ECE 661 – Homework 1

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Problem 1 - Solution

The origin in the physical space $\mathcal{R}^2$ is represented as

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Thus the HC in representational space $\mathcal{R}^3$ is

$$\begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}$$

for all $\mathrm{w} \in \mathcal{R}, \mathrm{w} \neq 0$

Problem 2 – Solution

No. Their HC in representational space is

$$\begin{pmatrix} u \\ v \\ 0 \end{pmatrix}$$

where u, v can be any real numbers. Thus all points at infinity are not the same.

Problem 3 – Solution

The matrix of a degenerate conic can be represented as

$$\mathrm{C} = \mathrm{l} \cdot \mathrm{m}^{\mathrm{T}} + \mathrm{m} \cdot \mathrm{l}^{\mathrm{T}}$$

where l and m are the two lines. Thus both $\mathrm{l} \cdot \mathrm{m}^{\mathrm{T}}$ and $\mathrm{m} \cdot \mathrm{l}^{\mathrm{T}}$ are rank 1 matrices because their columns are multiple of l and m. Thus we have

$$\mathrm{rank}(\mathrm{C}) \leq \mathrm{rank}\left(\mathrm{l} \cdot \mathrm{m}^{\mathrm{T}}\right) + rank\left(\mathrm{m} \cdot \mathrm{l}^{\mathrm{T}}\right) = 2$$

Problem 4 – Solution

(1)

$$l_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix}$$

$$l_2 = \begin{pmatrix} -6 \\ 8 \\ 1 \end{pmatrix} \times \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 12 \end{pmatrix}$$

$$\text{intersection } x_1 = l_1 \times l_2 = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 6 \\ 3 \\ 12 \end{pmatrix} = \begin{pmatrix} 24 \\ 72 \\ -30 \end{pmatrix} = -30 \cdot \begin{pmatrix} -0.8 \\ -2.4 \\ 1 \end{pmatrix}$$

Thus the intersection of l_1 and l_2 is (-0.8, -2.4).

(2)

$$l_2{}' = \begin{pmatrix} -10 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 20 \\ 0 \end{pmatrix}$$

$$\text{intersection } x_2 = l_1 \times l_2{}' = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} -6 \\ 20 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -108 \end{pmatrix} = -108 \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Thus the intersection of l_1 and $l_2{}'$ is (0, 0).

Problem 5 – Solution

$$l_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$l_2 = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 9 \end{pmatrix} = 3 \cdot \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$$\text{intersection } x_1 = l_1 \times l_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix}$$

Thus the two lines do not intersect and there is no intersection.

Problem 6 – Solution

The conic is $(x-5)^2 + (y-5)^2 = 1$ or represented as $x^2 + 0xy + y^2 - 10x - 10y + 49 = 0$

Thus the matrix of the conic in representational space is

$$\mathrm{C} = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ -5 & -5 & 49 \end{bmatrix}$$

The polar line is

$$l = Cx = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -5 \\ -5 & -5 & 49 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \\ 49 \end{pmatrix}$$

The x axes is

$$l_x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

The y axes is

$$l_y = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

The polar line intersects x axes at

$$x_0 = \begin{pmatrix} -5 \\ -5 \\ 49 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -49 \\ 0 \\ -5 \end{pmatrix} = -5 \cdot \begin{pmatrix} 9.8 \\ 0 \\ 1 \end{pmatrix}$$

The polar line intersects y axes at

$$y_0 = \begin{pmatrix} -5 \\ -5 \\ 49 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -49 \\ -5 \end{pmatrix} = -5 \cdot \begin{pmatrix} 0 \\ 9.8 \\ 1 \end{pmatrix}$$

Thus the polar line intersects x and y axes at (9.8, 0) and (0, 9.8).

Problem 7 – Solution

$l_1: 1x + 0y - 1 = 0$, so $l_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

$l_2: 0x + 1y - 1 = 0$, so $l_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$.

The intersection

$$x = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Thus the intersection of two lines is (1, 1).