Robotics 311 : How to build robots and make them move

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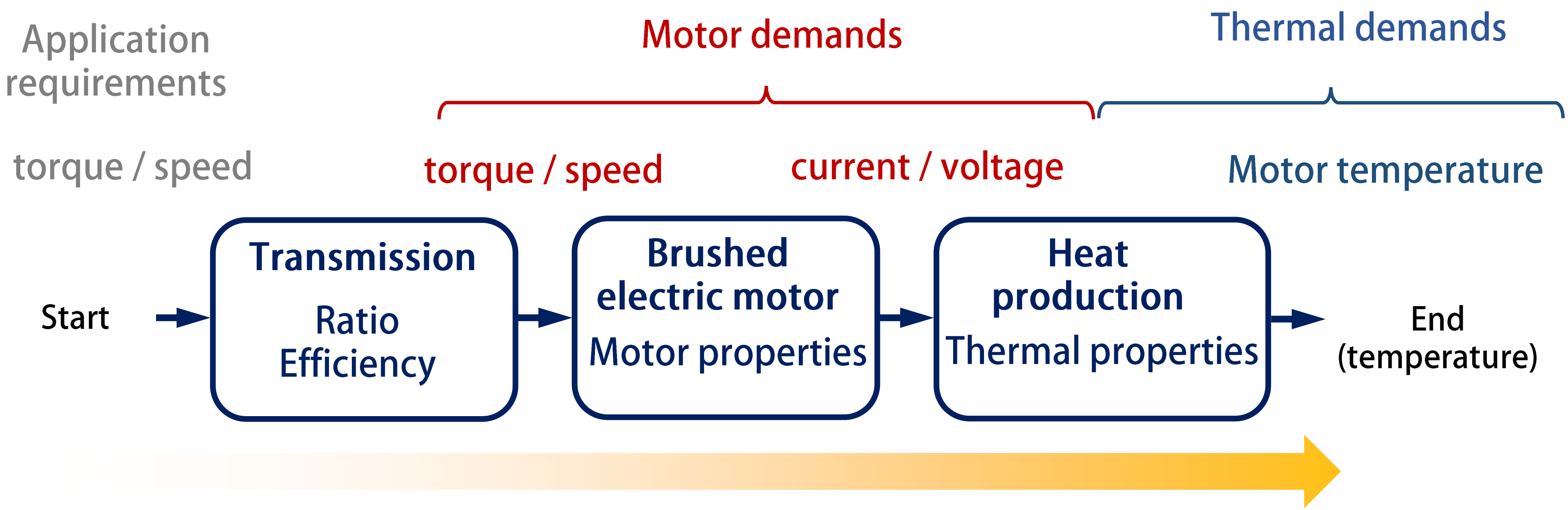
ROB 311 – Lecture 3

- Today:
  - Review design framework
  - Review motors and transmissions
  - Begin planar modeling of a ball-bot
  - Hopefully get to MATLAB example
  - Create a Github account

Design Analysis Framework

- We've begun with a framework for learning how to model the required capabilities of a robot's actuation system
- We use knowledge of the task requirements to guide the selection of a motor and transmission ratio, which impact physical design / control
- We are reviewing this framework before completing it using the ball-bot's task



Framework outcome: A motor that has the desired operating voltage, that is able to complete the task without overheating

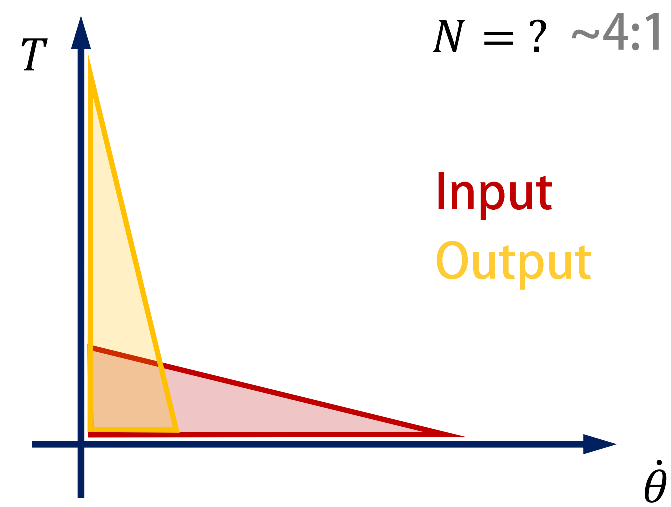
Modeling Transmissions

Velocity = $\dot{\psi}$
Torque = T_w

→ **Transmission**
Ratio = N
Efficiency = η →

Velocity = $\dot{\psi} \cdot N$
Torque = $\frac{T_w}{N \cdot \eta}$

- Transmissions are parameterized by their ratio and efficiency
- Efficiency depends on transmission type and stages (number of compound transmissions used)
- The ratio is often selected in tandem with the motor to produce the required torque-speed (or force-speed)
- Transmissions scale torque and speed inversely
- Usually input speed >> output speed
- Usually input torque << output torque
- Lets think about a cartoon example

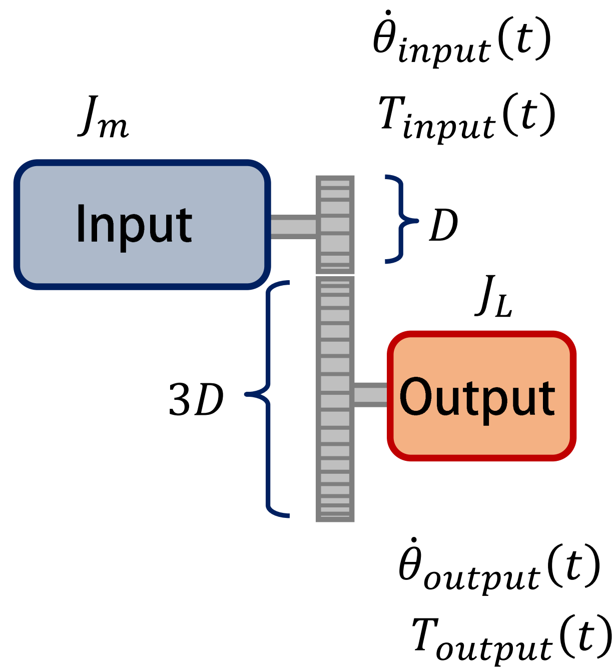


Modeling Transmissions

- Quick example using input/output:

$$\frac{3D}{D} = \frac{n_2}{n_1} = \frac{\dot{\theta}_{input}}{\dot{\theta}_{output}} = \frac{T_{output}}{T_{input}}$$

Notice torque and speed fractions reversed, also torque is ideal



- What is the equivalent inertia felt at the input? Or, how does J_L affect the inertia experience by the motor?
    - Look at systems kinetic energy

$$KE = \frac{1}{2} J_m \dot{\theta}_{input}^2 + \frac{1}{2} J_L \dot{\theta}_{output}^2 \qquad \dot{\theta}_{output} = \frac{\dot{\theta}_{input}}{N}$$

$$KE = \frac{1}{2} J_m \dot{\theta}_{input}^2 + \frac{1}{2} J_L \left(\frac{\dot{\theta}_{input}}{N}\right)^2 \qquad KE = \frac{1}{2}\left(\overbrace{J_m + \frac{J_L}{N^2}}^{\text{Known as: equivalent inertia}}\right)\dot{\theta}_{input}^2$$

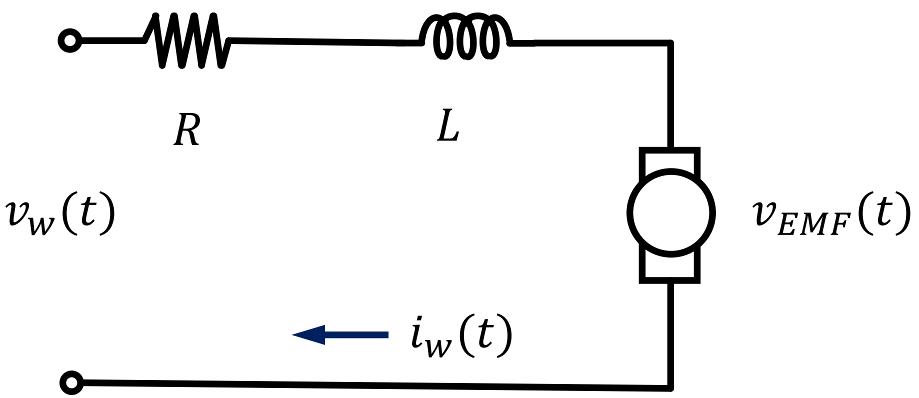
$\frac{J_L}{N^2}$ is reflected inertia on motor side

Modeling Brushed DC Motors

- Which is right for your application?
- We quantify required torque / speed and current / voltage to make decision
- Typically assessed in tandem with the transmission ratio

Modeling Brushed DC Motors

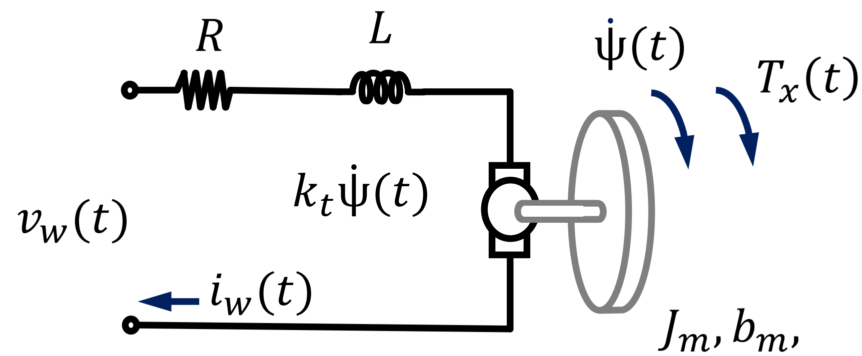
- DC brushed motors
  - Rules of thumb:
    - Voltage is proportional to speed (effort)
    - Current is proportional to torque (flow)



  - Think of a motor as a *transformer* – it transforms power in the form of current and voltage to power in the form of current and torque (plus loss as heat)

For brushed dc motors: $k_t = k_b = \frac{1}{k_v}$

Modeling Brushed DC Motors



- Governing equations

$$\underbrace{v_w(t)}_{\text{Voltage across winding}} = \underbrace{i_w(t)R}_{\text{Voltage across resistance}} + \underbrace{L\,\frac{d}{dt}i_w(t)}_{\text{Voltage across inductor}} + \underbrace{k_t\dot{\psi}(t)}_{\text{Solved second}}$$

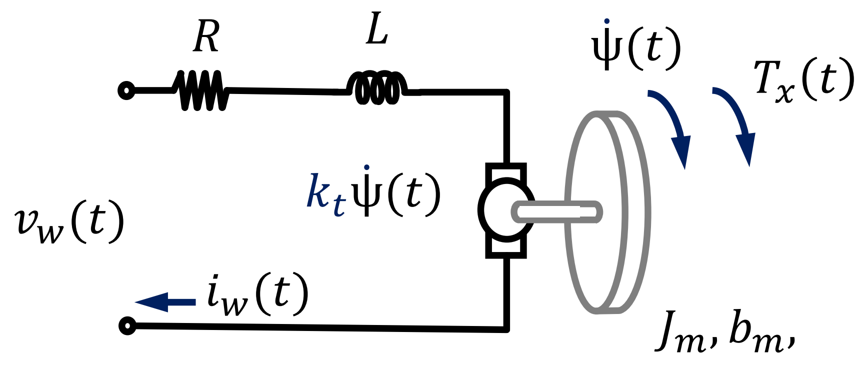
Solved second – using $i_w(t)$, solve for $v_w(t)$

$$\underbrace{k_t i_w(t)}_{\text{Motor torque}} = \underbrace{J_m\ddot{\psi}(t)}_{\text{Torque to accelerate inertia}} + \underbrace{b_m\dot{\psi}(t)}_{\text{Torque lost to friction}} + \underbrace{T_x(t)}_{\text{Motor output torque}}$$

Solved first – you know $\dot{\psi}(t)$ and derivatives → solve for $i_w(t)$

Modeling Brushed DC Motors

- At steady state, no damping (ideal)



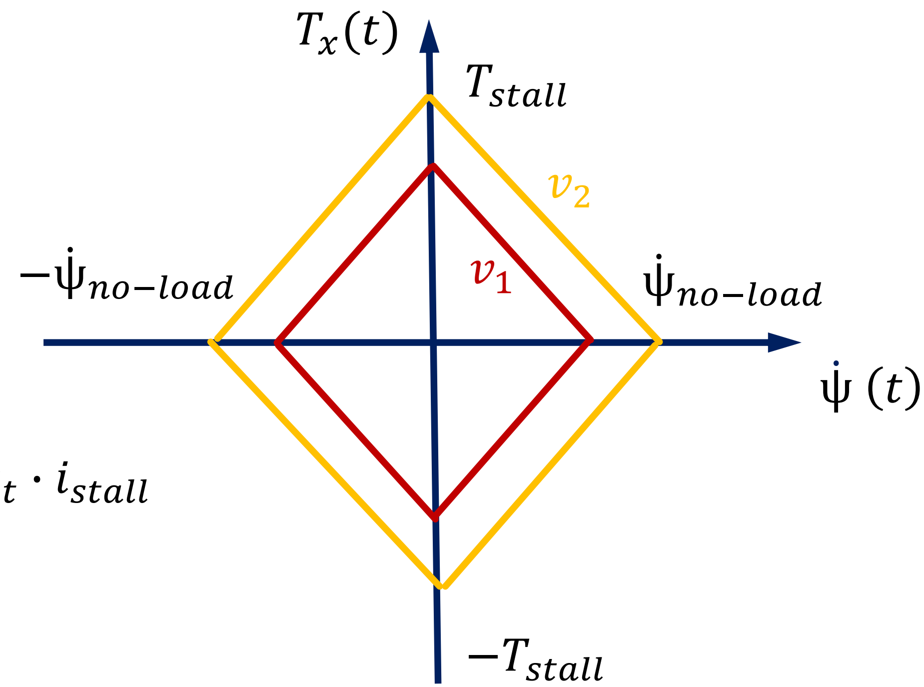
$$k_t i_w(t) = T_x(t) \rightarrow \; i_w(t) = \frac{T_x(t)}{k_t}$$

$$v_w(t) = i_w(t)R + k_t\dot{\psi}(t)$$

$$v_w(t) = \frac{RT_x(t)}{k_t} + k_t\dot{\psi}(t)$$

$$T_x(t) = \frac{{k_t}^2}{R}\dot{\psi}(t) + \frac{k_t}{R}v_w(t)$$

Slope of diamond



Stall torque → $T_{stall} = \frac{k_t}{R}v_w = k_t \cdot i_{stall}$

No-load speed → $\dot{\theta}_{no-load} = \frac{v_w}{k_t}$

Modeling Brushed DC Motors

	Mech.	Elec.
Power:	$T_x(t)\dot{\psi}(t)$	$v_w(t)i_w(t)$

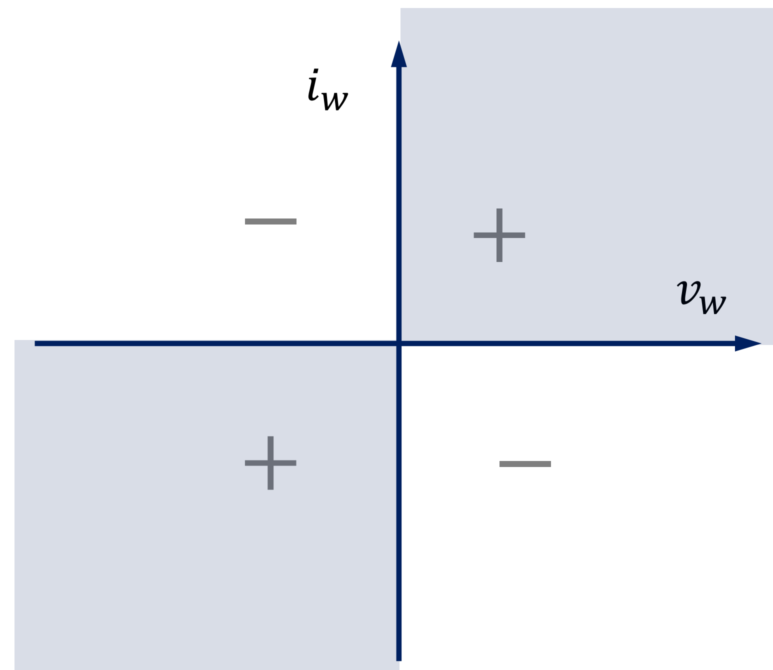
- Mechanical and electrical power:
- Power is the derivative of energy
- Positive electrical power:

$$\underbrace{v_w(t)}_{+} = \underbrace{i_w(t)R}_{+} + \underbrace{L\,\frac{d}{dt}i_w(t)}_{+/-} + \underbrace{k_t\dot{\psi}(t)}_{+}$$

$$\underbrace{v_w(t)}_{-} = \underbrace{i_w(t)R}_{-} + \underbrace{L\,\frac{d}{dt}i_w(t)}_{+/-} + \underbrace{k_t\dot{\psi}(t)}_{-}$$

Negative power: torque and velocity in opposite direction

Positive power: torque and velocity in same direction



Positive power: 'motoring'

Negative power: 'generating'

Modeling Brushed DC Motors

	Mech.	Elec.
Power:	$T_x(t)\dot{\psi}(t)$	$v_w(t)i_w(t)$

- Negative power:

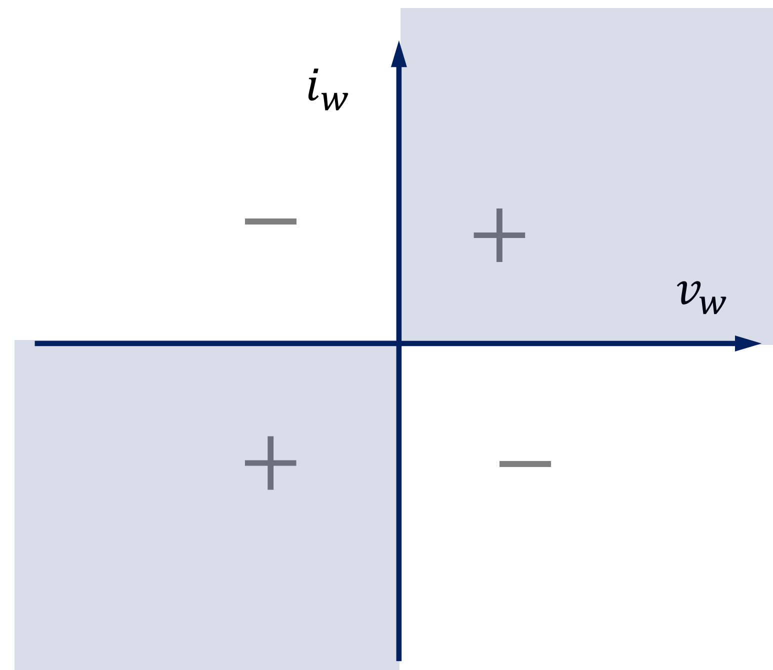
$$\underbrace{v_w(t)}_{+} = \underbrace{i_w(t)R}_{-} + \underbrace{L\frac{d}{dt}i_w(t)}_{+/-} + \underbrace{k_t\dot{\psi}(t)}_{+}$$

$$\underbrace{v_w(t)}_{-} = \underbrace{i_w(t)R}_{+} + \underbrace{L\frac{d}{dt}i_w(t)}_{+/-} + \underbrace{k_t\dot{\psi}(t)}_{-}$$

It is possible for negative mechanical power to have positive electrical power

Negative power: torque and velocity in opposite direction

Positive power: torque and velocity in same direction



Positive power: 'motoring'

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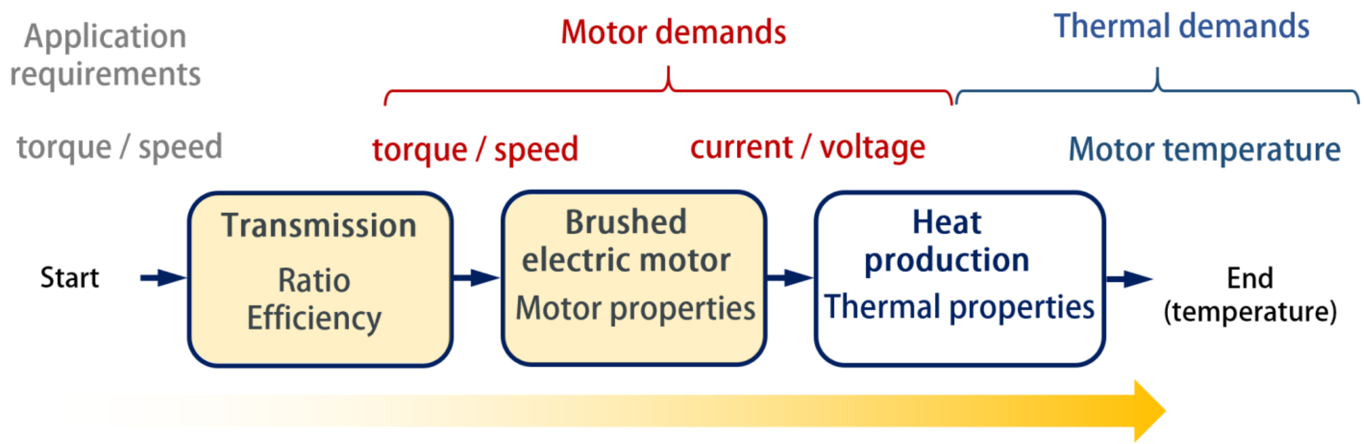
Beginning Mechanical Modeling



- We are stopping here and beginning to model a simplified ball-bot
- Then we will return to thermal analysis
- These procedures we will learn to model the mechanics / task are general
- We begin with Newton's 2nd Law:

$$Force = mass \cdot acceleration \qquad F = m \cdot \ddot{x}$$

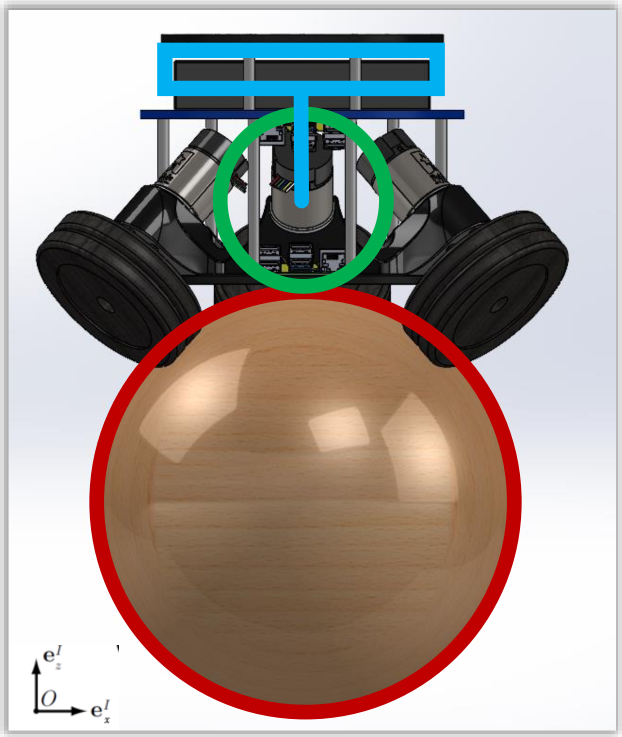
$$Torque = Moment\ of\ inertia \cdot angular\ acceleration \qquad T = J \cdot \ddot{\theta}$$

- But we need an understanding of the forces, properties, and motion

Mechanical Modeling of a Planar Ball-Bot

This analysis is acausal, meaning it uses knowledge of the future (derivatives)

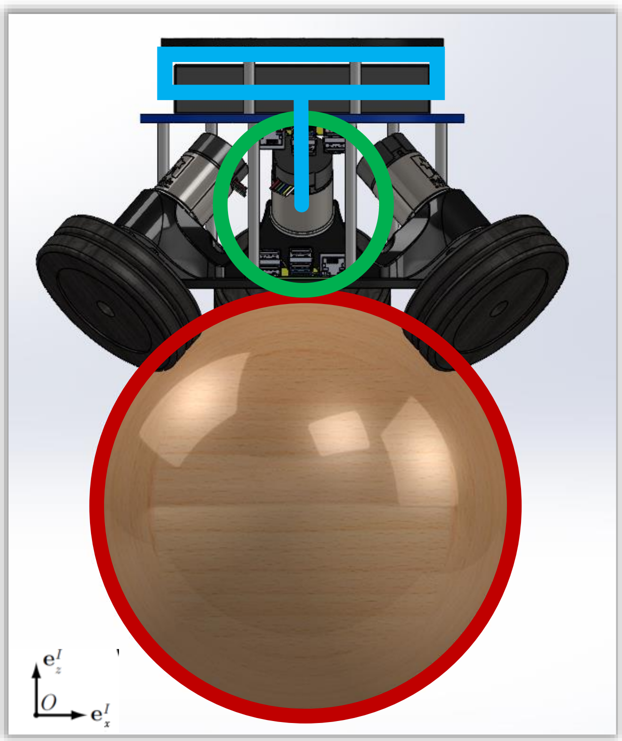
- We need a more detailed description of the torques required
- In this step, we will model the mechanics of the system → get torque
- Lets identify some major components and properties of the ball-bot in a single plane



- Lets model the ball bot as a single wheel instead of three
- Differences?
  - More torque will be required, since there's only one wheel
  - Serves as a conservative estimate
  - This image is the X-Z plane
  - What would be different if we modeled the Y-Z plane?
    - Nothing

Mechanical Modeling of a Planar Ball-Bot

- These components have physical properties
- These can be found using many tools: estimating, direct measurements, online resources, datasheets, intuition
- We measured / looked up:



Body / chassis:

- Distance to CoM: $L = 0.23\ m$
- Mass: $m_A = 1.71\ kg$
- Moment of inertia in x and y planes: $I_{x,y} = 0.01\ kgm^2$
- Moment of inertia in z plane: $I_z = 0.017\ kgm^2$

Wheel:

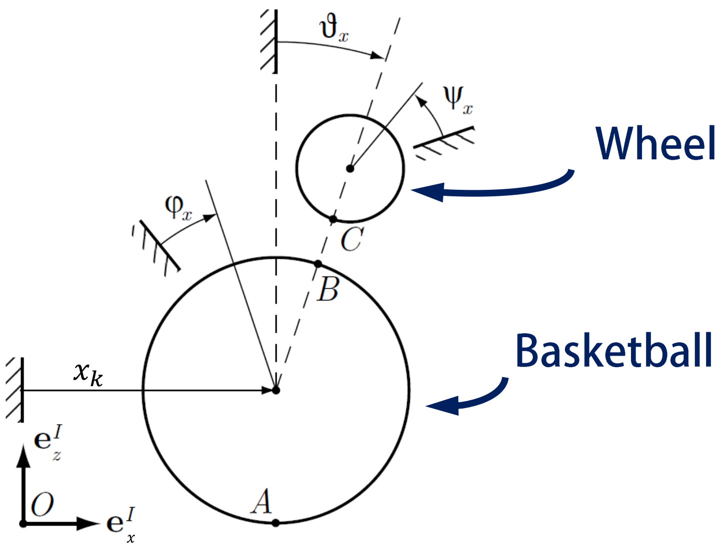
- Mass: $m_w = 0.29\ kg$
- Radius: $r_w = 0.1\ m$
- Moment of inertia: $I_w = 0.001\ kgm^2$

Ball:

- Mass: $m_k = 0.62\ kg$
- Radius: $r_k = 0.12\ m$
- Moment of inertia: $I_k = 0.004\ kgm^2$

Mechanical Modeling of a Planar Ball-Bot

- Now we want to think about the ball's motion
- We need to define parameters that describe this motion
- Both x and y planes are identical mechanically



Body / chassis motion:
- Lean rotation around the ball — ϑ_x or ϑ_y (rad)

Wheel motion:
- Rotation — ψ_x or ψ_y (rad)
- Rotation around the ball — ϑ_x or ϑ_y (rad)

Ball motion:
- Translation in x or y planes: — x_k or y_k (m)
- Rotation — φ_x or φ_y (rad)

Mechanical Modeling of a Planar Ball-Bot

- Now we want to think about the ball's motion
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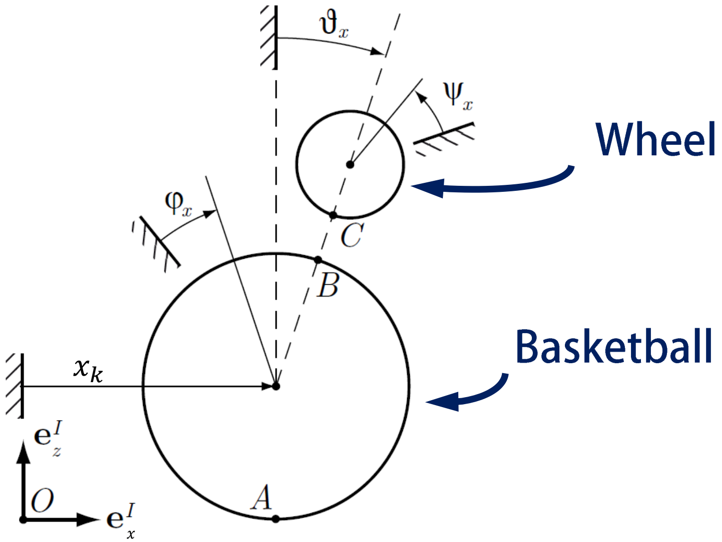


- Degrees of freedom: Four
  - x_k – Forward motion of ball
  - Theta - ϑ – Lean of body / chassis
  - Phi - φ – Rotation of ball
  - Psi - ψ – Rotation of wheel
- Constraints: Wheels do not slip

$$x_k = r_k \varphi$$

Basketball does not slip on the floor at A

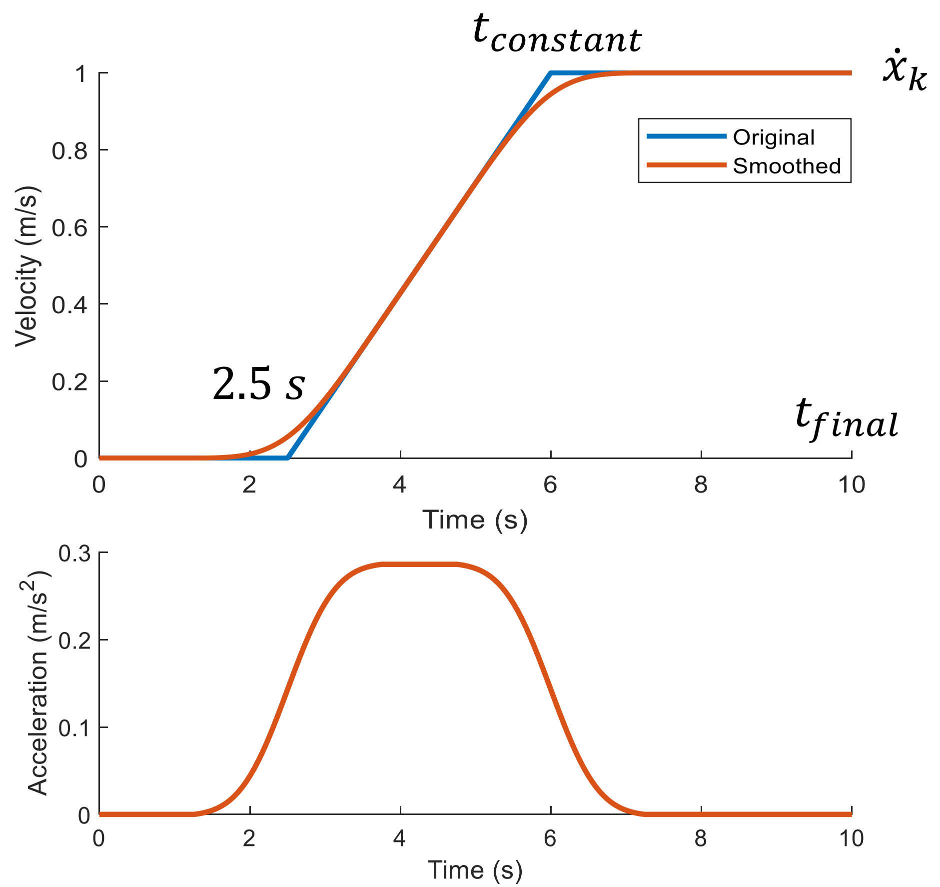
$$\psi = \frac{r_k}{r_w}(\varphi - \vartheta) - \vartheta$$

Tangential velocity constraint at B-C interface

- Two remaining DOFs:
  - ϑ and x_k for inverse modeling
  - ϑ and ψ for forward modeling

Mechanical Modeling of a Planar Ball-Bot

- We need to prescribe motion to the two remaining degrees of freedom
- Lets begin with x_k
- If we want to predict the torque required, we need to generate a motion profile.
- We choose this when spec'ing the task
  - How long do we want it to take?
  - Final velocity: $\dot{x}_k^{final} = 1\,\frac{m}{s}$
  - Ramp duration: $3.5\ s$
  - Resulting acceleration
  - What does this say about current and voltage needed?
    - Acceleration ~ Torque ~ Current
    - Velocity ~ Voltage



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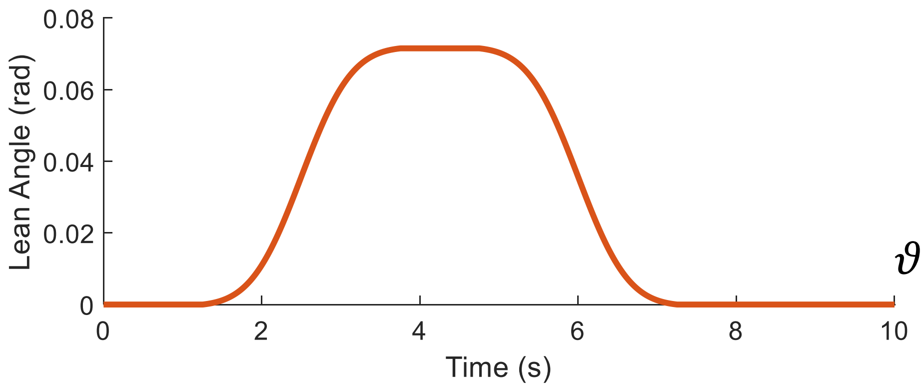
Mechanical Modeling of a Planar Ball-Bot

- We need to prescribe motion to the two remaining degrees of freedom
- We also need to provide information for the other DOF - ϑ – chassis lean angle
- Similarly, we choose this—how far do we want to be able to lean?
- We know it should lean with the applied torque
- Approach: scale the linear acceleration to an acceptable trajectory
- Lean angle is physically constrained – wheels will lose contact
- Lets use approx. 4 deg (0.07 rad) as an acceptable value

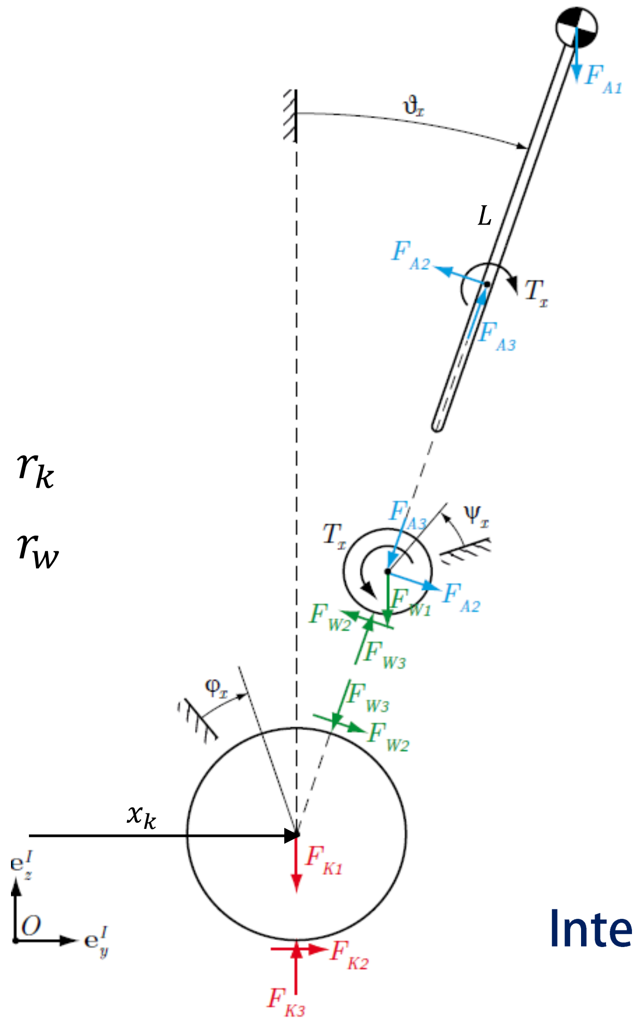


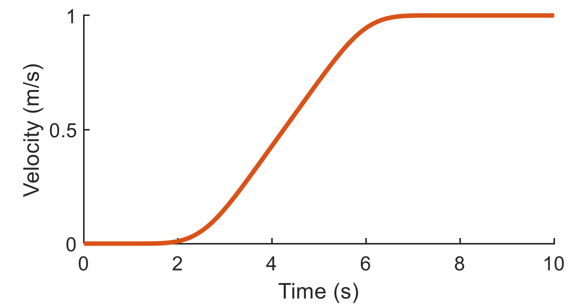
Mechanical Modeling of a Planar Ball-Bot

$$F_{K1} = g \cdot m_k$$

$$F_{W1} = g \cdot m_w$$

$$F_{A1} = g \cdot m_a$$





Ball radius r_k

Wheel radius r_w

Constraints: $x_k = r_k \varphi$

$$\psi = \frac{r_k}{r_w}(\varphi - \vartheta)$$

Ball velocity: $\dot{\varphi} = \frac{\dot{x}_k}{r_k}$

Intermediate var: $\gamma = L \cdot m_a + (r_k + r_w) \cdot m_w$

Tangential contact force : $F_{W2} = (m_a + m_w) \cdot (g \cdot \sin(\vartheta) - r_k \ddot{\varphi} \cos(\vartheta)) - \gamma \ddot{\vartheta}$

Mechanical Modeling of a Planar Ball-Bot

$$\gamma = L \cdot m_a + (r_k + r_w) \cdot m_w$$

$$\underbrace{F_{W2} = (m_a + m_w) \cdot (g \cdot \sin(\vartheta) - r_k \ddot{\varphi} \cos(\vartheta)) - \gamma \ddot{\vartheta}}_{\text{From free body diagram}}$$

$$\left.\begin{aligned} \dot{\varphi} &= \frac{\dot{x}_k}{r_k} \\ \psi &= \frac{r_k}{r_w}(\varphi - \vartheta) - \vartheta \end{aligned}\right\} \text{Constraints}$$

Required current

Output torque to roll ball / mass

Torque to accelerate motor inertia

$$i_w = \frac{\frac{T_x}{N\eta} + JN\ddot{\psi} + bN\dot{\psi}}{k_t}$$

Torque to overcome viscous loss in motor

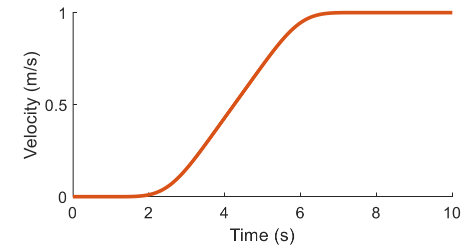
Required voltage

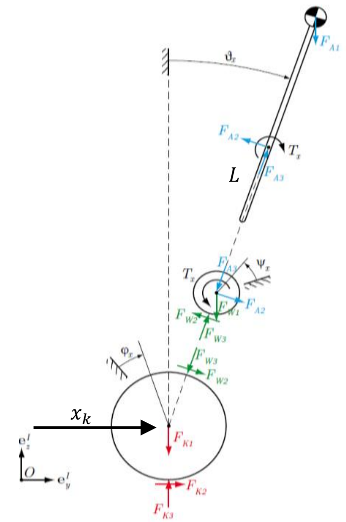
Voltage drop across resistance

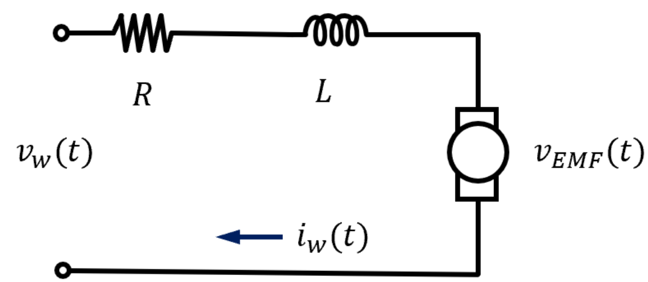
Back EMF voltage

$$v_w = i_w R + k_t N \dot{\psi} + L \frac{di_w}{dt}$$

Voltage drop across inductance







Mechanical Modeling of a Planar Ball-Bot

$$\gamma = L \cdot m_a + (r_k + r_w) \cdot m_w$$

$$F_{W2} = (m_a + m_w) \cdot (g \cdot \sin(\vartheta) - r_k \ddot{\varphi} \cos(\vartheta)) - \gamma \ddot{\vartheta}$$

$$\dot{\varphi} = \frac{\dot{x}_k}{r_k} \qquad \psi = \frac{r_k}{r_w}(\varphi - \vartheta) - \vartheta$$

$$i_w = \frac{\frac{T_x}{N\eta} + JN\ddot{\psi} + bN\dot{\psi}}{k_t} \qquad v_w = i_w R + k_t N \dot{\psi} + L\frac{di_w}{dt}$$