Robotics 311 : How to build robots and make them move

Prof. Elliott Rouse

GSI Yves Nazon MS

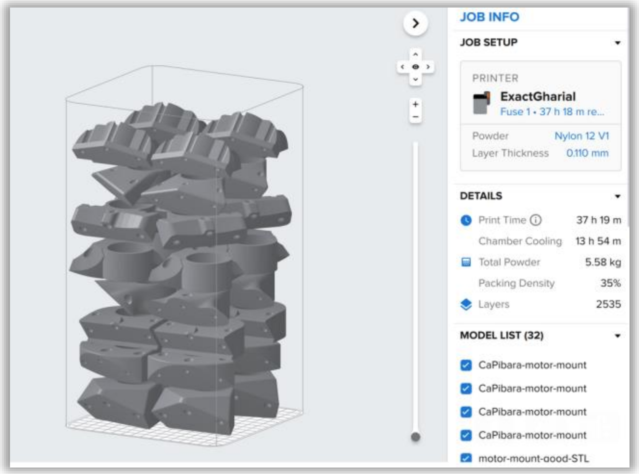
Fall 2022

Photo credit: Brenda Ahearn / U-M CoE

ROB 311 – Lecture 11

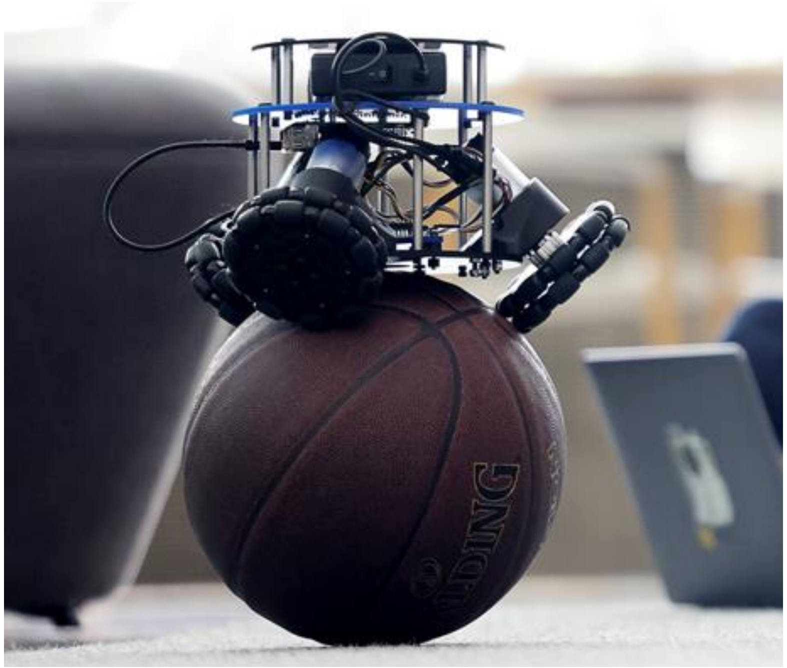
- Today:
  - Gears
  - Screws
  - Linkages

- Announcements
  - HW 2 due now
  - HW 3 posted later today
  - Your nylon SLS parts will be ready tomorrow



ROB 311 – Lab 6 tomorrow

- Goal: assemble and wire your ball-bot
- We will provide information on wiring (connectors only)
- Prof. Jenkins and CoE photography may stop by
- Tomorrow at the end of lab, your ball-bot should look something like this:



Manufacturing Types

'Rapid prototyping'

Spec'ing analysis completed →

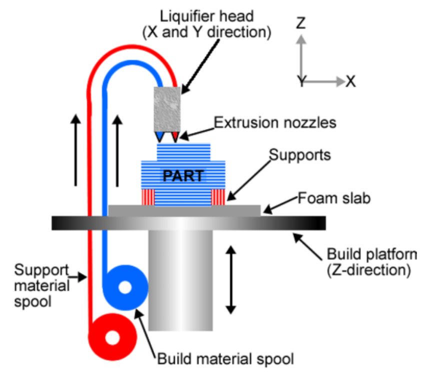
Additive Mfg.
3D
Plastics
Convenient

Laser Cutter
2D
Acrylic, wood,
Convenient

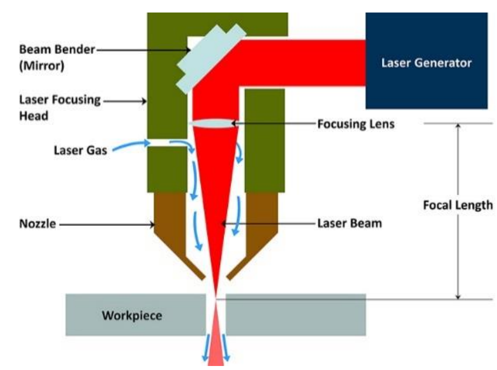
Water Jet Cutter
2D
Anything
Less convenient

→ Layout next...

- We've learned how to spec our robot to complete some desired function
- We're learning rapid prototyping methods and design
- For the sake of lab, we began with manufacturing
- We have already covered
  - 3D printing
  - Laser cutting
  - Most of water jetting

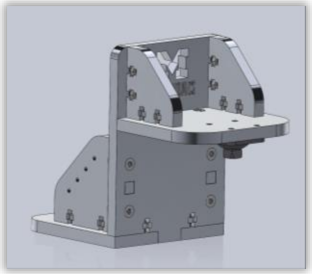


3D printer anatomy

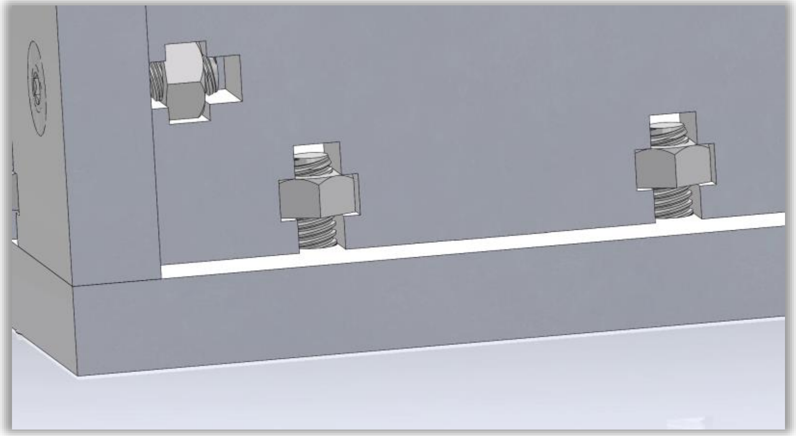


Laser cutter anatomy

Best Practices – Fasteners and Tabbing Text



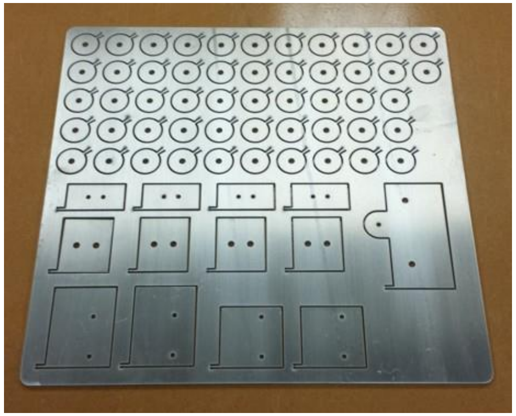
- Similar to a laser cutter, nuts can be easily captured using a hex or square nut
- Nut dimensions can be found from McMaster-Carr
- Extra space is needed to ensure fit
    - Oversize slot for nuts by 0.75+ mm to ensure fit
    - Easy way to make assembly / disassembly convenient

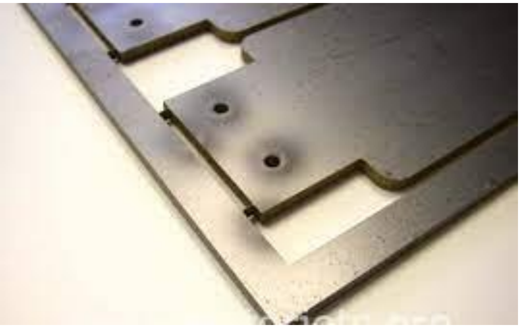


- You also may wish to add small, filled features
- For example, the inside of an ‘o ‘ or an ’A’
- This can be done by adding ~1 mm tabs
- Tab width depends on the quality of the water jet
- Demo!

Best Practices – Tabbing

- Once a part is cut from the material, it falls into the water bath
- Sometimes this is inconvenient
- Tabbing is a solution that enables the parts to stay connected to the work piece
- A tab is a small piece of material that connects your part to the work piece
  - A bridge from the main work piece to your part
- Tabbing can be added manually in ProtoMax LAYOUT or in Solidworks / your .DXF file
- This is good practice if you are cutting many small parts

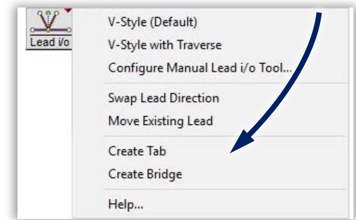


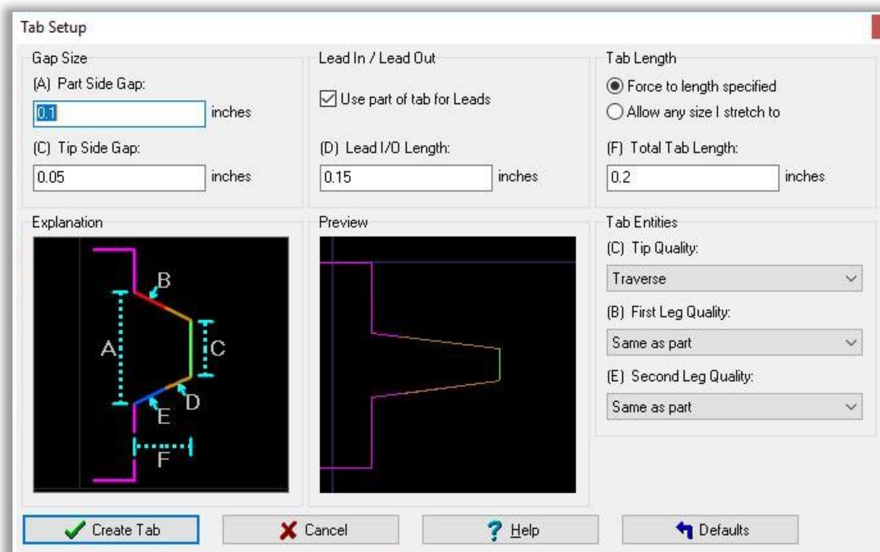


Best Practices – Tabbing

- To create a tab, open your part in ProtoMax LAYOUT
- First, right click on the Lead I/O button and select Create Tab
- Then click where you want the tab
- Once Create Tab is selected, options will appear to setup the tab
- Setup parameters include:
    - Part side gap (A)
    - Tip side gap (C)
    - Lead I/O (D)
    - Total tab length (F)
- This process can be repeated to add multiple tabs
- ~1 – 3 mm size tabs

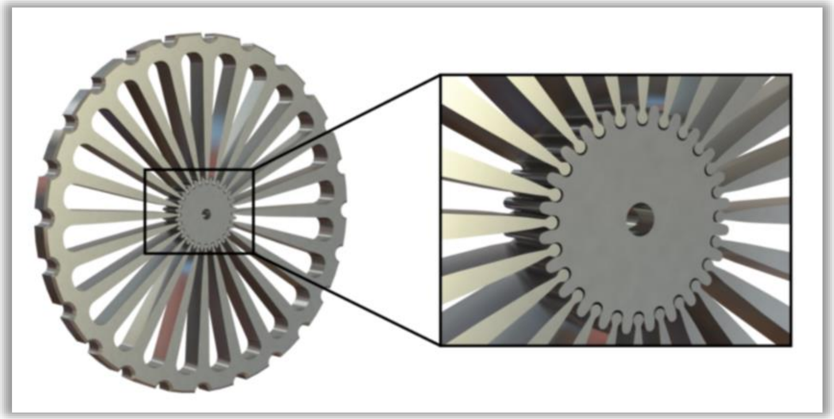
Click here





Best Practices – Flexures

- Water jet cutters can be used to design flexible elements, like laser cutters
- Flexures from metal are more like springs
- Spring design is outside the scope of this course—but interesting!
- Research example
  - Developed the equations to predict the mechanics
  - Fit inside output pulley
  - Cut with Wire-EDM → water jet?



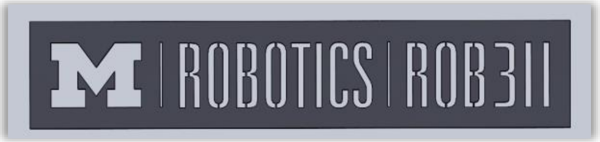
Example torsion springs that could be cut with a water jet

$$\theta_{des} = \sqrt[3]{\frac{8tnL^3\sigma_d^3}{27E^2kr}}$$

t – spring thickness (m)
n – number of flexures
L – radial length of flexure (m)
σ_d – design stress (Pa)
E – elastic modulus (Pa)
k – spring stiffness (Nm/rad)
r – contact radius

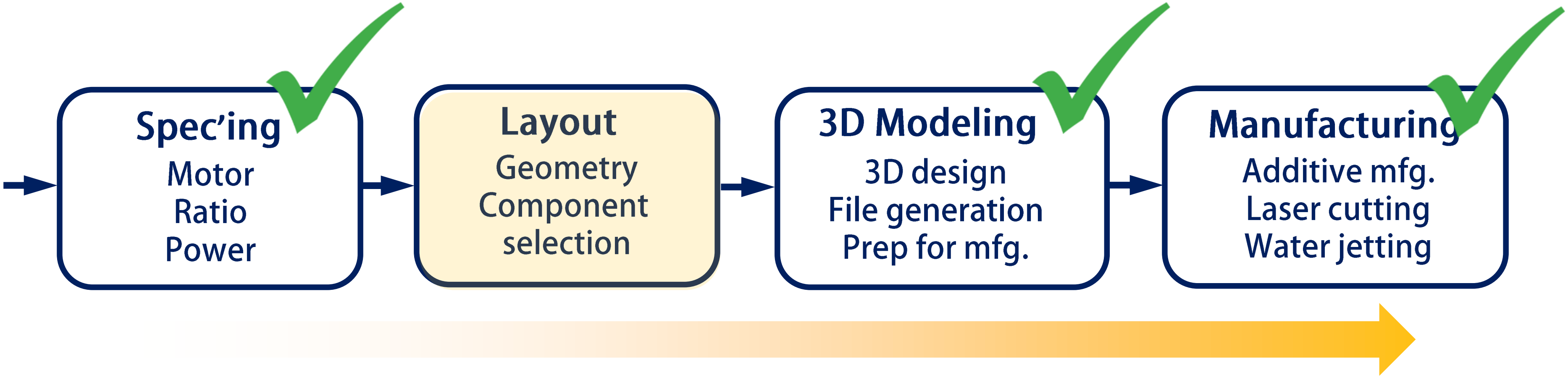
Manufacturing – in-class example

- We're basically finished with manufacturing!
- I'd like to walk through an example together that combines many tools we learned
- This will serve as the quiz for this week
- Goal: Create extruded boss of M | Robotics and ROB 311
- In addition, you can use these in your ball-bot design



- Steps:
  - Export M | Robotics logo as .DXF from AI
  - Create a 3/16" plate in Solidworks as an extruded boss
  - Import the .DXF in Solidworks
  - Modify the sketch to include ROB 311 (by using an pasted/edited B and I)
  - Add tabs to hold inside letters in place
  - Scale the sketch and extrude cut the profile through the plate

Manufacturing Types



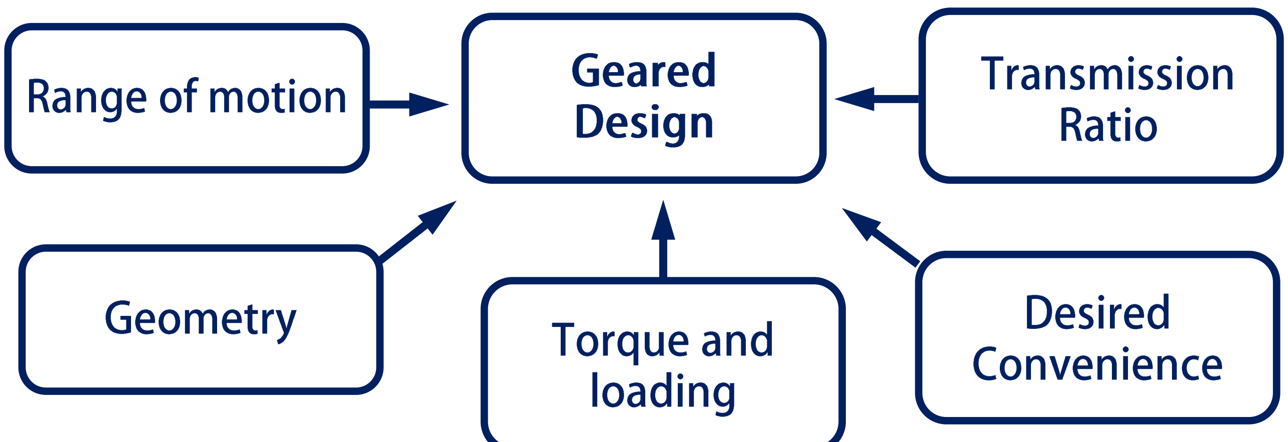
- We've learned how to spec and make robots, now lets talk about design layouts
- This is often moving motion from one place to another (kinematics)
- In robots, motion moving from the actuator to the end effector
- It begins with understanding the geometry of your robot and transmissions
- Very application specific!
- Coming up:
  - Introduce transmissions and linkages
  - In-depth example of ball-bot geometry and kinematics
  - Move to mechatronics, ball-bot dynamics, and control

Understanding Motion

- To determine layout, first we need to understand the required motion in our robot design
- Motion can be rotational (more common) or linear (more difficult)
- At this point, you know your desired motor and transmission ratio
- We need to package this ratio in the proper form factor for your robot
- Many types of transmissions
  - Gearing (spur, bevel, worm, etc.)
  - Belt drives
  - Screws
  - Linkages
- This lecture will step through these transmission types
- Often there are geometric constraints in addition to ratio constraints
  - Does motion need to be somewhere specific?
  - For the ball-bot, this is very important

Designing Geared Transmissions

- First, obtain any specific geometric information related to your application
- For the ball-bot
  - Locations of wheels
  - Deep dive in kinematics next lecture
- Required information before beginning design
- The more you know about your application, the easier design will be
- Many types of geared transmissions



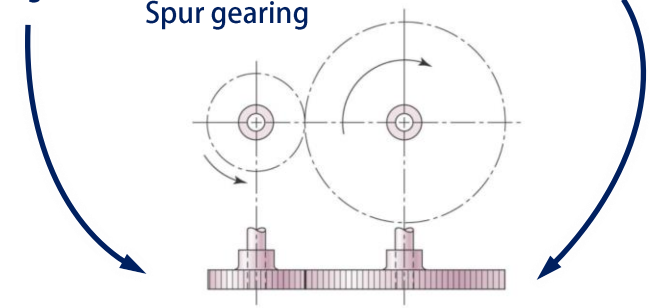
Gearing

- Backdrivable and non-backdrivable: whether negative power can be transmitted through the transmission.
  - Efficiency
  - Pressure angle / friction cone
  - Implications?   No energy regeneration
- **Spur gears** – teeth parallel to the axis of rotation, transmitting motion from one shaft to another (parallel) shaft
- **Bevel gears** – teeth on conical surfaces, which transmit motion between two perpendicular shafts
- **Worms and worm gears** – high ratio gearing that transmits motion between two perpendicular / offset shafts

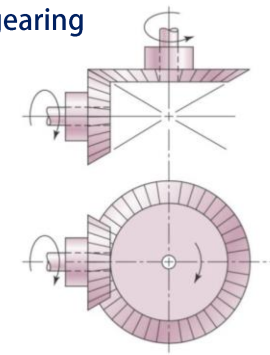
Input / motor side (small gear)

Output / robot side (large gear)

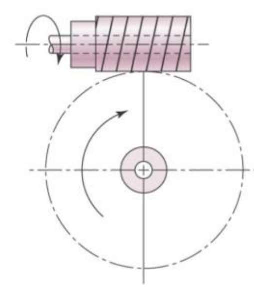
Spur gearing



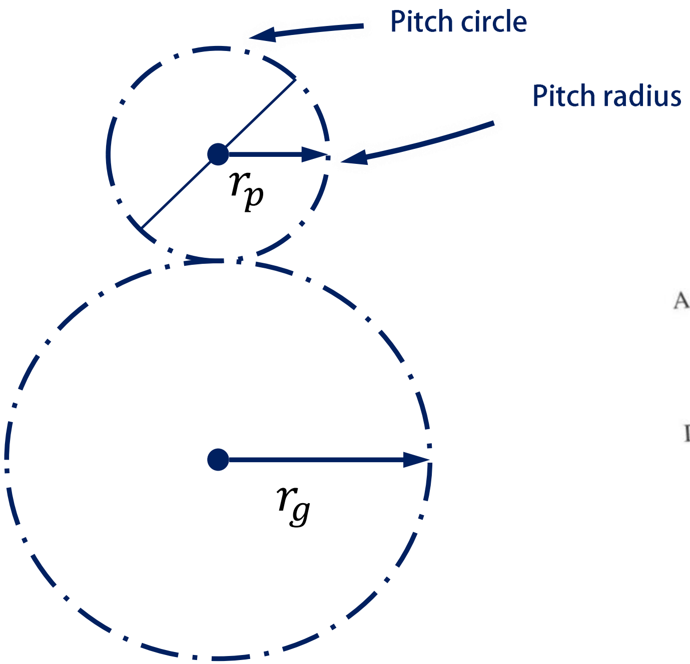
Bevel gearing



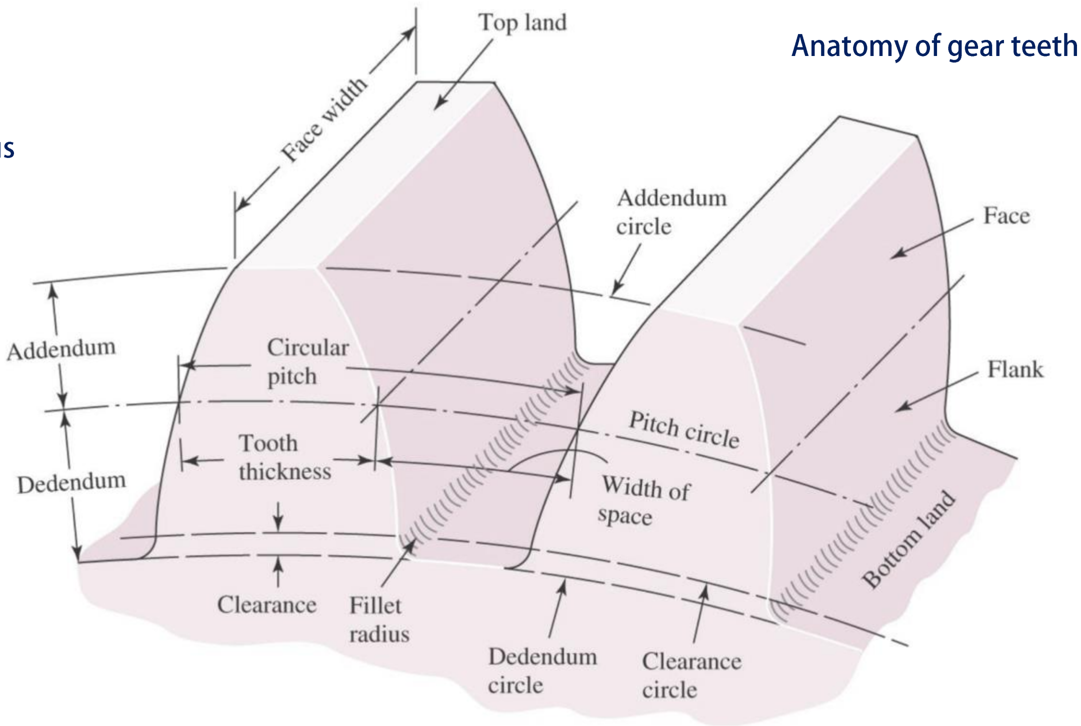
Worm gearing



Gearing

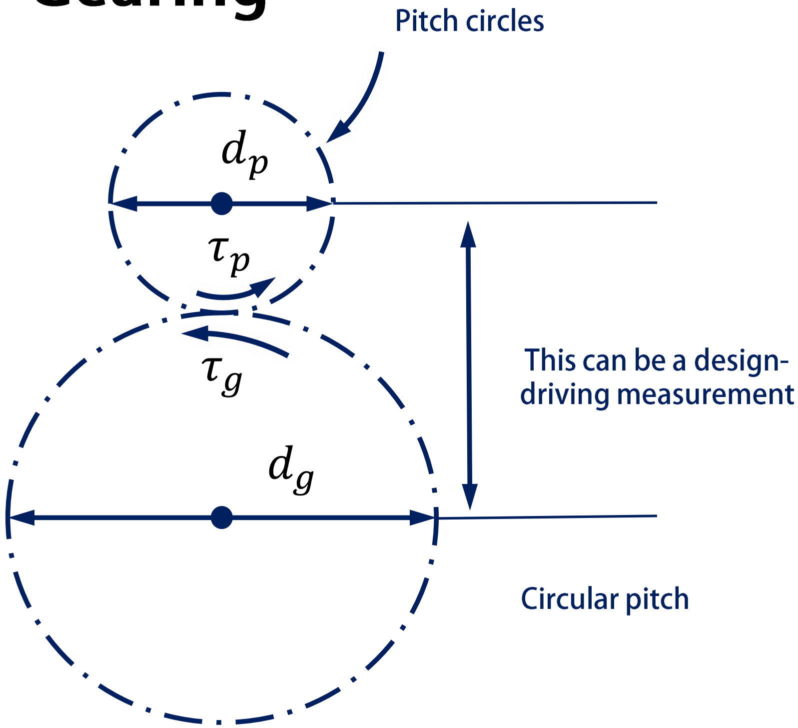


Anatomy of gear teeth



- Pitch diameter, p – theoretical diameter / circle upon which all calculations are based
- Diametral pitch, P – the ratio of the number of teeth to the pitch diameter in units of teeth/m
- Backlash – amount of angular play in transmission (tooth space > tooth width)
- The small gear is often known as the *pinion* and the larger is the *gear*

Gearing



Diametral pitch

Circular pitch

$$P = \frac{n}{d}$$

$$p = \frac{\pi d}{n}$$

Pertains to individual gears

$$N = \frac{d_p}{d_g} = \frac{n_p}{n_g} = \left|\frac{\omega_g}{\omega_p}\right| = \frac{\tau_g}{\eta \tau_p}$$

Pertains to gearsets

- Pinion teeth, n_p – number of teeth on pinion
- Gear teeth, n_g – number of teeth on gear
- Transmission ratio, N – ratio of input speed to output (also diameters, torque, ...)
- Conjugate action – defines that the ratio of velocity is inversely proportional to the pitch radii

Gearing

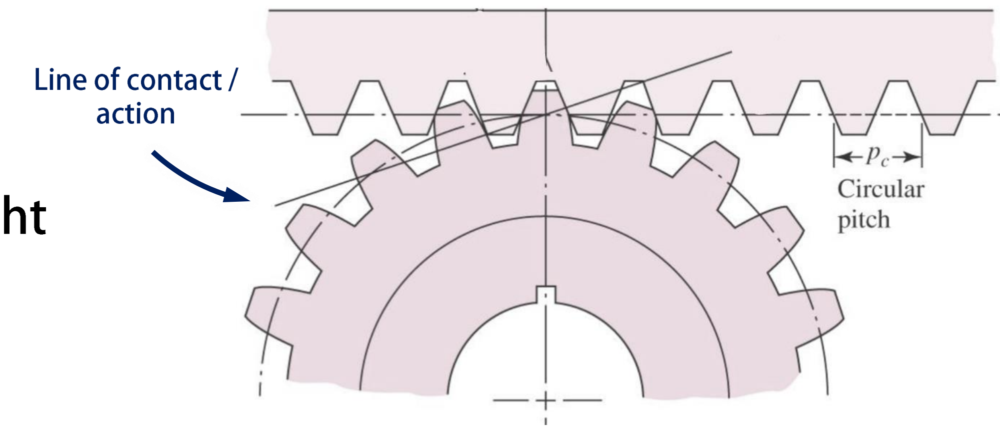
- Often, you can buy a motor with a gearhead, known as a gearmotor
- Multiple ratios available for a given motor
- This can make adding the required ratio more convenient
- Gearing components already selected

Rated Voltage	Stall Current	No-Load Current	Gear Ratio	No-Load Speed (RPM)	Extrapolated Stall Torque		Max Power (W)	 Without Encoder	 With Encoder
					(kg · cm)	(oz · in)			
12 V	5.5 A	0.2 A	1:1 (no gearbox)	10,000	0.5	7	–	–	item #4750
			6.3:1	1600	3.0	42	12	item #4747	item #4757
			10:1	1000	4.9	68	12	item #4748	item #4758
			19:1	530	8.5	120	12	item #4741	item #4751
			30:1	330	14	190	12	item #4742	item #4752
			50:1	200	21	290	10	item #4743	item #4753
			70:1	150	27	380	10*	item #4744	item #4754
			100:1	100	34	470	8*	item #4745	item #4755
			131:1	76	45	630	6*	item #4746	item #4756
			150:1	67	49	680	6*	item #2829	item #2828

Pololu 37D gearmotor selection

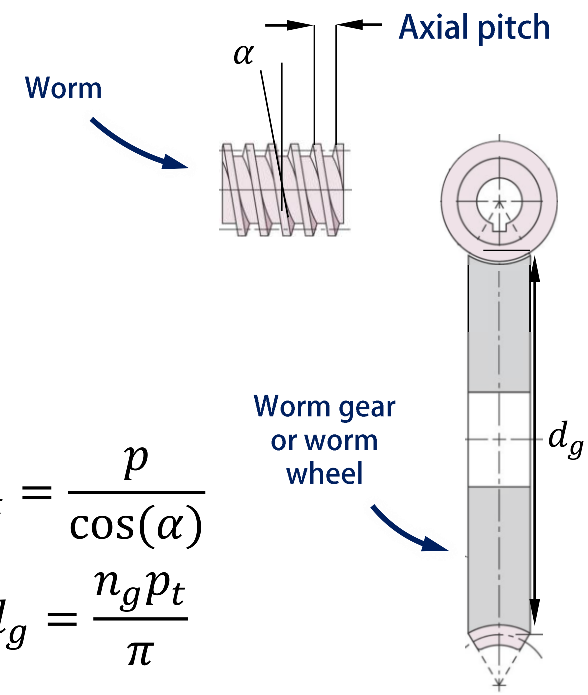
Gearing

- Rack and pinion – spur gear transmission with an infinite pitch diameter for the gear
- Gears with infinite diameter (straight gears) are known as *racks*
- Can be plastics, brass, or steels



Worm Gears

- Worm gears are used for extremely high ratio transmissions
- Specified by axial pitch, gear diameter, and transverse circular pitch (p_t)
- Backlash less noticeable (high ratio)
- Rotate angular motion 90°
- Can be plastics, brass, or steels

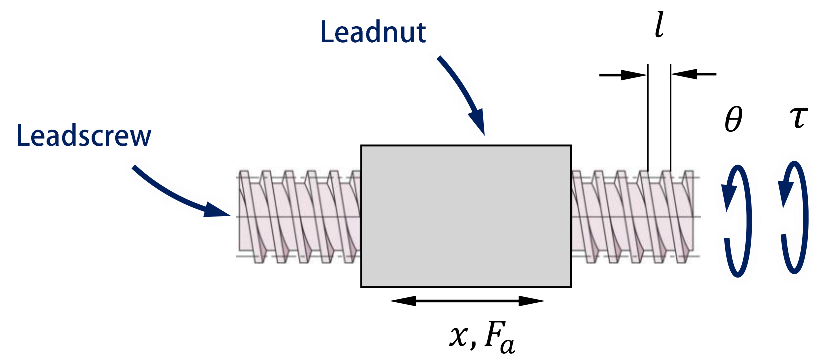


$$p_t = \frac{p}{\cos(\alpha)}$$

$$d_g = \frac{n_g p_t}{\pi}$$

Screws

- Screws turn rotary motion into linear motion
- Useful in a wide array of robotics applications
- Lead screws are low cost and useful
- Can be purchased as a set with specified dimensions
- More information required to know full transmission ratio
- Ball screws can be used for highly efficient motion (expensive)



$$\tau = \frac{F_a l}{2\pi\eta}$$

η	Efficiency
F_a	Thrust force
l	Screw lead
τ	Driving torque

$$\dot{x} = \frac{l\dot{\theta}}{2\pi}$$

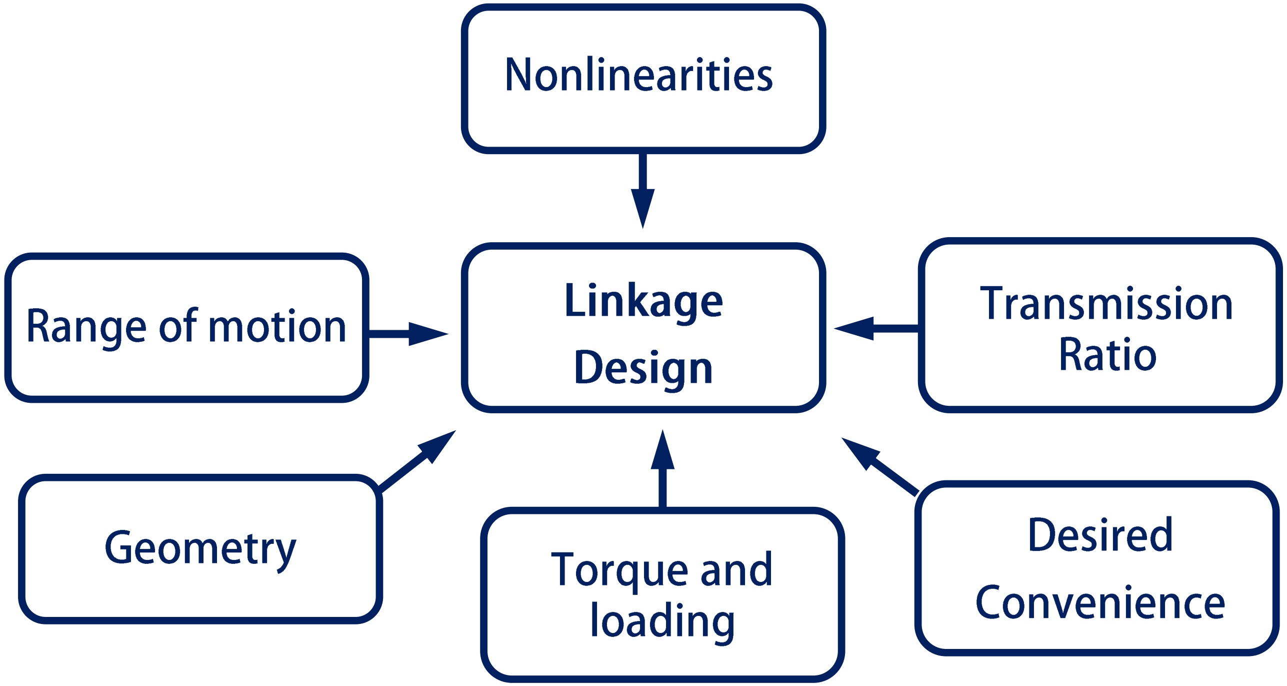
$\dot{x}$	Nut linear velocity
$\dot{\theta}$	Shaft angular velocity

leadscrew

ballscrew

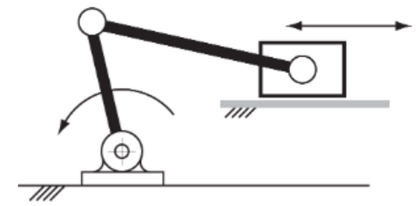
Linkages

- Linkages are commonly used in robotic systems
- They have many uses (rotary to rotary, rotary to linear)
- They have nonlinear transmission / velocity ratios
- Similarly, geometric information about your application is critical

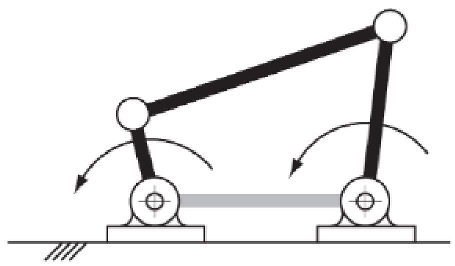


Linkages

- Example linkage types (more [here]())
- First step is determining the input and output links
- Determine transmission ratio and kinematics as a function of starting configuration and link lengths
- Kinematics / transmission ratio determined using geometry
- L_3 is input, L_1 is output

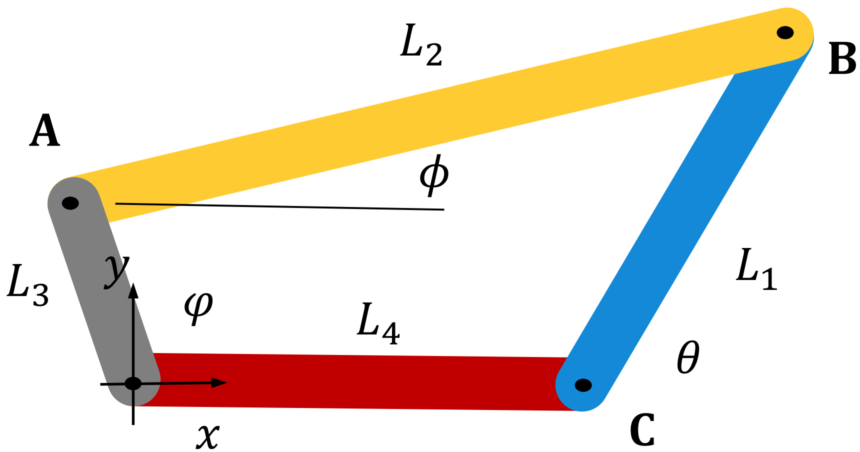


Slider crank



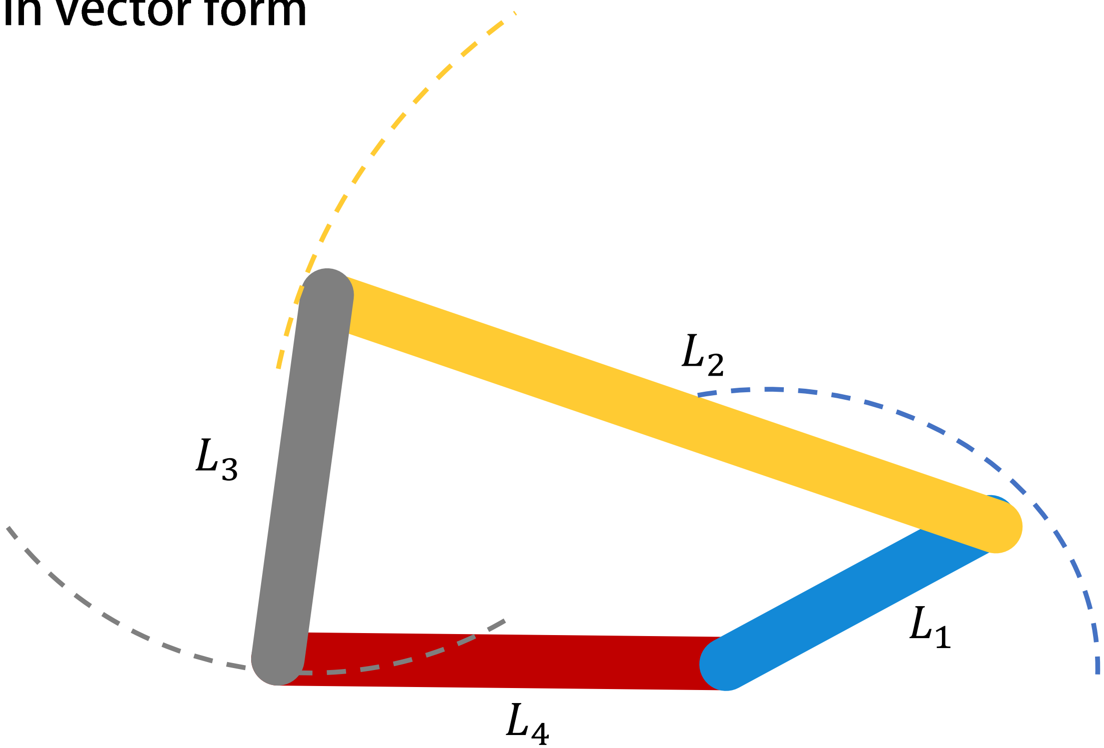
Parallelogram 4-bar

Many more…



Linkages

- Solve the geometric equations indicating all points must be on the circular arcs of their respective base links
- Analysis known as kinematic synthesis
- Quantify how input velocity scales output velocity
- Lets look in vector form



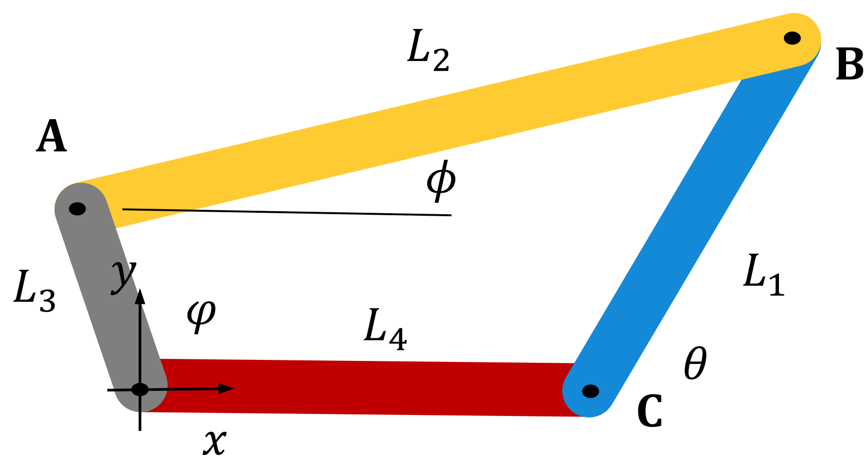
Linkages

- A series of constraints and geometry are needed to design a four-bar mechanism
- The next few slides introduce these constraints

$$\mathbf{A} = \begin{bmatrix} L_3\cos(\varphi) \\ L_3\sin(\varphi) \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} L_4 + L_1\cos(\theta) \\ L_1\sin(\theta) \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} L_4 \\ 0 \end{bmatrix} \quad (\mathbf{B}-\mathbf{A})\cdot(\mathbf{B}-\mathbf{A}) - {L_2}^2 = 0$$

Geometric constraint equation

$$\left(2L_1L_4 - 2L_3L_1\cos(\varphi)\right)\cos(\theta) - \left(2L_3L_1\sin(\varphi)\right)\sin(\theta) + \ldots$$
$$\ldots + \left({L_1}^2 + {L_3}^2 + {L_4}^2 - {L_2}^2 - 2L_3L_4\cos(\varphi)\right) = 0$$



L_3 Input
L_1 Output
L_4 Ground

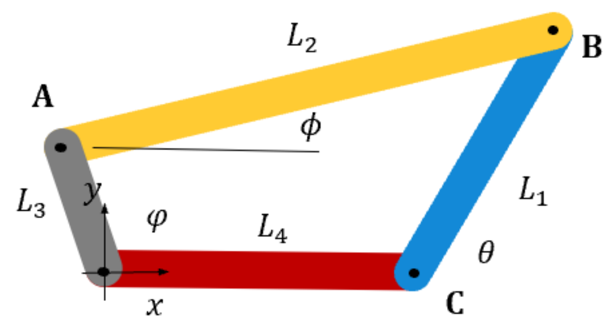
Linkages

Position constraint equations

$$\mathbf{A} + (\mathbf{B} - \mathbf{A}) = \mathbf{C} + (\mathbf{B} - \mathbf{C})$$

$$\begin{bmatrix} L_3\cos(\varphi) \\ L_3\sin(\varphi) \end{bmatrix} + \begin{bmatrix} L_2\cos(\phi) \\ L_2\sin(\phi) \end{bmatrix} = \begin{bmatrix} L_4 \\ 0 \end{bmatrix} + \begin{bmatrix} L_1\cos(\theta) \\ L_1\sin(\theta) \end{bmatrix}$$

$$\phi = \operatorname{atan}\left(\frac{L_1\sin(\theta) - L_3\sin(\varphi)}{L_4 + L_1\cos(\theta) - L_3\cos(\varphi)} \right)$$



Velocity constraint equations

$$\dot{\mathbf{A}} + \frac{d}{dt}(\mathbf{B} - \mathbf{A}) = \frac{d}{dt}(\mathbf{B} - \mathbf{C})$$

$$\begin{bmatrix} -L_3\sin(\varphi) \\ L_3\cos(\varphi) \end{bmatrix} \dot{\varphi} + \begin{bmatrix} -L_2\sin(\phi) \\ L_2\cos(\phi) \end{bmatrix} \dot{\phi} = \begin{bmatrix} -L_1\sin(\theta) \\ L_1\cos(\theta) \end{bmatrix} \dot{\theta}$$

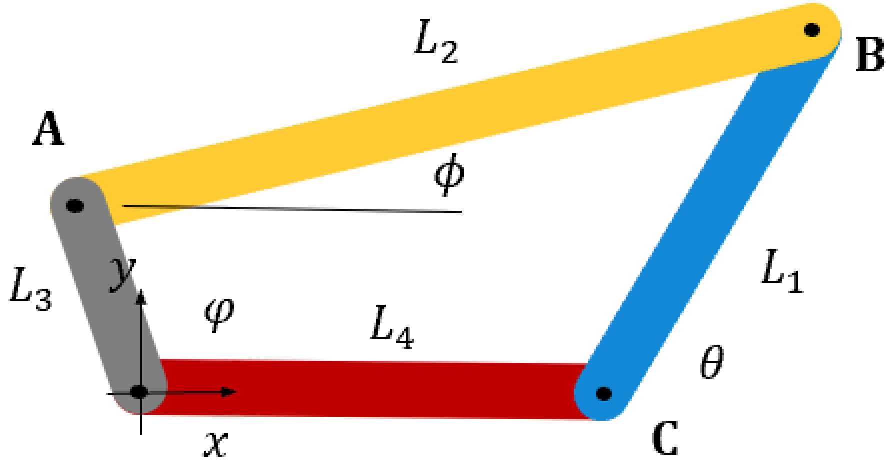
Linkages

- Describing the transmission ratio of a four-bar linkage

$$N = \frac{\dot{\varphi}}{\dot{\theta}} = \frac{-L_3 L_1 \cos(\theta)\sin(\varphi) - L_1 L_4 \sin(\theta) + L_3 L_1 \sin(\theta)\cos(\varphi)}{L_3 L_4 \sin(\varphi) + L_3 L_1 \cos(\theta)\sin(\varphi) - L_3 L_1 \sin(\theta)\cos(\varphi)}$$

More on this in 2 slides

Kinematically varying transmission ratio



L_3 Input

L_1 Output

L_4 Ground

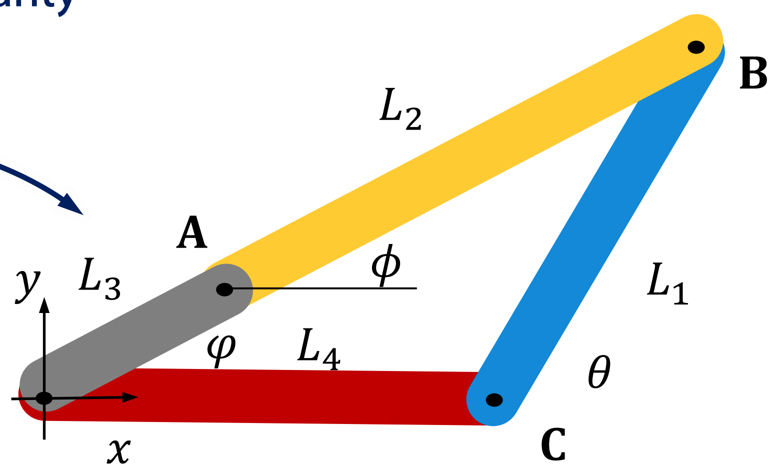
Singularities

- Singularities can occur when two links are co-linear
- Causes the loss of a degree of freedom in the linkage
- The applied torque from L_3 cannot apply torque to L_1
- The slope of the input-output kinematics / ratio goes to infinity
- Avoid singularities by 30° or more

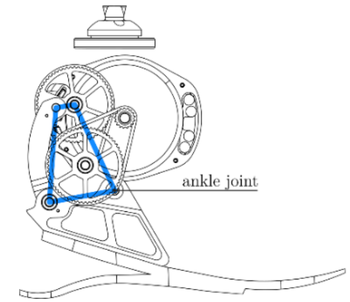
$$N = \frac{\dot{\varphi}}{\dot{\theta}}$$

Changes sign at the singularity

Singularity configuration



Linkages



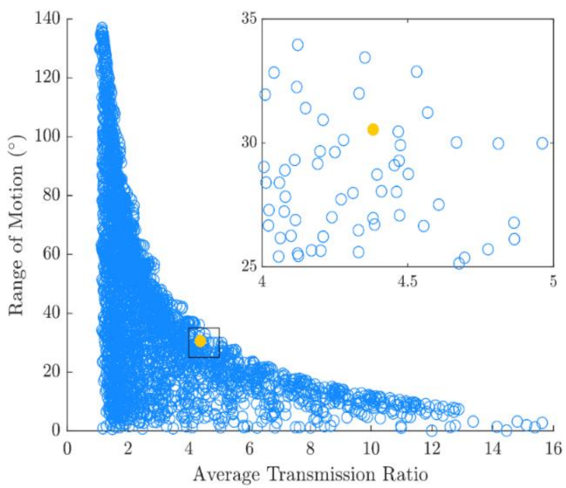
Open-Source Leg v1
www.opensourceleg.com

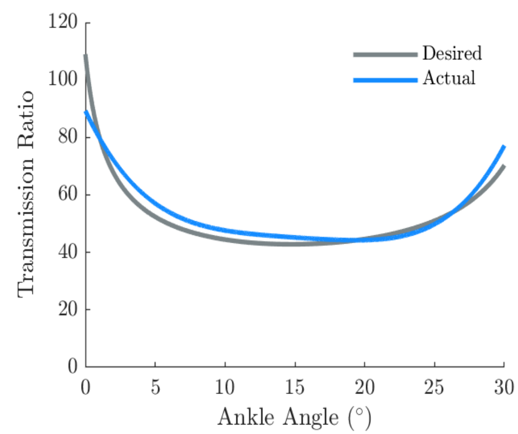
- There are many combinations that are viable
- They will have different average transmission ratios and range of motion
- Can we determine the transmission ratio empirically? Yes!

$$N = \frac{\dot{\varphi}}{\dot{\theta}} = \frac{d\varphi/_{dt}}{d\theta/_{dt}} = \frac{d\varphi}{d\theta}$$

Transmission ratio is also the slope of the output-input kinematics

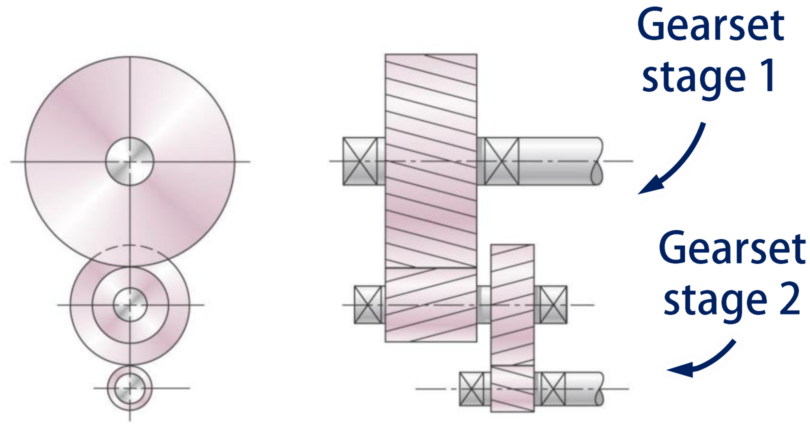
- Research example: singularities, multiple solutions and link lengths





Compound Transmissions

- Sometimes, larger ratios are needed
- This can be accomplished by stacking transmissions
- Known as *compound transmissions*
- Shown as gears, but could be any type
- Ratios are multiplied
- Efficiencies are multiplied
- Extends to an arbitrary number of stages



Gearset 1: N_1, η_1 → **Gearset 2**: N_2, η_2

Compound transmission

$$N_{total} = N_1 \cdot N_2$$
$$\eta_{total} = \eta_1 \cdot \eta_2$$