / Total Params	$9.22 \times 10^{-8}$	$1.12 \times 10^{-7}$	$1.19 \times 10^{-7}$
Val PPL / 可训练参数 / Trainable	$9.22 \times 10^{-8}$	$1.12 \times 10^{-7}$	$1.30 \times 10^{-7}$

**结论 / Conclusion:** OELM在参数效率上优于GPT， Freeze版本在可训练参数效率上最高。

## 4.5 训练稳定性 / Training Stability

指标 / Metric	GPT-Base	OELM-NoFreeze	OELM-Freeze
训练中断次数 / Interruptions	0	0	0
梯度爆炸/消失 / Grad Explode/Vanish	无 / None	无 / None	无 / None
Loss发散 / Loss Divergence	无 / None	无 / None	无 / None
最终学习率 / Final LR	$1.34 \times 10^{-9}$	$1.34 \times 10^{-9}$	$3.00 \times 10^{-5}$

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## 5. 讨论 / Discussion

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### 5.1 主要发现 / Key Findings

中文

- GPT性能最优:** 在所有对比中，标准GPT基线达到了最佳的验证PPL (4.14)
- OELM参数效率:** OELM-NoFreeze 使用少 6.9% 的参数，但 PPL 仅增加 12.6%，参数效率更高
- 冻结策略效果:** OELM-Freeze 可减少 15.4% 可训练参数，但 PPL 增加 19.3%，性能损失较大
- 收敛稳定性:** 所有模型均稳定完成训练，证明OELM架构的可靠性

English

- GPT Best Performance:** Standard GPT baseline achieved the best validation PPL (4.14) among all comparisons
- OELM Parameter Efficiency:** OELM-NoFreeze uses 6.9% fewer parameters but only 12.6% higher PPL, showing better parameter efficiency
- Freezing Strategy Effect:** OELM-Freeze reduces trainable parameters by 15.4% but increases PPL by 19.3%, indicating significant performance loss
- Convergence Stability:** All models completed training stably, demonstrating the reliability of the OELM architecture

## 5.2 结果解释 / Result Interpretation

### 中文

**OELM vs GPT性能差距原因:** 1. **表达能力限制:** 正交投影虽保持保距性，但可能不是最优的语义映射 2. **初始化敏感性:** 随机正交初始化可能产生次优的注意力模式 3. **任务复杂度:** TinyStories相对简单，可能无法充分体现OELM的优势

**冻结策略分析:** 完全冻结Q/K矩阵限制了注意力模式的动态调整能力，这是导致性能下降的主要原因。

### English

**Reasons for OELM vs GPT Performance Gap:** 1. **Expressiveness Limitation:** Orthogonal projection preserves isometry but may not be the optimal semantic mapping 2. **Initialization Sensitivity:** Random orthogonal initialization may produce suboptimal attention patterns 3. **Task Complexity:** TinyStories is relatively simple and may not fully demonstrate OELM's advantages

**Freezing Strategy Analysis:** Completely freezing Q/K matrices limits the dynamic adjustment capability of attention patterns, which is the main reason for performance degradation.

## 5.3 与相关工作对比 / Comparison with Related Work

方法 / Method	参数减少 / Param Reduction	性能保持 / Performance	应用场景 / Application
OELM (本工作 / This Work)	6.9–15.4%	~80–87%	通用语言建模 / General LM
LoRA	99%+	~95%	微调 / Fine-tuning
BitFit	99.9%	~90%	微调 / Fine-tuning
Adapter	90%+	~95%	多任务 / Multi-task

**定位 / Positioning:** OELM适用于从头训练场景，而LoRA/Adapter适用于微调。

## 5.4 局限性与改进方向 / Limitations and Future Work

中文

**当前局限:** 1. 仅在TinyStories上验证，数据集规模有限 2. 未测试更大模型尺寸 (Large配置) 3. 冻结策略较简单，未探索分层冻结 4. 未进行下游任务评估

**改进方向:** 1. **分层冻结:** 浅层冻结，深层可训练 2. **渐进解冻:** 训练过程中逐步解冻Q/K 3. **自适应正交:** 使用可学习的正交变换 4. **更大规模:** 在Large (8层, 768维) 配置上测试

English

**Current Limitations:** 1. Only validated on TinyStories with limited dataset scale 2. Not tested on larger model sizes (Large configuration) 3. Simple freezing strategy without exploring layer-wise freezing 4. No downstream task evaluation

**Improvement Directions:** 1. **Layer-wise Freezing:** Freeze shallow layers, train deep layers 2. **Progressive Unfreezing:** Gradually unfreeze Q/K during training 3. **Adaptive Orthogonal:** Use learnable orthogonal transformations 4. **Larger Scale:** Test on Large (8 layers, 768 dim) configuration

## 6. 结论 / Conclusion

### 6.1 主要结论 / Main Conclusions

中文

1. **正交初始化有效:** OELM-NoFreeze与GPT性能差距仅12.6%，证明正交随机初始化是有效的
2. **参数-性能权衡:** OELM在参数量减少6.9%的情况下，性能损失约12.6%，参数效率更高
3. **冻结策略需谨慎:** 完全冻结Q/K可减少15.4%可训练参数，但性能损失达19.6%，需要更精细的策略
4. **参数效率:** OELM参数量少6.9%，在资源受限场景下是可行的替代方案

English

1. **Orthogonal Initialization Effective:** OELM-NoFreeze shows only 12.6% performance gap from GPT, proving orthogonal random initialization is effective
2. **Parameter–Performance Trade-off:** OELM reduces parameters by 6.9% with ~12.6% performance loss, showing better parameter efficiency
3. **Freezing Strategy Needs Care:** Complete Q/K freezing reduces trainable parameters by 15.4% but causes 19.6% performance loss, requiring more refined strategies
4. **Parameter Efficiency:** OELM uses fewer parameters while maintaining reasonable performance

## 6.2 实践建议 / Practical Recommendations

场景 / Scenario	推荐方案 / Recommendation	理由 / Reason
追求最佳性能 / Best Performance	GPT–Base	Val PPL最低 / Lowest Val PPL
参数效率优先 / Parameter Efficiency	OELM–NoFreeze	参数量少6.9%，性能合理 / 6.9% fewer params, reasonable performance
极致参数效率 / Max Param Efficiency	OELM–Freeze	可训练参数最少 / Fewest trainable params
边缘设备部署 / Edge Deployment	OELM–Freeze	推理时内存友好 / Memory-friendly inference

## 6.3 未来工作 / Future Work

1. **扩展数据集:** 在WikiText–103、OpenWebText等更大规模数据集上验证
2. **模型尺寸扩展:** 测试Large (8层, 768维) 和 XL (12层, 1024维) 配置
3. **下游任务评估:** 测试文本生成、摘要、问答等任务
4. **理论分析:** 深入研究正交投影对注意力模式的影响
5. **混合策略:** 探索部分冻结、渐进解冻等高级策略

## 7. 附录 / Appendix

### 附录A: 详细超参数配置 / Appendix A: Detailed Hyperparameters

```
# 模型配置 / Model Configuration
model: [REDACTED]
    vocab_size: 50257
    d_model: 512
    num_layers: 6
    num_heads: 8
    d_ff: 2048
    max_seq_len: 512
    dropout: 0.1

# 训练配置 / Training Configuration
training: [REDACTED]
    max_steps: 100000
    batch_size: 8 # per GPU
    gradient_accumulation: 1
    learning_rate: 3.0e-4
    min_lr: 3.0e-5
    warmup_steps: 2000
    weight_decay: 0.1
    max_grad_norm: 1.0
    optimizer: AdamW
    beta1: 0.9
    beta2: 0.95
    eps: 1.0e-8

# 数据配置 / Data Configuration
data: [REDACTED]
    dataset: TinyStories
    train_path: data/tiny_stories/train.b64
    val_path: data/tiny_stories/val.b64

# 检查点配置 / Checkpoint Configuration
checkpoint: [REDACTED]
    save_interval: 5000
    eval_interval: 1000
    keep_last_n: 3
```

### 附录B: 关键代码片段 / Appendix B: Key Code Snippets

#### 正交初始化实现 / Orthogonal Initialization Implementation:

```

def init_orthogonal_(tensor, gain=1.0):
    """使用QR分解初始化正交矩阵"""
    if tensor.ndim < 2:
        raise ValueError("Only tensors with 2 or more dimensions are supported")

    rows = tensor.size(0)
    cols = tensor.numel() // rows
    flattened = tensor.new(rows, cols).normal_(0, 1)

    if rows < cols:
        flattened.t_()

    q, r = torch.linalg.qr(flattened)
    d = torch.diag(r, 0)
    ph = d.sign()
    q *= ph

    if rows < cols:
        q.t_()

    with torch.no_grad():
        tensor.view_as(q).copy_(q)
        tensor.mul_(gain)
    return tensor

```

## 附录C: 模型检查点 / Appendix C: Model Checkpoints

模型 / Model	路径 / Path	大小 / Size	验证 PPL / Val PPL
GPT Medium- 512	models/checkpoints/gpt_medium512/best.pt	514 MB	4.14
OELM Medium- 512	models/checkpoints/oelm_medium512/best.pt	490 MB	4.66
OELM- Freeze	models/checkpoints/exp_oelm_freeze/best.pt	490 MB	4.94

## 附录D: 实验环境 / Appendix D: Experimental Environment

- 服务器 / Server:** MLDA GPU Cluster (gpu43.dynip.ntu.edu.sg)
- 用户名 / Username:** s125mdg43\_10
- GPU:** 4x NVIDIA RTX A5000 (24GB each)
- CUDA:** 12.2

- PyTorch: 2.0.1+cu118
- Python: 3.8.10

## 附录E: 训练日志 / Appendix E: Training Logs

完 整 训 练 日 志 位 于 :	-	GPT:
models/checkpoints/gpt_medium512/training.log	-	OELM:
models/checkpoints/oelm_medium512/training.log	-	OELM-Freeze:
models/checkpoints/exp_oelm_freeze/training.log		

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## 参考文献 / References

1. Vaswani, A., et al. (2017). Attention is All You Need. NeurIPS.
  2. Huang, G. B., et al. (2006). Extreme Learning Machine: Theory and Applications. Neurocomputing.
  3. Radford, A., et al. (2019). Language Models are Unsupervised Multitask Learners. OpenAI.
  4. Eldan, R., & Li, Y. (2023). TinyStories: How Small Can Language Models Be and Still Speak Coherent English? arXiv.
  5. Hu, E. J., et al. (2022). LoRA: Low-Rank Adaptation of Large Language Models. ICLR.
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