Robotics 311 : How to build robots and make them move

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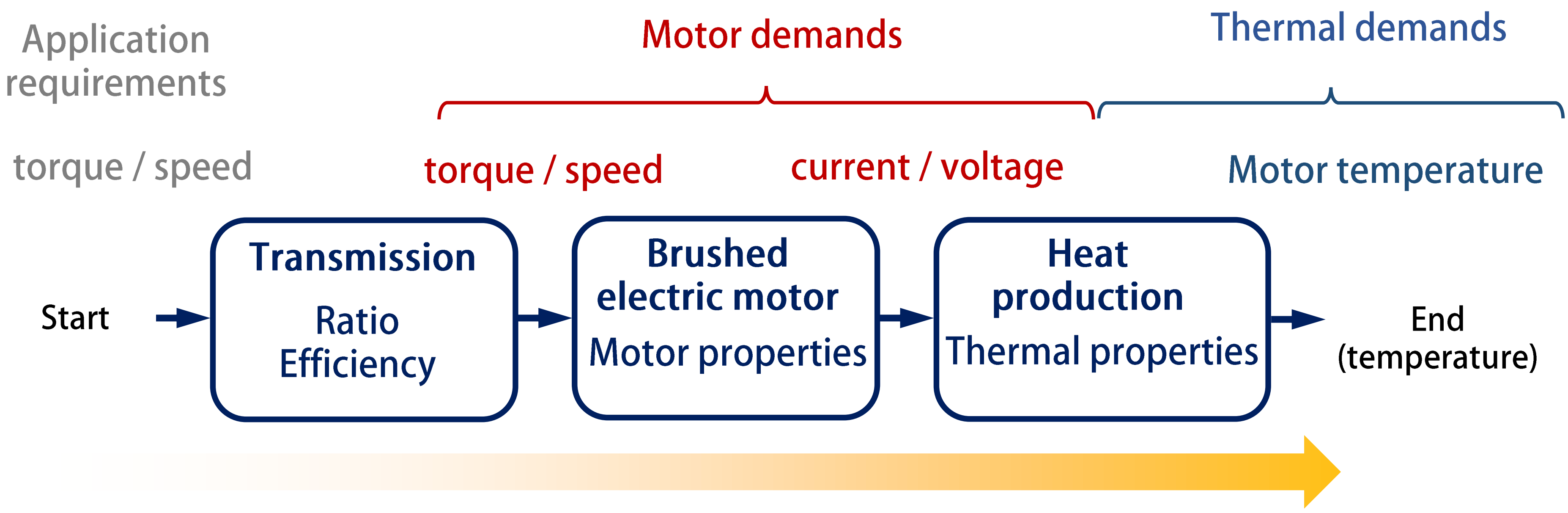
ROB 311 – Lecture 2

- Welcome to ROB 311!
- Today:
  - Review design framework
  - Discuss the initial blocks in the flow chart (transmissions and motors)
  - Discuss governing equations for brushed dc motors
  - Do a simple example and an example using the ball-bot

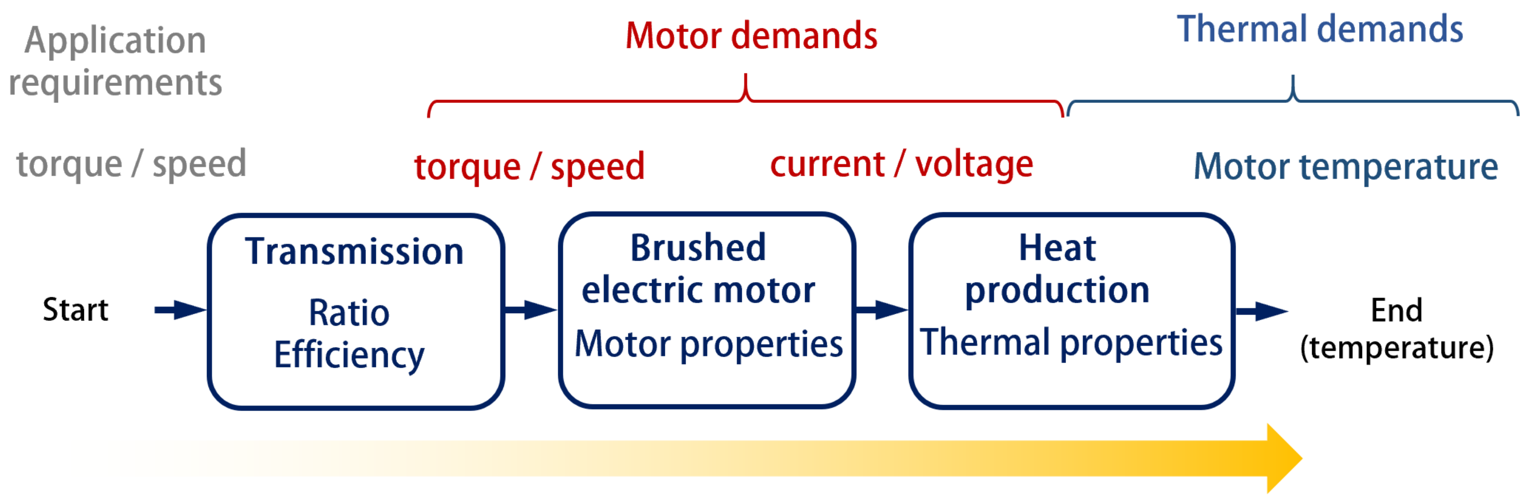
Design Analysis Framework

- As a reminder,
  - We have a description of the required torque to roll the ball / fight friction
  - We have a max velocity goal, say 1 m/s
- We use this torque and velocity to understand the demand on the motion, structural, and mechatronics components



Framework outcome: A motor that has the desired operating voltage, that is able to complete the task without overheating

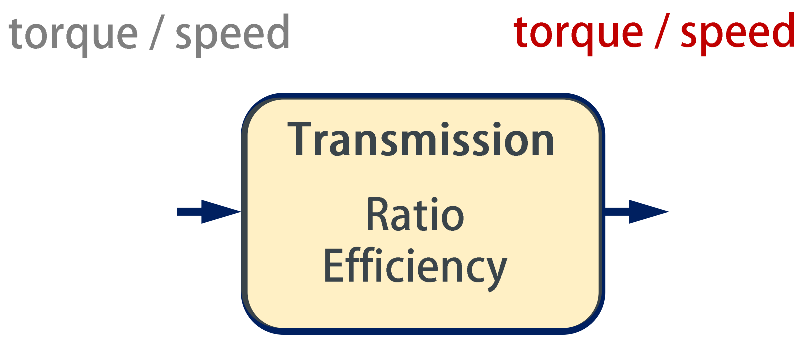
Design Analysis Framework



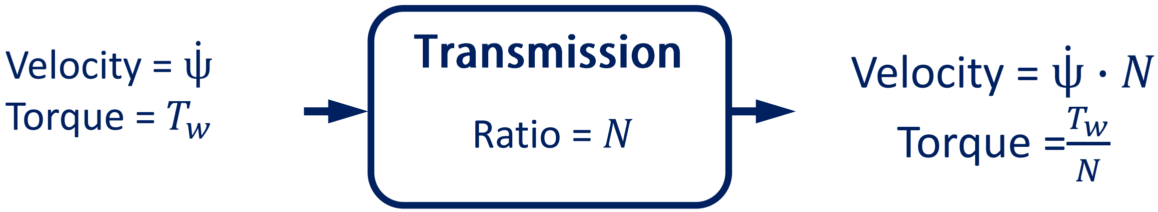
- What constraints might we have on the design, other than torque-velocity?
  - Battery voltage – we will operate on 12 V
  - Max current – the motor drivers can take 1-2 A continuous
- A motor-transmission combination is viable if
  - The required current and voltage demanded are within the constraints
  - The rise in motor temperature is acceptable

Modeling Transmissions

- Lets begin with the first block – transmissions
- Transmission exchange effort (torque or force) for flow (velocity)
- Transmissions convert one torque-speed regime to another torque-speed regime



- It is parameterized by two coefficients – the ratio and efficiency
- In an ideal transmission, power is conserved, and efficiency is 100%



Modeling Transmissions

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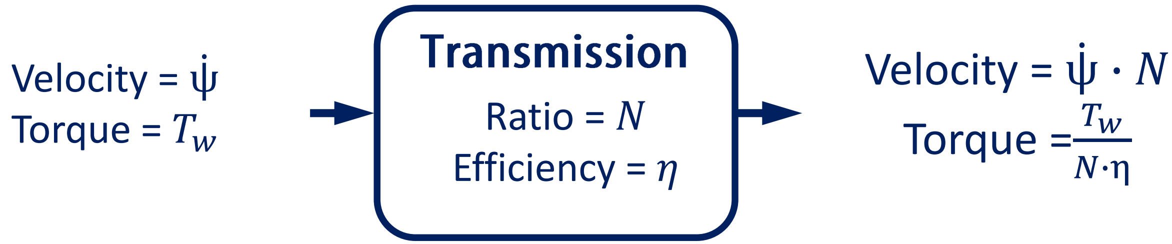
- In an ideal transmission, power is conserved, and efficiency is 100%

Velocity = $\dot{\psi}$
Torque = T_w
→ **Transmission**
Ratio = N
→ Velocity = $\dot{\psi} \cdot N$
Torque = $\frac{T_w}{N}$

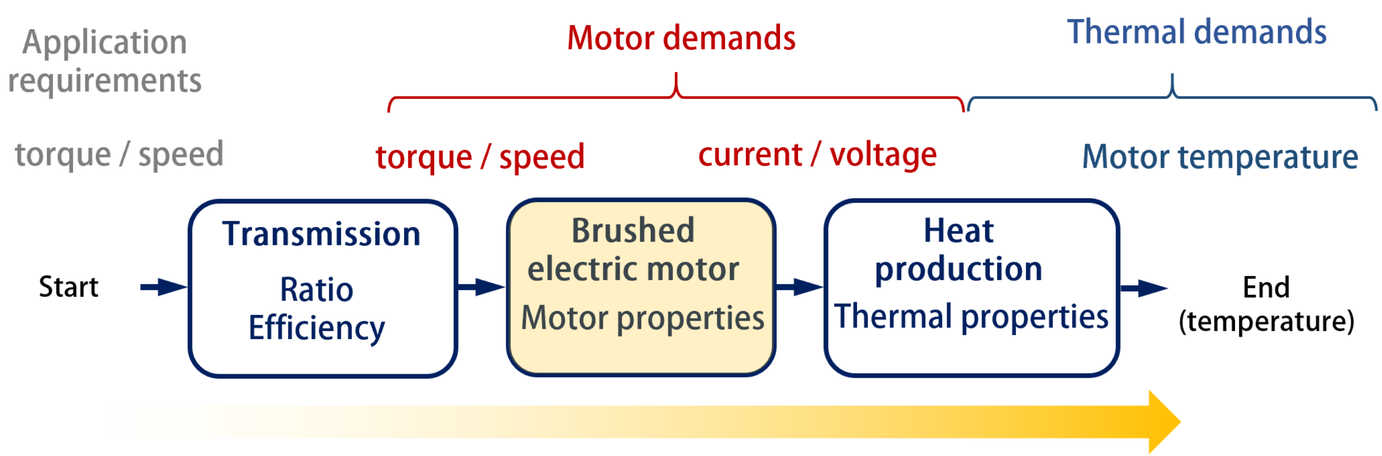
- Lets think about what transmissions do—why have them?
  - They trade torque for speed, which can be advantageous
- Why does a transmission on a bicycle matter so much?
  - The efficiency of the human body depends on:
    - Posture and speed

- We need our model of a transmission to account for inefficiency
  - Simplified models of inefficiency include it as a constant (0% - 100%)

Modeling Transmissions

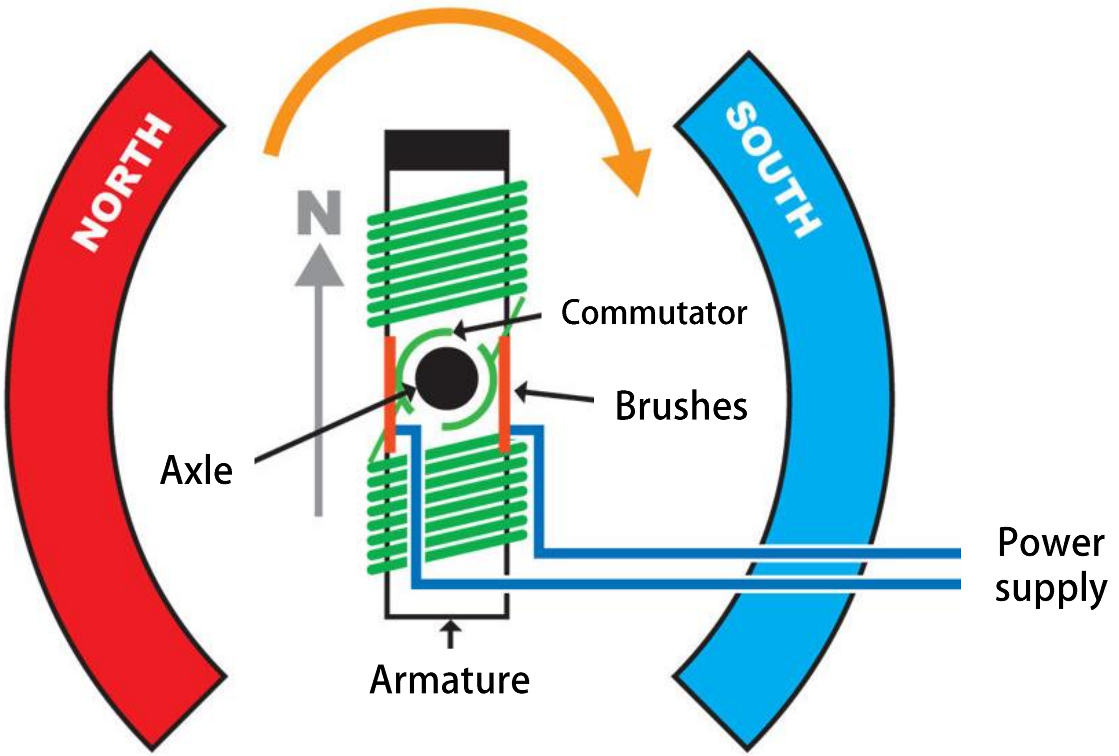


- Inefficiency reduces the torque transferred through the transmission
- Common values of η are 0.8 – 0.95, depending on type / stages
- At this point, if we know the torque and velocity requirements, we can predict how these requirement are seen at the motor (after the transmission)



Modeling Brushed DC Motors

- This brings us to modeling brushed dc electric motors
- Our goal is to use the required torque-velocity at the motor input to determine the required current-voltage that must be provided by the power supply
- How do brushed motors work?



Modeling Brushed DC motors

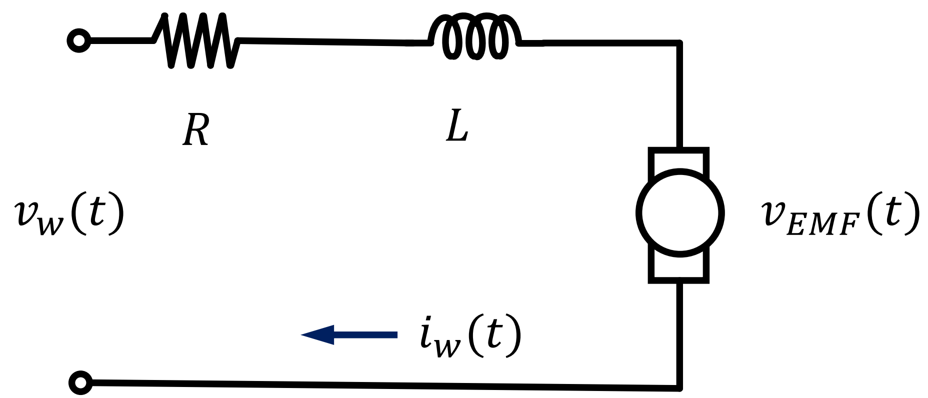
- Brushed motor considerations:
  - Brushes make mechanical contact to provide electrical power
  - Pros: Simple to implement
  - Cons:
    - Greater friction
    - Lower speeds
    - Result: Lower power / size → lower specific power (W/kg)
- Brushless motors exist that eliminate the need for mechanical brushes
  - Brushless motors have three windings (instead of one)
  - Brushless motors "spin" the current in the windings instead of the rotor
  - They require a computer to spin the currents in windings, known as a "brushless motor drive"
  - Higher speeds meaning higher specific power (W/kg)

Modeling Brushed DC Motors

- Which is right for your application?
- We quantify required torque / speed and current / voltage to make decision
- Typically assessed in tandem with the transmission ratio

Modeling Brushed DC Motors

- DC brushed motors
  - Rules of thumb:
    - Voltage is proportional to speed (effort)
    - Current is proportional to torque (flow)

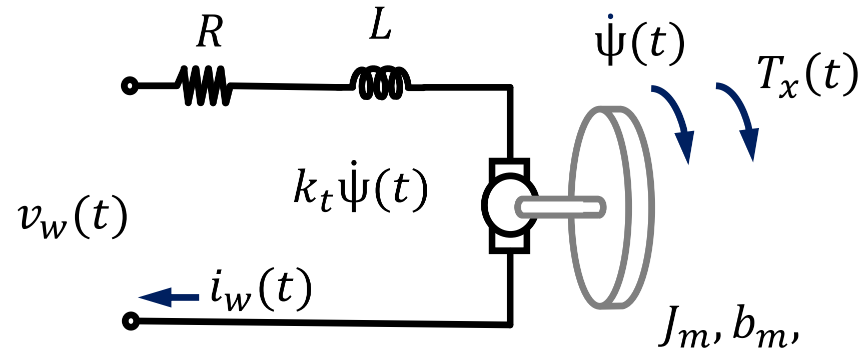


  - Think of a motor as a *transformer* – it transforms power in the form of current and voltage to power in the form of current and torque (plus loss as heat)

For brushed dc motors: $k_t = k_b = \frac{1}{k_v}$

Modeling Brushed DC Motors

- Governing equations



$$\underbrace{v_w(t)}_{\text{Voltage across winding}} = \underbrace{i_w(t)R}_{\text{Voltage across resistance}} + \underbrace{L\,\frac{d}{dt}i_w(t)}_{\text{Voltage across inductor}} + \underbrace{k_t\dot{\psi}(t)}_{\text{Solved second}}$$

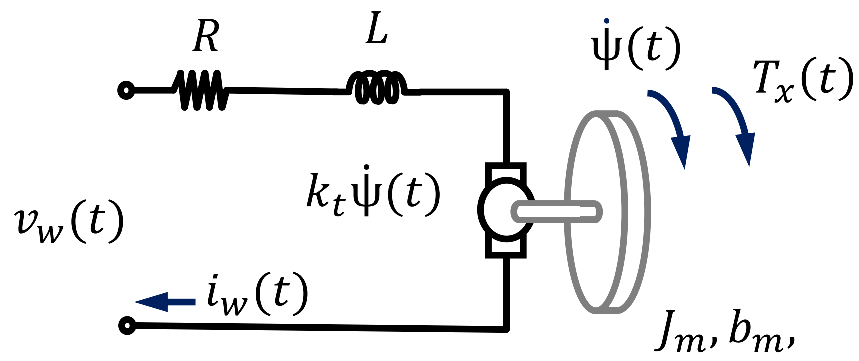
Solved second – using $i_w(t)$, solve for $v_w(t)$

$$\underbrace{k_t i_w(t)}_{\text{Motor torque}} = \underbrace{J_m\ddot{\psi}(t)}_{\text{Torque to accelerate inertia}} + \underbrace{b_m\dot{\psi}(t)}_{\text{Torque lost to friction}} + \underbrace{T_x(t)}_{\text{Motor output torque}}$$

Solved first – you know $\dot{\psi}(t)$ and derivatives → solve for $i_w(t)$

Modeling Brushed DC Motors

- Exercise



$$v_w(t) = i_w(t)R + L\frac{d}{dt}i_w(t) + k_t\dot{\psi}(t)$$

$$k_t i_w(t) = J_m\ddot{\psi}(t) + b_m\dot{\psi}(t) + T_x(t)$$

- Compute voltage and current at <u>steady state</u>:

$$R = 0.2\ \Omega$$

$$k_t = 0.1\frac{Nm}{A}$$

$$\dot{\psi} = 20\frac{rad}{s}$$

$$T_x(t) = 2\ Nm$$

$$b_m = 0.02\ Nms/rad$$

$$i_w(t) = \frac{0.02 \cdot 20 + 2}{0.1} = 24\ A$$

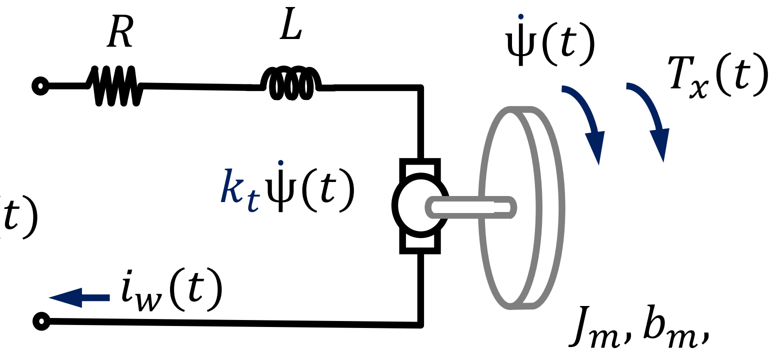
$$v_w(t) = 24 \cdot 0.2 + 0.1 \cdot 20 = 6.8\ V$$

Modeling Brushed DC Motors

- At steady state, no damping (ideal)

$$k_t i_w(t) = T_x(t) \rightarrow \; i_w(t) = \frac{T_x(t)}{k_t}$$

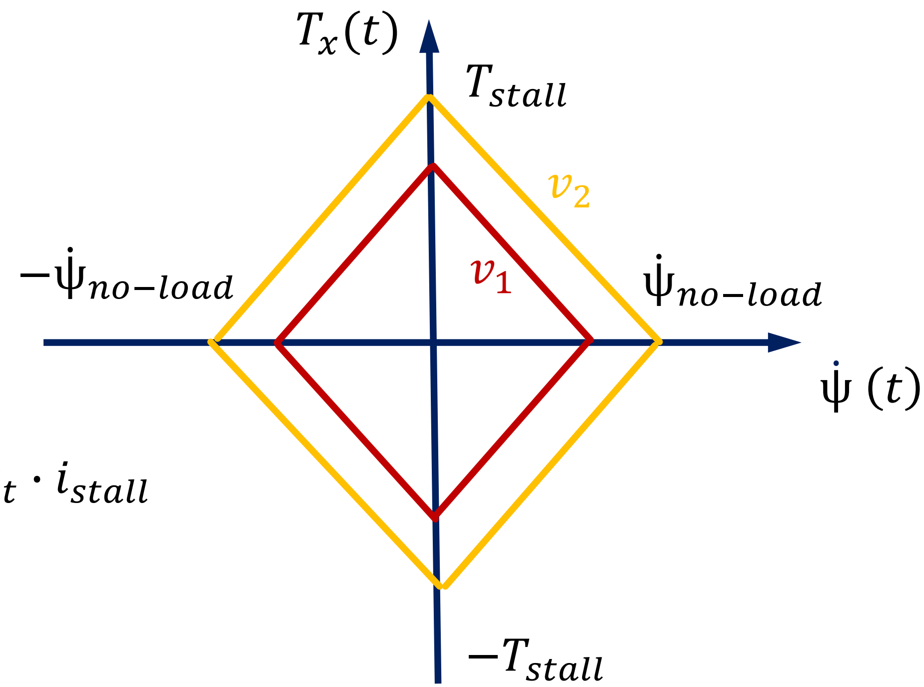
$$v_w(t) = i_w(t)R + k_t\dot{\psi}(t)$$



$$v_w(t) = \frac{RT_x(t)}{k_t} + k_t\dot{\psi}(t)$$

$$T_x(t) = \frac{{k_t}^2}{R}\dot{\psi}(t) + \frac{k_t}{R}v_w(t)$$

Slope of diamond



Stall torque → $T_{stall} = \frac{k_t}{R}v_w = k_t \cdot i_{stall}$

No-load speed → $\dot{\theta}_{no-load} = \frac{v_w}{k_t}$

Modeling Brushed DC Motors

	Mech.	Elec.
Power:	$T_x(t)\dot{\theta}_1(t)$	$v_w(t)i_w(t)$

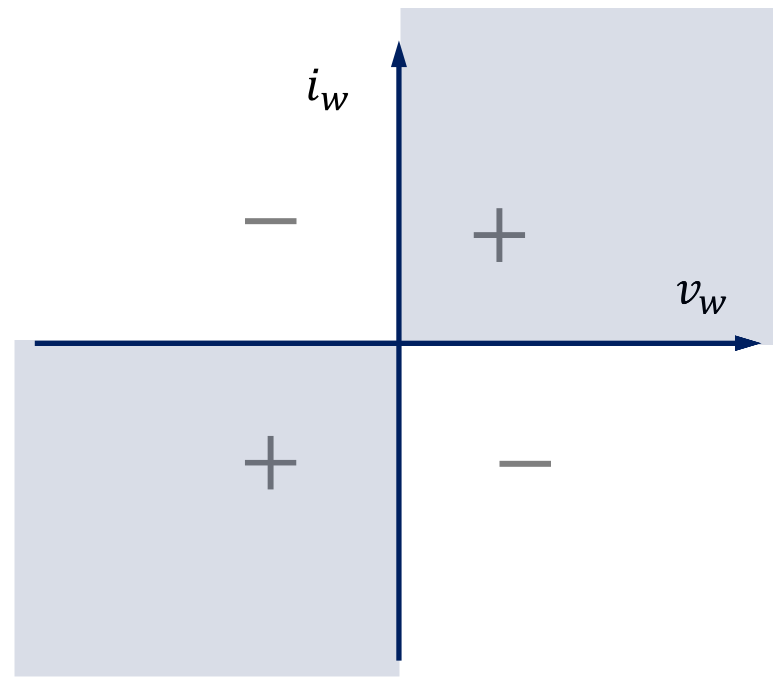
- Mechanical and electrical power:
- Power is the derivative of energy
- Positive electrical power:

$$\underbrace{v_w(t)}_{+} = \underbrace{i_w(t)R}_{+} + \underbrace{L\frac{d}{dt}i_w(t)}_{+/-} + \underbrace{k_t\dot{\psi}(t)}_{+}$$

$$\underbrace{v_w(t)}_{-} = \underbrace{i_w(t)R}_{-} + \underbrace{L\frac{d}{dt}i_w(t)}_{+/-} + \underbrace{k_t\dot{\psi}(t)}_{-}$$

Negative power: torque and velocity in opposite direction

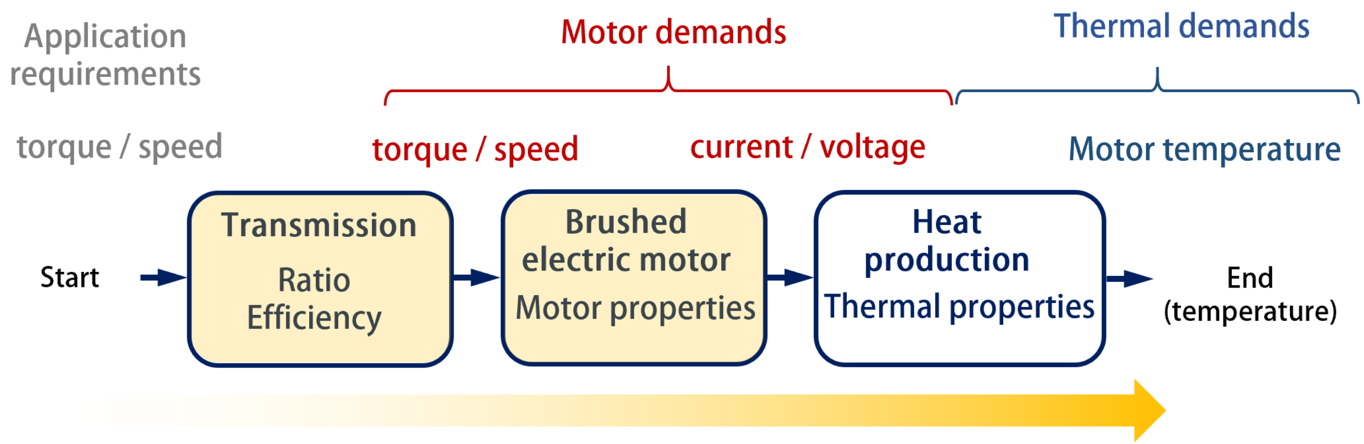
Positive power: torque and velocity in same direction



Positive power: 'motoring'

Negative power: 'generating'

Modeling Brushed DC Motors



- So far, we have learned how to model a transmission and motor
- Next, we will examine how heat affects component selection
- But before that, we will go over a simple physics-based model of a ball-bot
- We will focus on Newton's second law:

$$Force = mass \cdot acceleration \qquad F = m \cdot \ddot{x}$$

$$Torque = Moment\ of\ inertia \cdot angular\ acceleration \qquad T = J \cdot \ddot{\theta}$$

- To do this, we need an understanding of the forces and torques

Mechanical Modeling of a Planar Ball-Bot

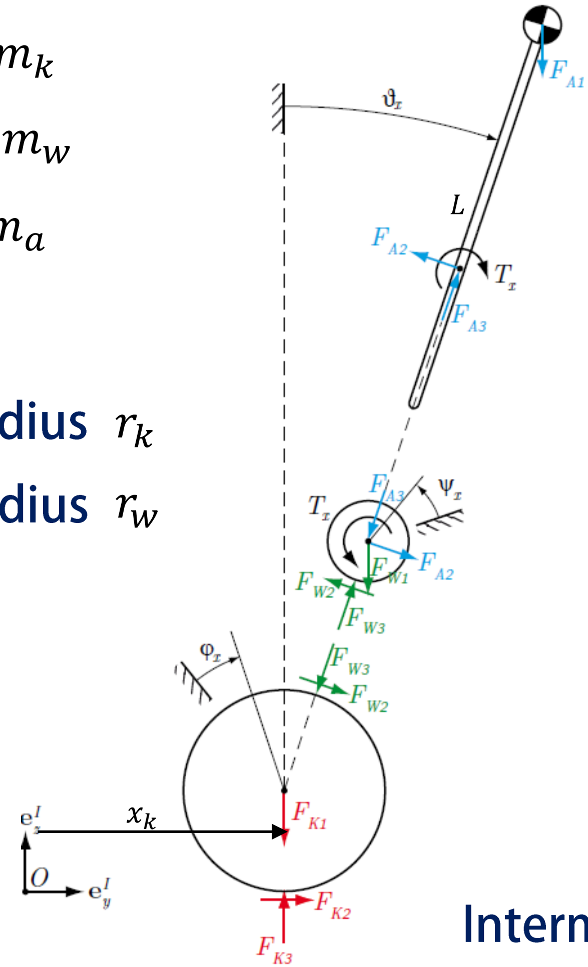
$$F_{K1} = g \cdot m_k$$

$$F_{W1} = g \cdot m_w$$

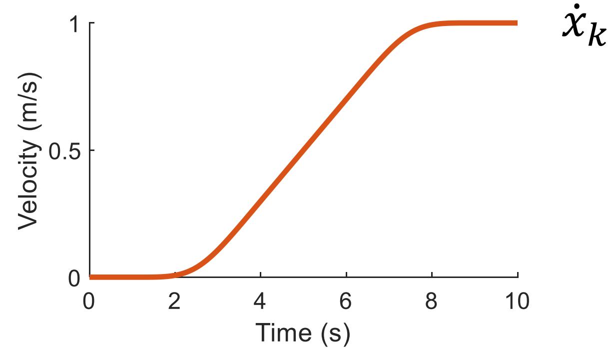
$$F_{A1} = g \cdot m_a$$

Ball radius r_k

Wheel radius r_w



- If we want to predict the torque required, we need to generate a motion profile.



Ball velocity: $\dot{\varphi} = \dfrac{\dot{x}_k}{r_k}$

Intermediate var: $\gamma = L \cdot m_a + (r_k + r_w) \cdot m_w$

Contact force : $F_{W2} = (m_a + m_w) \cdot (g \cdot \sin(\vartheta) - r_k \ddot{\varphi} \cos(\vartheta)) - \gamma \ddot{\vartheta}$