Robotics 311 : How to build robots and make them move

Prof. Elliott Rouse

GSI Yves Nazon MS

Fall 2022

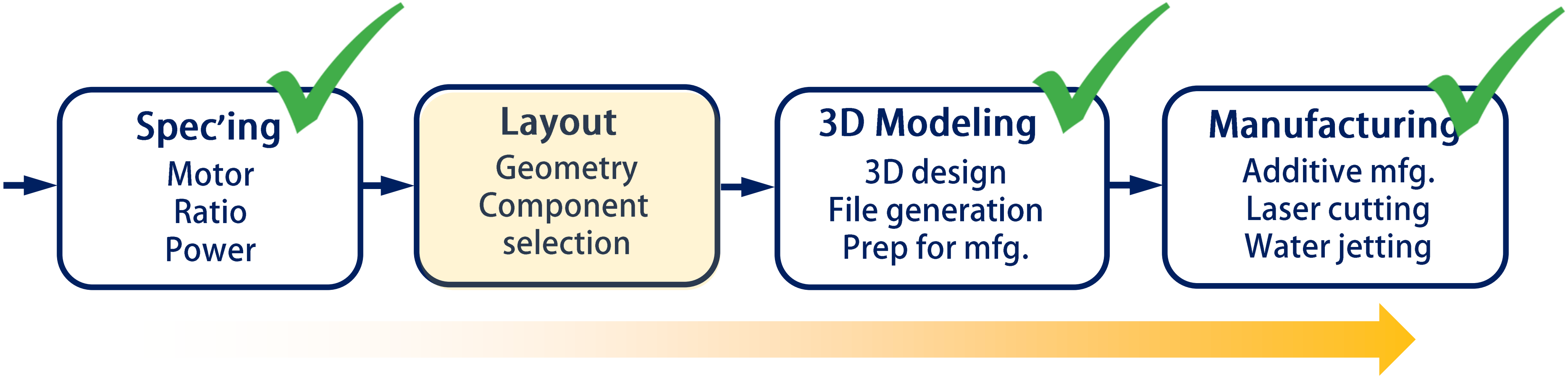
Photo credit: Brenda Ahearn / U-M CoE

ROB 311 – Lecture 12

- Today:
  - Review transmission types
  - Discussion layout and dimensions
  - Full planar model
  - Torque conversion

- Announcements
  - HW 3 will be posted…

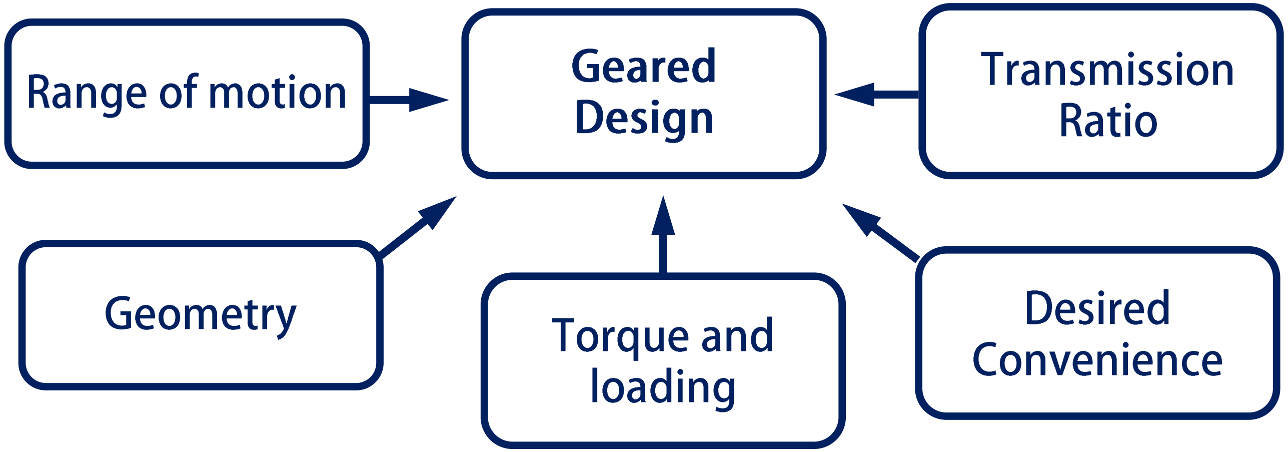
Manufacturing Types



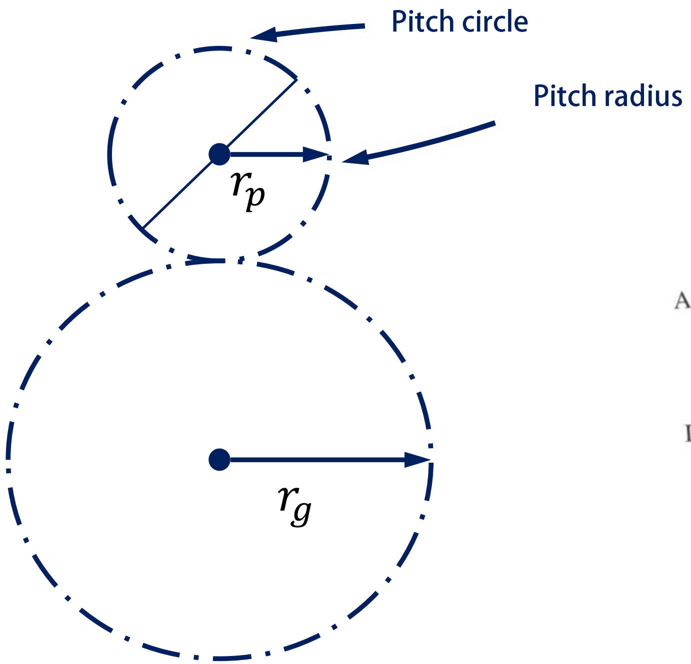
- We've learned how to spec and make robots, now lets talk about design layouts
- This is often moving motion from one place to another (kinematics)
- In robots, motion moving from the actuator to the end effector
- It begins with understanding the geometry of your robot and transmissions
- Very application specific!
- Coming up:
  - Introduce transmissions and linkages
  - In-depth example of ball-bot geometry and kinematics
  - Move to mechatronics, ball-bot dynamics, and control

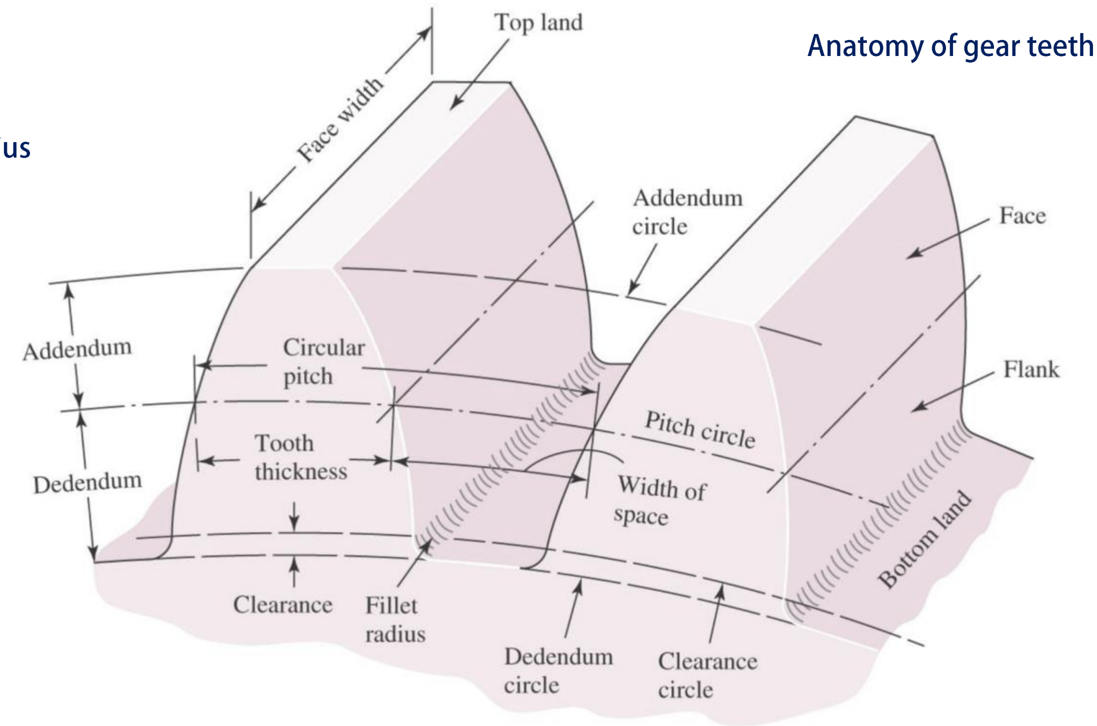
Designing Geared Transmissions

- First, obtain any specific geometric information related to your application
- For the ball-bot
  - Locations of wheels
  - Deep dive in kinematics next lecture
- Required information before beginning design
- The more you know about your application, the easier design will be
- Many types of geared transmissions



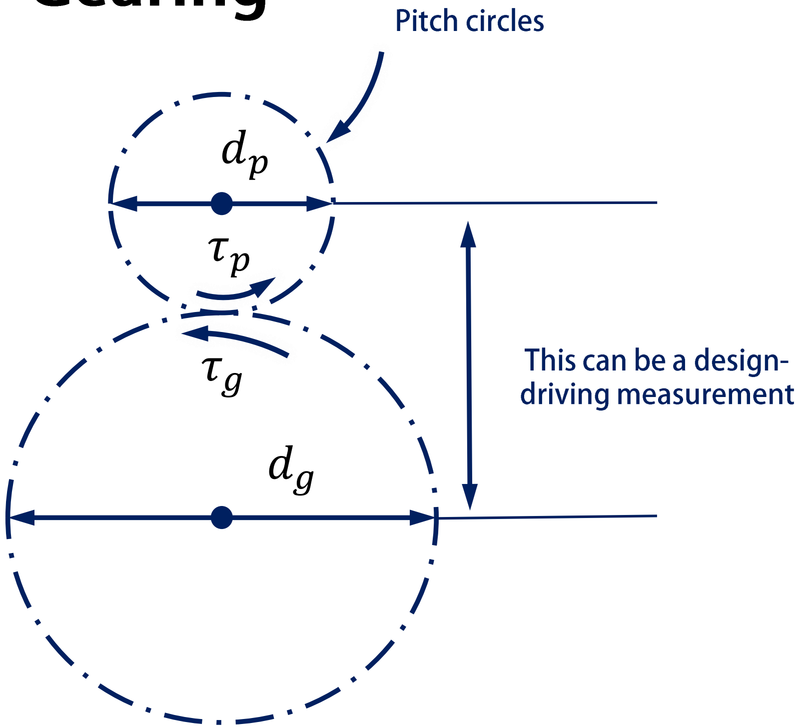
Gearing





- Pitch diameter, p – theoretical diameter / circle upon which all calculations are based
- Diametral pitch, P – the ratio of the number of teeth to the pitch diameter in units of teeth/m
- Backlash – amount of angular play in transmission (tooth space > tooth width)
- The small gear is often known as the *pinion* and the larger is the *gear*

Gearing



Diametral pitch

$$P = \frac{n}{d}$$

Circular pitch

$$p = \frac{\pi d}{n}$$

Pertains to individual gears

$$N = \frac{d_p}{d_g} = \frac{n_p}{n_g} = \left|\frac{\omega_g}{\omega_p}\right| = \frac{\tau_g}{\eta \tau_p}$$

Pertains to gearsets

- Pinion teeth, n_p – number of teeth on pinion
- Gear teeth, n_g – number of teeth on gear
- Transmission ratio, N – ratio of input speed to output (also diameters, torque, …)
- Conjugate action – defines that the ratio of velocity is inversely proportional to the pitch radii

Gearing

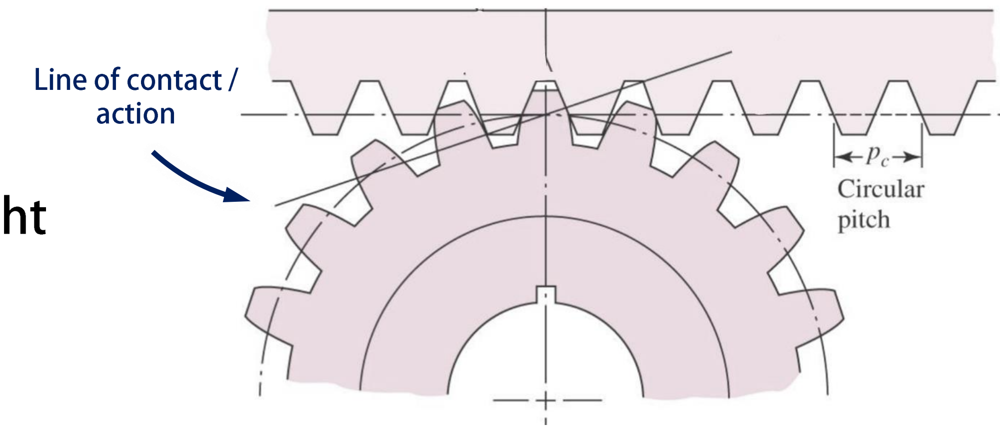
- Often, you can buy a motor with a gearhead, known as a gearmotor
- Multiple ratios available for a given motor
- This can make adding the required ratio more convenient
- Gearing components already selected

Rated Voltage	Stall Current	No-Load Current	Gear Ratio	No-Load Speed (RPM)	Extrapolated Stall Torque (kg · cm)	(oz · in)	Max Power (W)	 Without Encoder	 With Encoder
12 V	5.5 A	0.2 A	1:1 (no gearbox)	10,000	0.5	7	–	–	item #4750
			6.3:1	1600	3.0	42	12	item #4747	item #4757
			10:1	1000	4.9	68	12	item #4748	item #4758
			19:1	530	8.5	120	12	item #4741	item #4751
			30:1	330	14	190	12	item #4742	item #4752
			50:1	200	21	290	10	item #4743	item #4753
			70:1	150	27	380	10*	item #4744	item #4754
			100:1	100	34	470	8*	item #4745	item #4755
			131:1	76	45	630	6*	item #4746	item #4756
			150:1	67	49	680	6*	item #2829	item #2828

Pololu 37D gearmotor selection

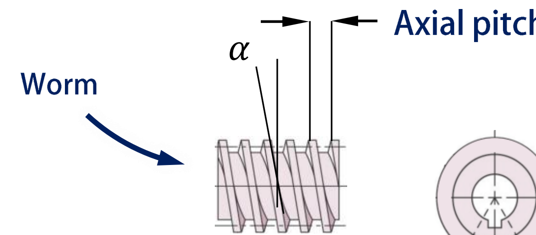
Gearing

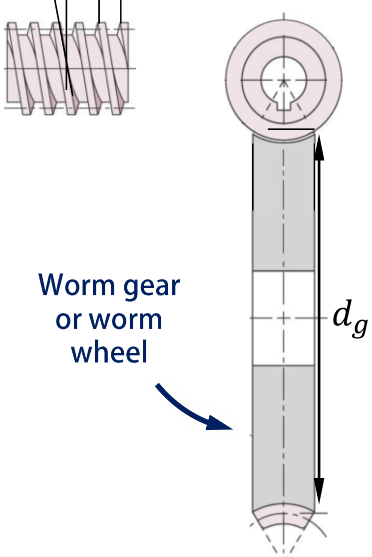
- Rack and pinion – spur gear transmission with an infinite pitch diameter for the gear
- Gears with infinite diameter (straight gears) are known as *racks*
- Can be plastics, brass, or steels



Worm Gears

- Worm gears are used for extremely high ratio transmissions
- Specified by axial pitch, gear diameter, and transverse circular pitch (p_t)
- Backlash less noticeable (high ratio)
- Rotate angular motion 90°
- Can be plastics, brass, or steels



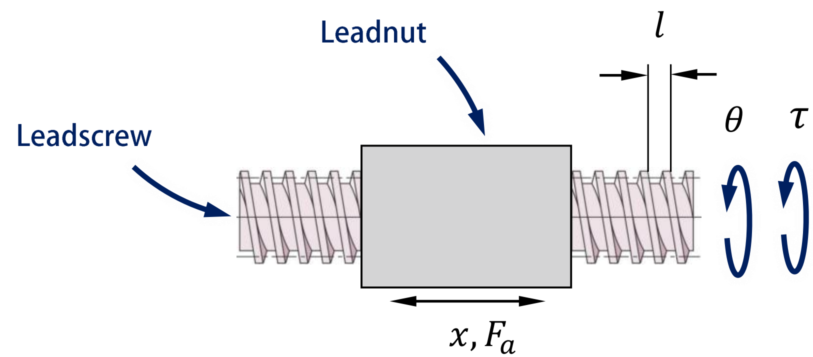


$$p_t = \frac{p}{\cos(\alpha)}$$

$$d_g = \frac{n_g p_t}{\pi}$$

Screws

- Screws turn rotary motion into linear motion
- Useful in a wide array of robotics applications
- Lead screws are low cost and useful
- Can be purchased as a set with specified dimensions
- More information required to know full transmission ratio
- Ball screws can be used for highly efficient motion (expensive)



$$\tau = \frac{F_a l}{2\pi\eta}$$

η	Efficiency
F_a	Thrust force
l	Screw lead
τ	Driving torque

$$\dot{x} = \frac{l\dot{\theta}}{2\pi}$$

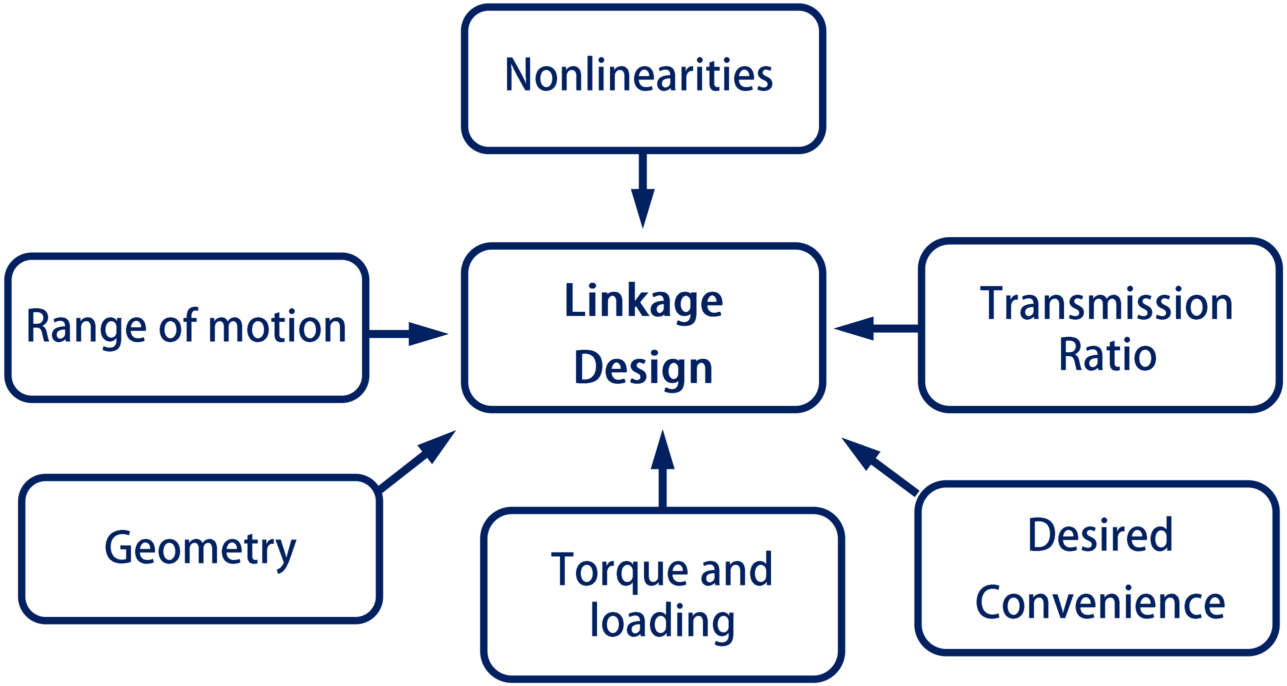
$\dot{x}$	Nut linear velocity
$\dot{\theta}$	Shaft angular velocity

leadscrew

ballscrew

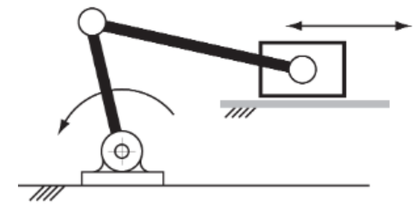
Linkages

- Linkages are commonly used in robotic systems
- They have many uses (rotary to rotary, rotary to linear)
- They have nonlinear transmission / velocity ratios
- Similarly, geometric information about your application is critical

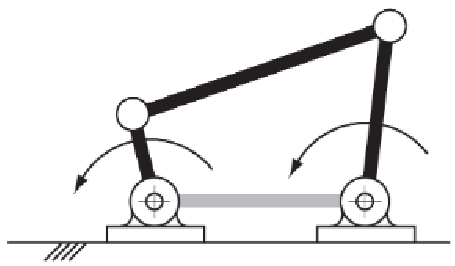


Linkages

- Example linkage types (more here)
- First step is determining the input and output links
- Determine transmission ratio and kinematics as a function of starting configuration and link lengths
- Kinematics / transmission ratio determined using geometry
- L_3 is input, L_1 is output

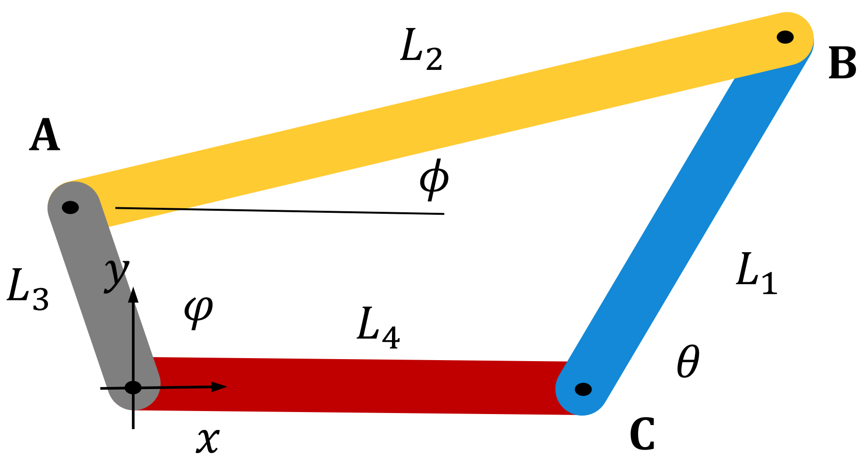


Slider crank



Parallelogram 4-bar

Many more…



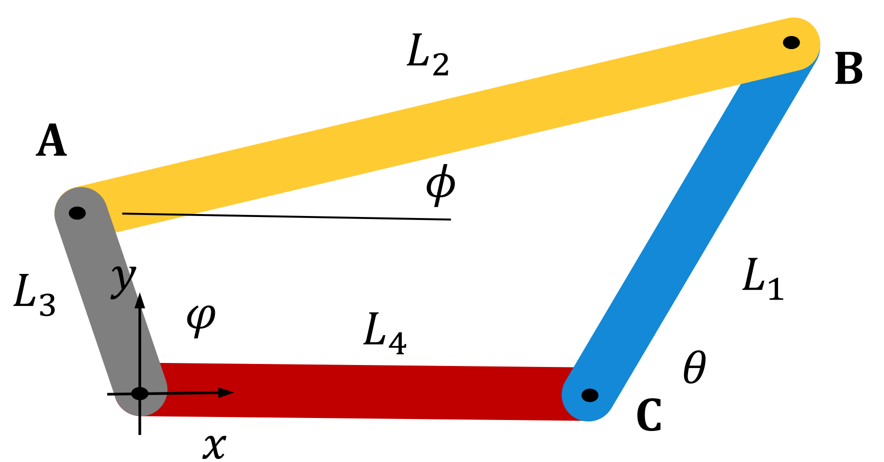
Linkages

- A series of constraints and geometry are needed to design a four-bar mechanism
- The next few slides introduce these constraints

$$\mathbf{A} = \begin{bmatrix} L_3\cos(\varphi) \\ L_3\sin(\varphi) \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} L_4 + L_1\cos(\theta) \\ L_1\sin(\theta) \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} L_4 \\ 0 \end{bmatrix} \quad (\mathbf{B}-\mathbf{A})\cdot(\mathbf{B}-\mathbf{A}) - {L_2}^2 = 0$$

$$\big(2L_1L_4 - 2L_3L_1\cos(\varphi)\big)\cos(\theta) - \big(2L_3L_1\sin(\varphi)\big)\sin(\theta) + \dots$$
$$\dots + \left({L_1}^2 + {L_3}^2 + {L_4}^2 - {L_2}^2 - 2L_3L_4\cos(\varphi)\right) = 0$$

Geometric constraint equation



L_3 Input
L_1 Output
L_4 Ground

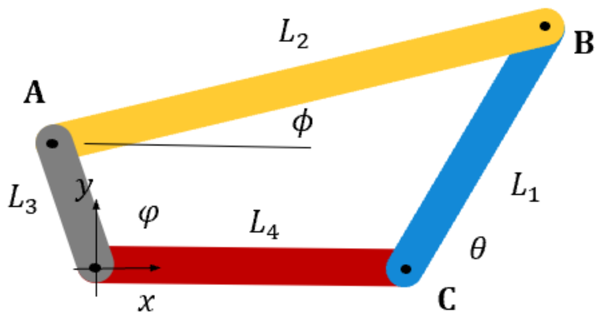
Linkages

Position constraint equations

$$\mathbf{A} + (\mathbf{B} - \mathbf{A}) = \mathbf{C} + (\mathbf{B} - \mathbf{C})$$

$$\begin{bmatrix} L_3\cos(\varphi) \\ L_3\sin(\varphi) \end{bmatrix} + \begin{bmatrix} L_2\cos(\phi) \\ L_2\sin(\phi) \end{bmatrix} = \begin{bmatrix} L_4 \\ 0 \end{bmatrix} + \begin{bmatrix} L_1\cos(\theta) \\ L_1\sin(\theta) \end{bmatrix}$$

$$\phi = \operatorname{atan}\left(\frac{L_1\sin(\theta) - L_3\sin(\varphi)}{L_4 + L_1\cos(\theta) - L_3\cos(\varphi)}\right)$$



Velocity constraint equations

$$\dot{\mathbf{A}} + \frac{d}{dt}(\mathbf{B} - \mathbf{A}) = \frac{d}{dt}(\mathbf{B} - \mathbf{C})$$

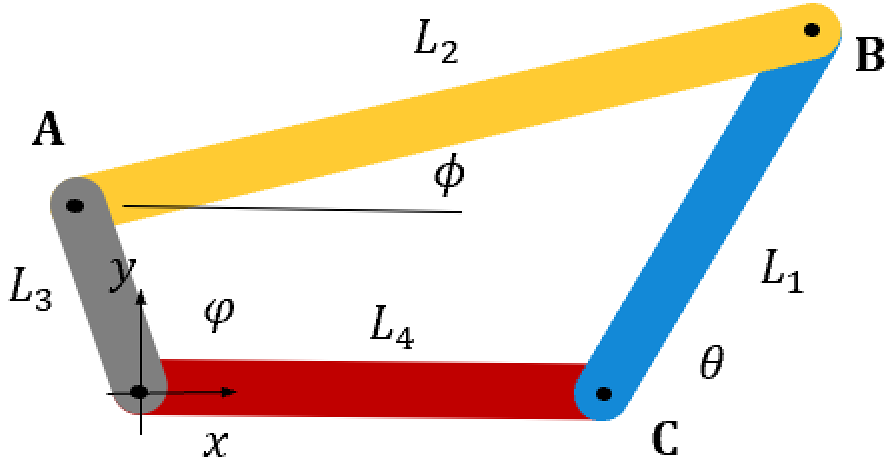
$$\begin{bmatrix} -L_3\sin(\varphi) \\ L_3\cos(\varphi) \end{bmatrix}\dot{\varphi} + \begin{bmatrix} -L_2\sin(\phi) \\ L_2\cos(\phi) \end{bmatrix}\dot{\phi} = \begin{bmatrix} -L_1\sin(\theta) \\ L_1\cos(\theta) \end{bmatrix}\dot{\theta}$$

Linkages

- Describing the transmission ratio of a four-bar linkage

$$N = \frac{d\varphi}{d\theta} = \frac{-L_3 L_1 \cos(\theta)\sin(\varphi) - L_1 L_4 \sin(\theta) + L_3 L_1 \sin(\theta)\cos(\varphi)}{L_3 L_4 \sin(\varphi) + L_3 L_1 \cos(\theta)\sin(\varphi) - L_3 L_1 \sin(\theta)\cos(\varphi)}$$

Kinematically varying transmission ratio



L_3 Input
L_1 Output
L_4 Ground

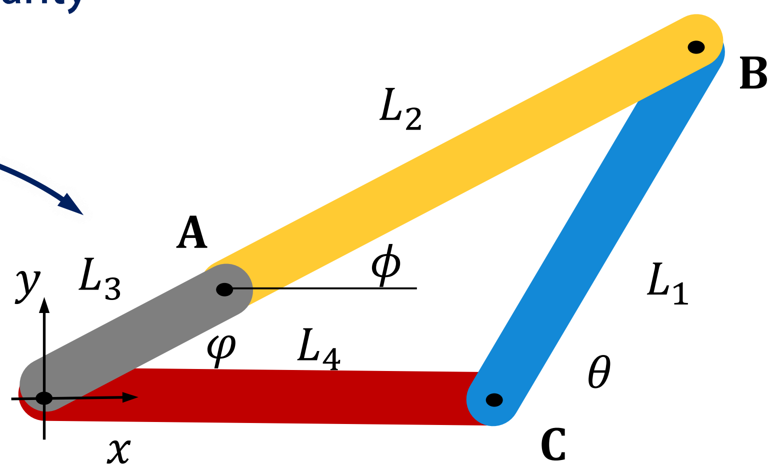
Singularities

- Singularities can occur when two links are co-linear
- Causes the loss of a degree of freedom in the linkage
- The applied torque from L_3 cannot apply torque to L_1
- The slope of the input-output kinematics / ratio goes to infinity
- Avoid singularities by 30° or more

$$N = \frac{d\varphi}{d\theta}$$

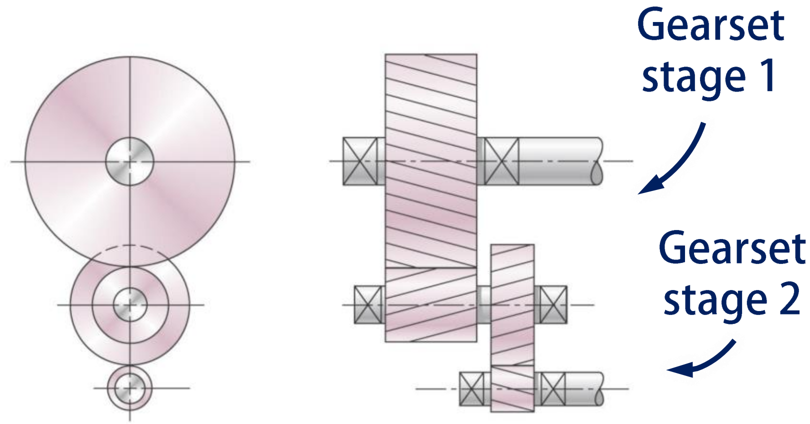
Changes sign at the singularity

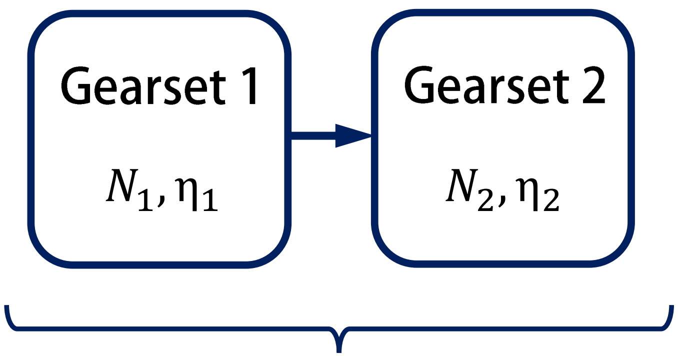
Singularity configuration



Compound Transmissions

- Sometimes, larger ratios are needed
- This can be accomplished by stacking transmissions
- Known as *compound transmissions*
- Shown as gears, but could be any type
- Ratios are multiplied
- Efficiencies are multiplied
- Extends to an arbitrary number of stages

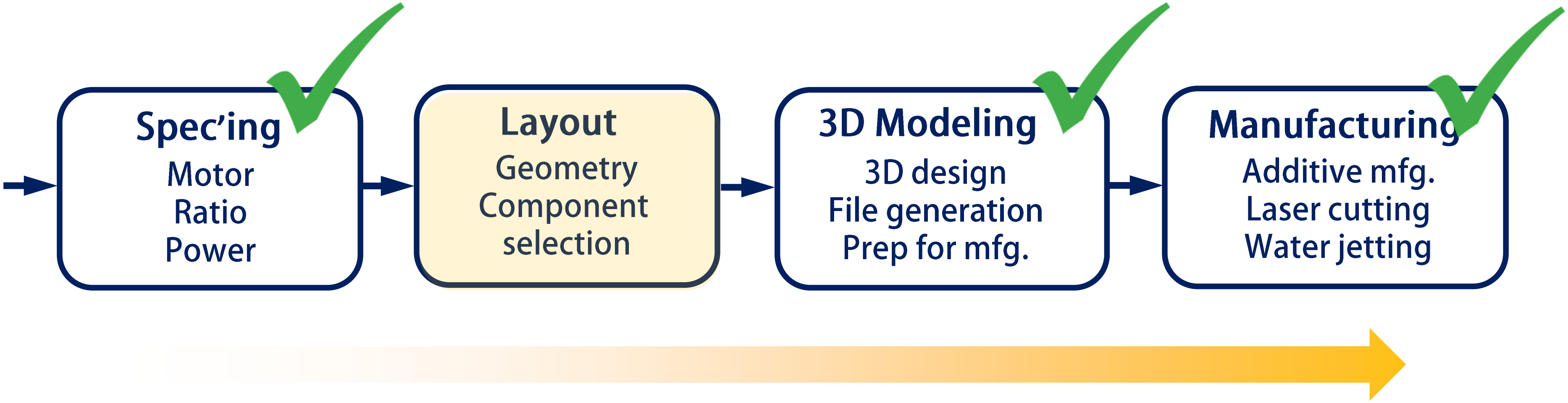




Compound transmission

$$N_{total} = N_1 \cdot N_2$$
$$\eta_{total} = \eta_1 \cdot \eta_2$$

Manufacturing Types

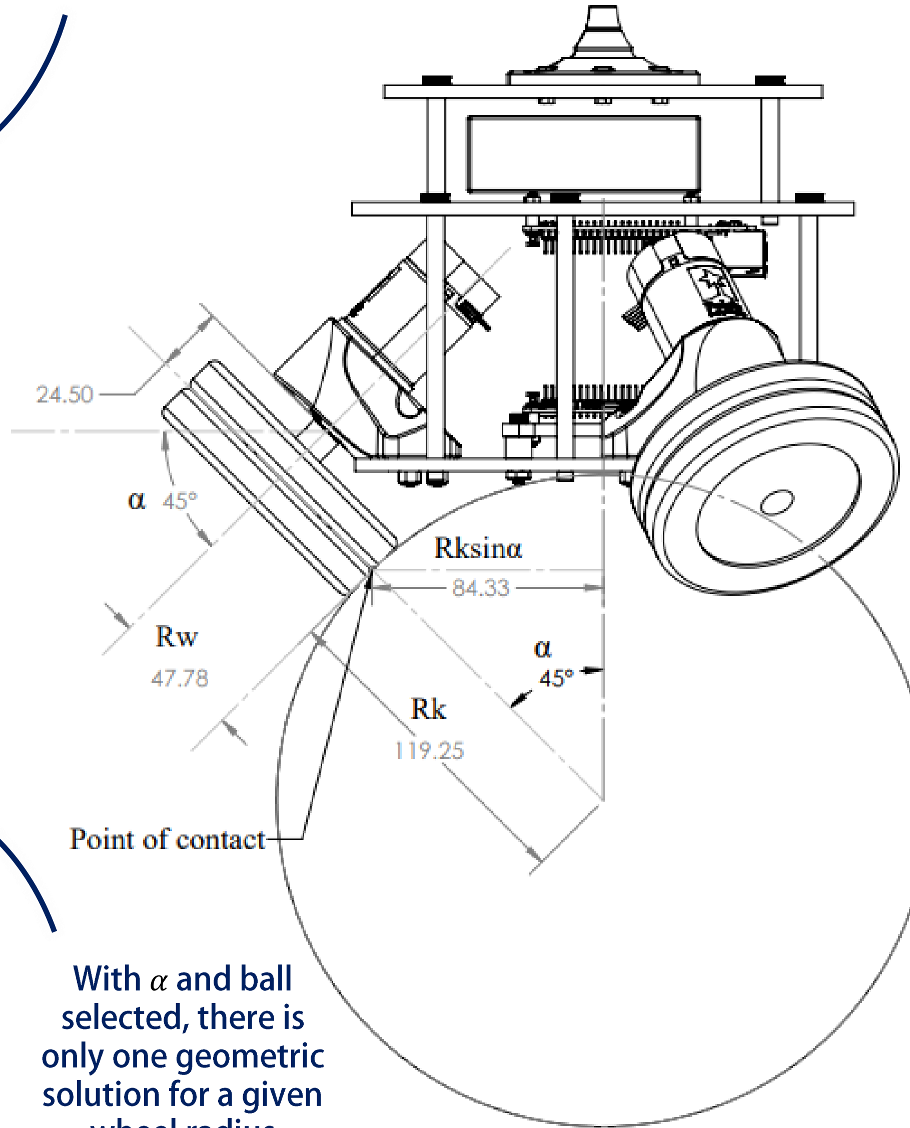


- To determine the layout, we need to know the geometry of our robot
- The geometry of the ball-bot is more complex / important than usual
- To describe the geometry, we need an understanding of how motion is applied to the ball-bot
- This requires some in-depth descriptions of the ball-bot
  - Torque / motion applied from the wheels to the basketball
- This is also critical for control

Ball-Bot Layout

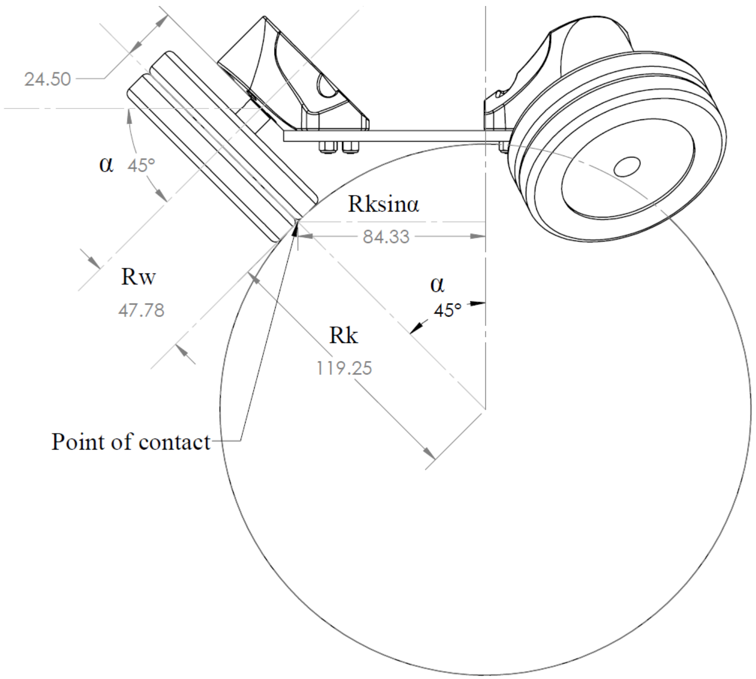
Balances base width with radial force / friction increases

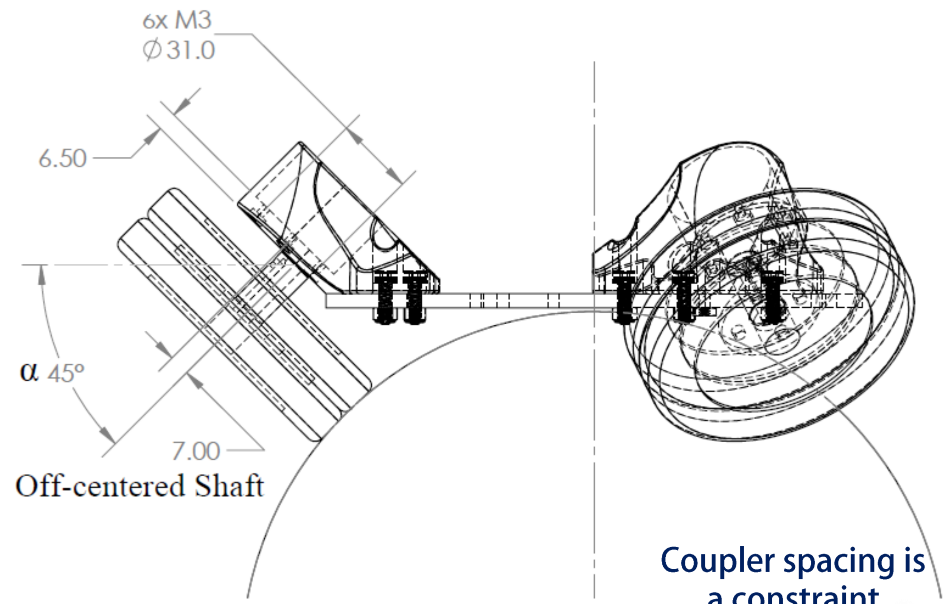
- Major design decisions are α angle, ball radius, and wheel radius
- We chose $\alpha = 45^{\circ}$
  - As α is increased, radial force increases → more friction
  - Larger α designs have a greater lean angle
- We chose a basketball for its cost, size, and texture
- Basketball radius (R_k) is 119.25 mm
- Wheel radius (R_W) is 47.8 mm
- Coupler to wheel center distance is 24.5 mm
- Wheel contacts spaced on circle with radius 84.3 mm from Z axis $(R_k \sin(\alpha))$



With α and ball selected, there is only one geometric solution for a given wheel radius

Ball-Bot Layout

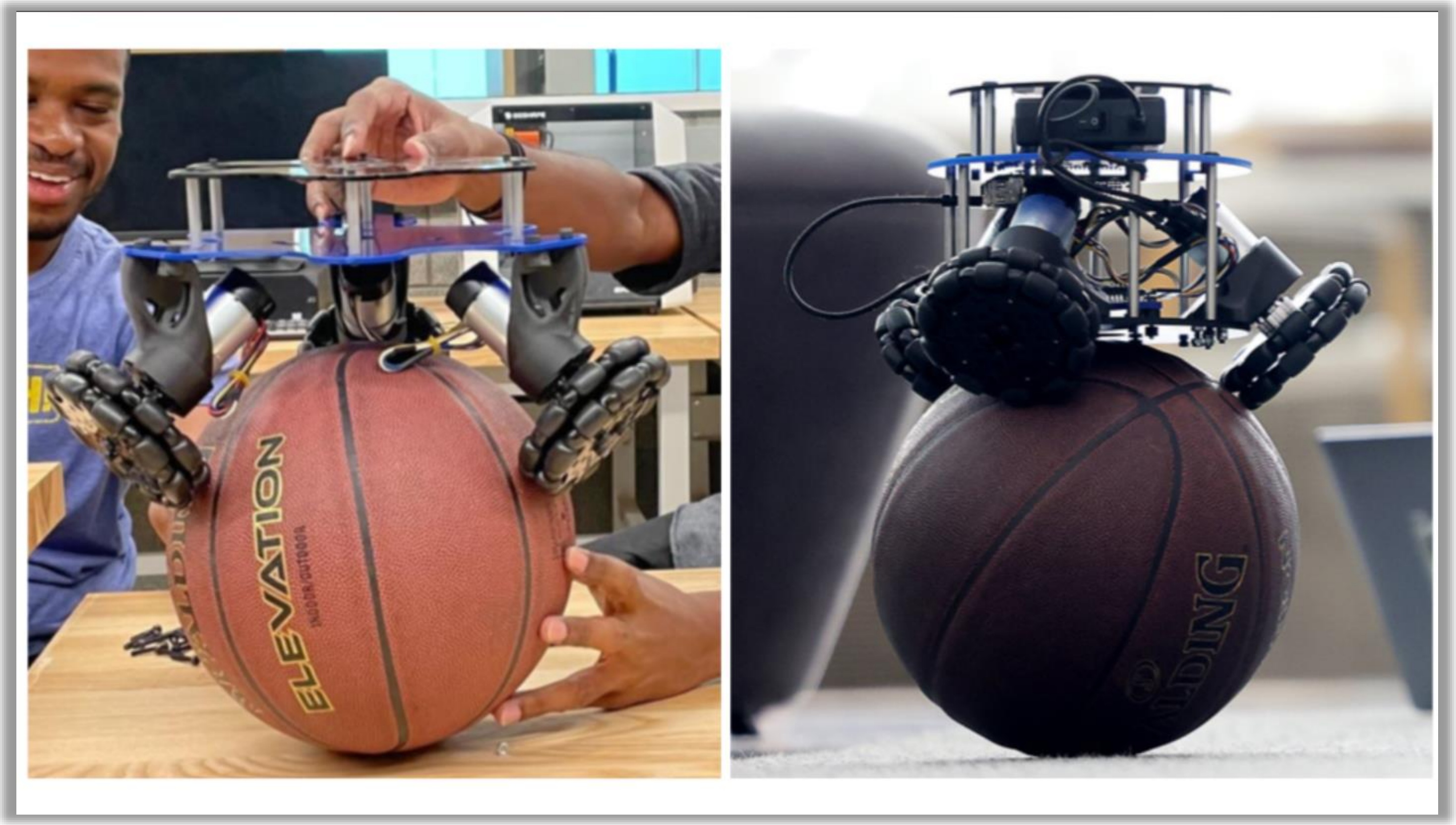




Coupler spacing is a constraint

- Now we know many of the dimensions, driven by α, ball radius, wheel radius, and coupler spacing
- To design the motor mount sketches, we need to know more about the motor dimensions
- Shaft is off center by 7 mm
- These dimensions define how your sketches were provided
- Now lets discuss motion and torque transfer

Ball-Bot Layout

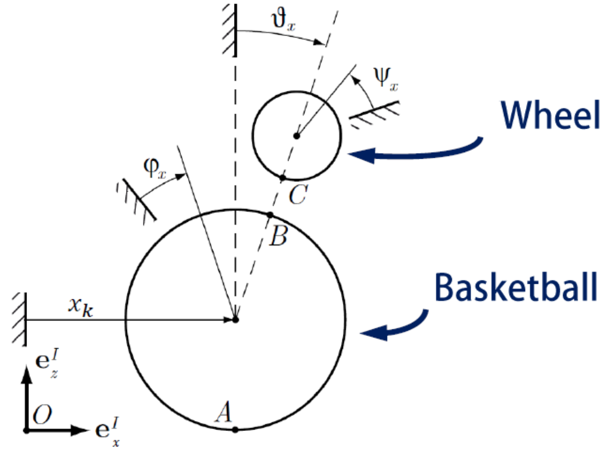


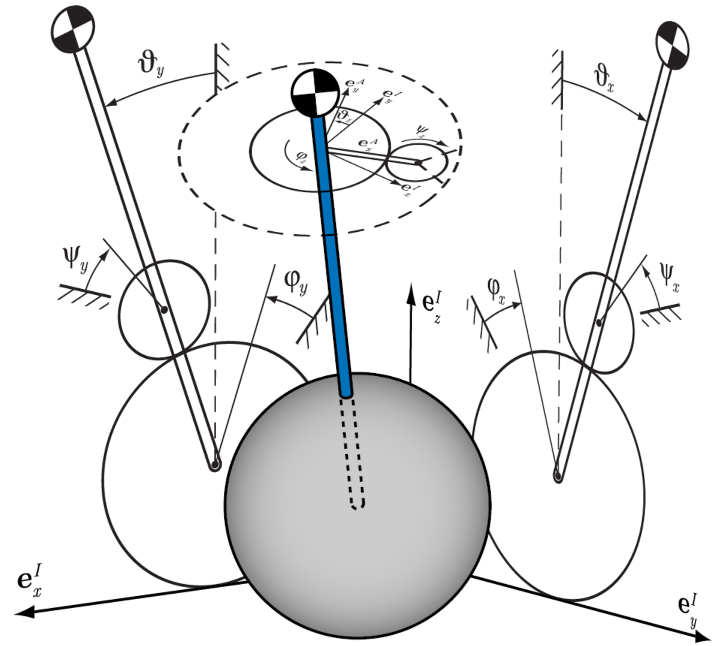
Design v1
Mounts are under
the base plate

Design v2
Mounts are above
the base plate

- What effect did this have? Lowering the center of mass and stiffer mounts

Full Planar Model

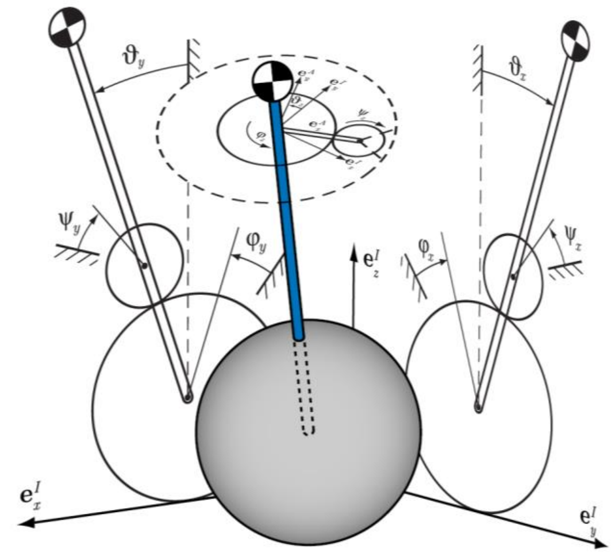




- When spec'ing, we developed a planar model of the ball-bot
- We will build on our planar model to describe the 3D motion / torque of the robot
- We project the 3D ball-bot into three planes
  - X-Z plane and Y-Z plane, and X-Y plane
- Each plane contains one virtual wheel
- This is simplification is helpful analytically

Assumptions and Rolling Physics

- No slip: The contact points at the wheel-ball and ball-ground interfaces do not slip
  - Velocity at ball-ground interface = 0
- No deformation: We will not consider deformation of the ball
- Consider a ball rolling with some velocity
- Tangential velocity from rolling is $r\dot{\theta}$

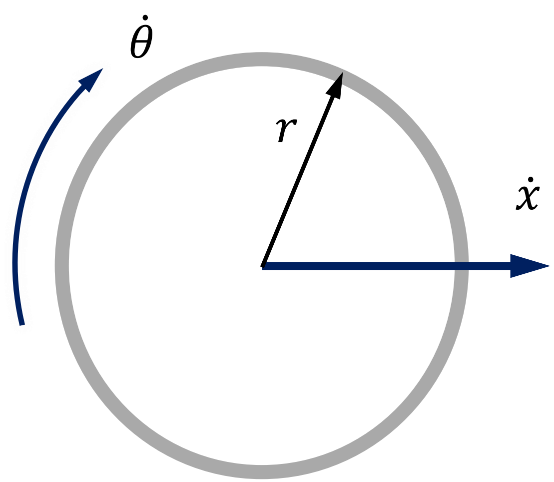


What is the x-component of velocity at the top of the ball?

$$\dot{x}_{top} = \dot{x} + r\dot{\theta}$$

How does the no-slip condition relate rolling and linear velocities?

$$r\dot{\theta} = \dot{x}$$

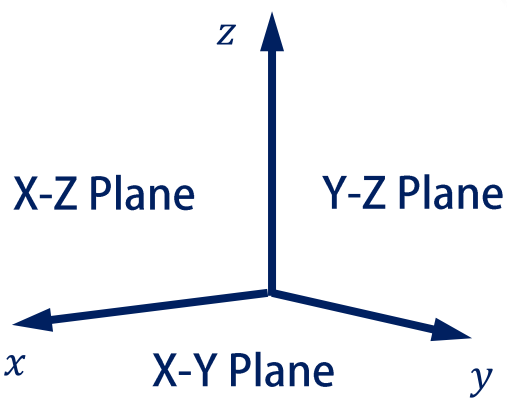


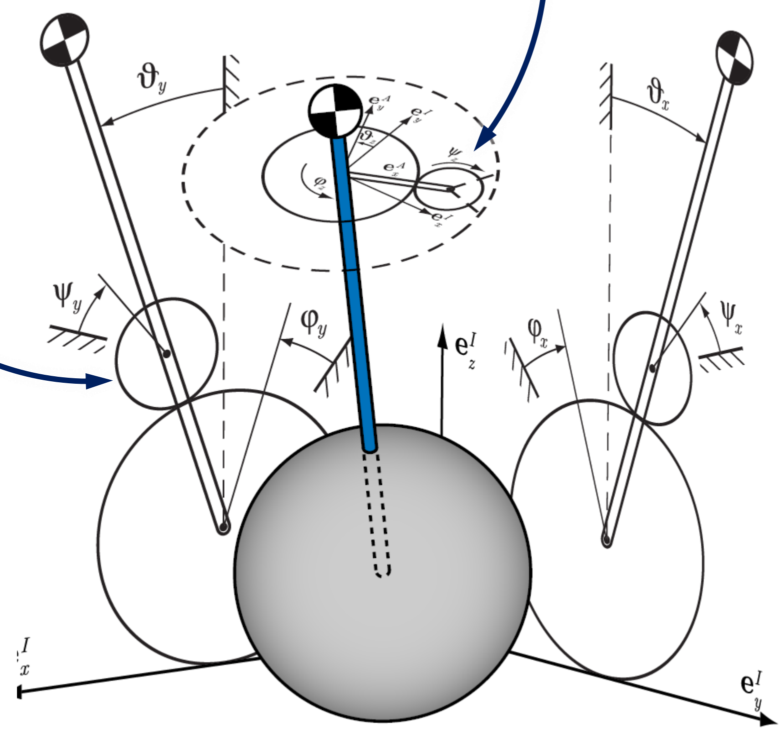
Full Planar Model

The X-Z and Y-Z planes have a single wheel beneath the center of mass

The X-Y plane contains a single wheel that rotates the ball-bot

ϑ_{axis}	Body rotation
ψ_{axis}	Wheel rotation
φ_{axis}	Ball rotation





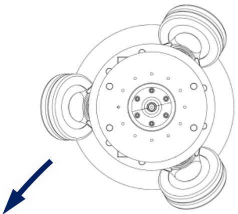
Since we want to balance and drive, the rotation of the ball (i.e. X-Y plane) may be irrelevant

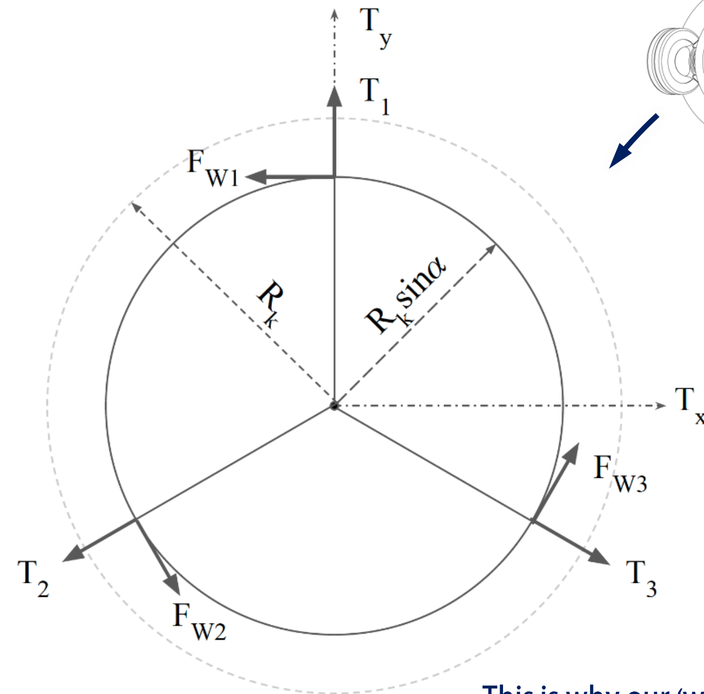
- Each plane has two DOFs: Rotations of the chassis and ball
- X-Z and Y-Z planes are required for balance and motion—we will focus on these
- The torques and motion of the virtual wheels are different from the real wheels

Torque Transfer

F_{W1}	Wheel-ball force 1	R_x	Ball radius
F_{W2}	Wheel-ball force 2	T_1	Motor 1 torque
F_{W3}	Wheel-ball force 3	T_2	Motor 2 torque
		T_3	Motor 3 torque

- How does the motor torque get transferred to each plane?
- Lets look at how the geometry affects resolving torques between coordinate systems
  - Planar torques are around the x and y axes
  - Wheel torques are T_1, T_2, and T_3
- Wheels on contact circle defined by $R_k \sin(\alpha)$
- We aligned Motor 1 torque to be along y axis
- We defined α at 45° and each torque vector spaced at 120°

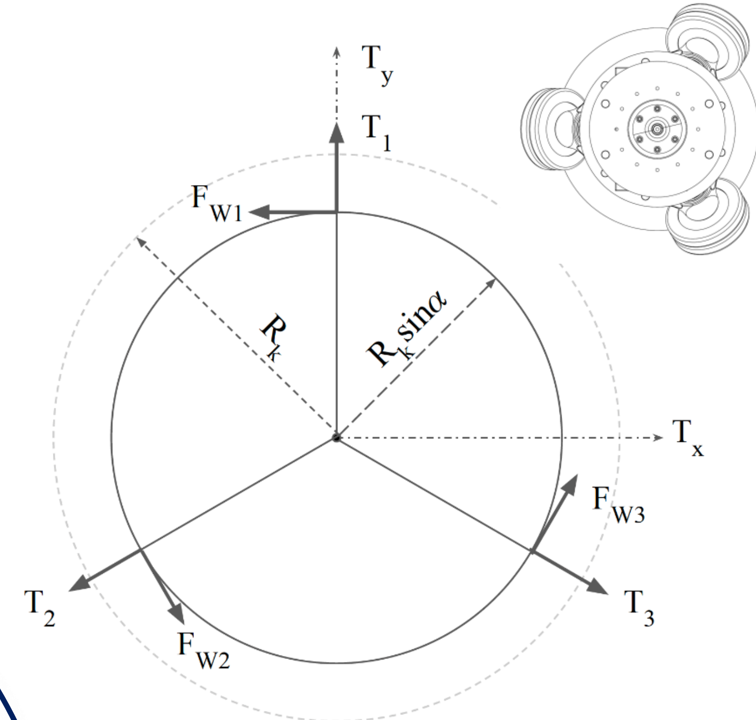




This is why our 'worst case' approximation during modeling for motor selection was helpful. We planned for this! (Lecture 3)

Will this have consequences on the motor? Yes, we're defining one of the axis torques to be fully borne by one motor, while splitting the torques from the other plane to two motors

Virtual and Real Wheel Contact



- Torque on the wheel - cross product

$$T_{Wi} = r_{Wi} \times F_{Wi} \;\; \text{for} \;\; i = 1,2,3$$

$$T_{Wj} = r_{Wj} \times F_{Wj} \;\; \text{for} \;\; j = x, y, z$$

- Virtual wheel contact points in (X-Z and Y-Z planes)

$$r_{Wx} = R_k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad r_{Wy} = R_k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad r_{kWz} = R_k \begin{bmatrix} 0 \\ \sin(\alpha) \\ 0 \end{bmatrix}$$

We begin with contact points then discuss forces

- Real wheel contact points

$$r_{W1} = R_k \begin{bmatrix} 0 \\ \sin(\alpha) \\ \cos(\alpha) \end{bmatrix} \qquad r_{W2} = R_k \begin{bmatrix} \frac{-\sqrt{3}}{2}\sin(\alpha) \\ -\frac{1}{2}\sin(\alpha) \\ \cos(\alpha) \end{bmatrix} \qquad r_{W3} = R_k \begin{bmatrix} \frac{\sqrt{3}}{2}\sin(\alpha) \\ -\frac{1}{2}\sin(\alpha) \\ \cos(\alpha) \end{bmatrix}$$

Virtual and Real Forces

- Torque on the wheel - cross product

$$T_{Wi} = r_{Wi} \times F_{Wi} \text{for } \ i = 1,2,3$$
$$T_{Wj} = r_{Wj} \times F_{Wj} \ \text{ for } \ j = x, y, z$$

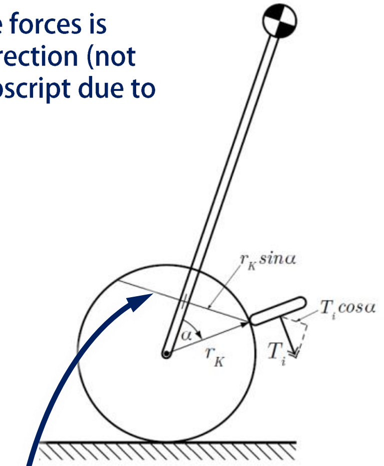
- Virtual wheel forces

$$F_{Wx} = \frac{T_x}{r_w}\begin{bmatrix}0\\1\\0\end{bmatrix} \quad F_{Wy} = \frac{T_y}{r_w}\begin{bmatrix}1\\0\\0\end{bmatrix} \quad F_{Wz} = \frac{T_z}{r_w}\begin{bmatrix}-1\\0\\0\end{bmatrix}$$

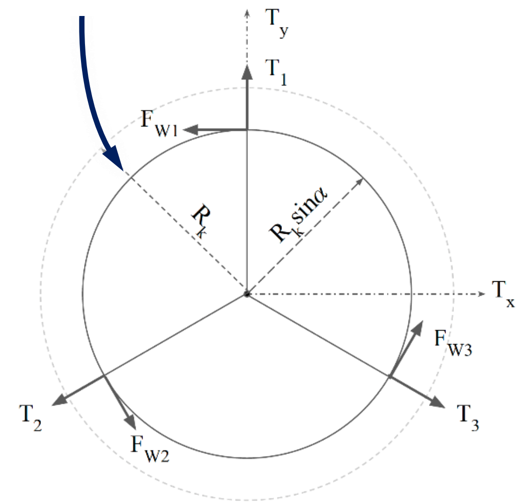
- Real wheel forces

$$F_{W1} = \frac{T_1}{r_w}\begin{bmatrix}-1\\0\\0\end{bmatrix} \quad F_{W2} = \frac{T_2}{r_w}\begin{bmatrix}1/2\\-\sqrt{3}/2\\0\end{bmatrix} \quad F_{W3} = \frac{T_3}{r_w}\begin{bmatrix}1/2\\\sqrt{3}/2\\0\end{bmatrix}$$

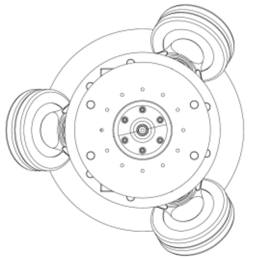
The direction of the forces is along the vector direction (not the component subscript due to the cross product)



Same circle



Relating Virtual and Real Torques



- Torque in both coordinate systems is conserved

$$T_1 + T_2 + T_3 = T_x + T_y + T_z$$

- Written in terms of force and perpendicular distance

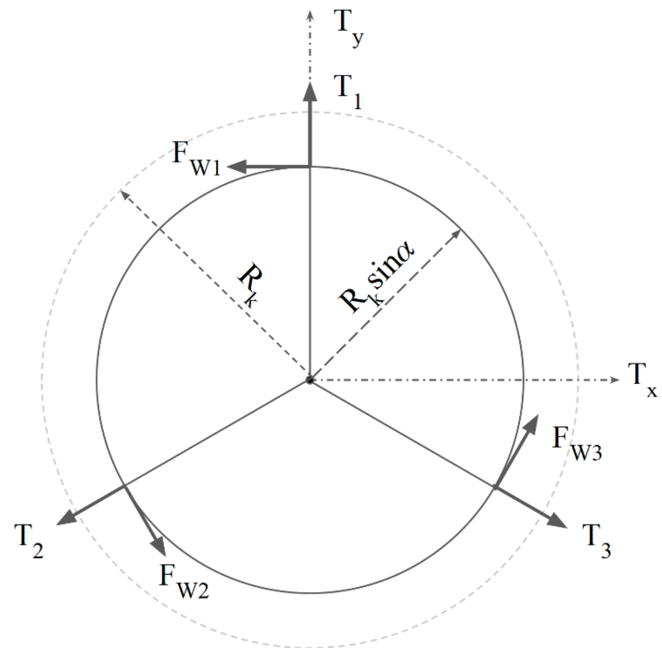
$$r_{W1} \times F_{W1} + r_{W2} \times F_{W2} + r_{W3} \times F_{W3} \ldots$$
$$= r_{Wx} \times F_{Wx} + r_{Wy} \times F_{Wy} + r_{Wz} \times F_{Wz}$$

- Solving for torques

$$T_1 = \frac{1}{3}\left(T_z - \frac{2T_y}{\cos(\alpha)}\right)$$

$$T_2 = \frac{1}{3}\left(T_z + \frac{1}{\cos(\alpha)}\left(-\sqrt{3}T_x + T_y\right)\right)$$

$$T_3 = \frac{1}{3}\left(T_z + \frac{1}{\cos(\alpha)}\left(\sqrt{3}T_x + T_y\right)\right)$$



Determined by solving conservation of torque using eqns. from previous slides

Full Planar Model

- We now can describe torque from the wheels to full planar model
- Can we develop a similar analysis for velocity transformation?
- The real and virtual wheels are the same size
- Do the real wheels and virtual wheels have the same velocity?
  - No, wheel orientation affects velocity
  - Wheels only spin with the component of linear velocity perpendicular to wheel axis
- More on this next lecture