COMPARISON OF BRUSHLESS DC MOTORS WITH TRAPEZOIDAL AND SINUSOIDAL BACK-EMF

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ABSTRACT

For analysis of electromechanical systems with brushless DC (BLDC) motors and power electronic drives it is essential to have a sufficiently accurate model of the machine. This paper compares the BLDC motors with sinusoidal and trapezoidal back-EMF waveforms and presents an improved detailed model for the 120^o BLDC motor-inverter system with trapezoidal back-EMF. The established model is shown to be more accurate in comparison to the typical models that assume sinusoidal back-EMF. The presented studies are based on a typical industrial BLDC and include measurements and simulations using various detailed models developed in two different simulation packages.

Index Terms— Detailed Model, brushless dc motor, trapezoidal back-EMF

1. INTRODUCTION

Brushless dc (BLDC) motors are increasingly used in industrial automation, instrumentation, and many equipment and servo applications. Such motors generally have good torque-speed characteristics, fast dynamic response, high efficiency, long life, and etc. A typical BLDC motor consists of a permanent magnet synchronous machine (PMSM) (with either sinusoidal or trapezoidal back-EMF) driven by an inverter as depicted in Fig. 1. The detailed modeling of such machine-inverter system, wherein the switching of each transistor and diode is represented, has been investigated in the literature quite well [1], [2] and may be readily carried out using a number of available simulation packages [3], [4], [5]. However, it is often assumed that the machine's back-EMF is perfectly sinusoidal, where in practice may not be true for many motors. Including the back EMF harmonics increases the model accuracy and is needed for further studies and derivation of average-value models [6], [7].

To develop a detailed model that predicts the performance of BLDC motor-drive system with trapezoidal back-EMF, an appropriate simulation package must be used such that the back-EMF can be modified to include the desired amount of harmonics [4], [5].

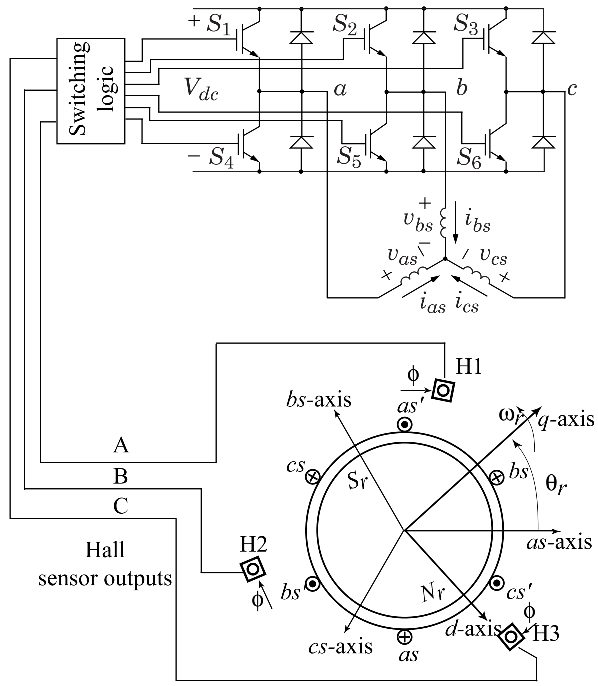

Figure 1. Schematic diagram of a typical VSI-driven BLDC motor-drive system.

Herein the typical voltage-source-inverter-driven (VSI-driven) BLDC motors are considered where the inverter operates using the 120^o commutation method [2]. According to this switching logic, each phase is allowed to be open-circuited for a fraction of a revolution, giving rise to complicated commutation-conduction patterns of the stator currents [2]. The steady state analysis of such motors has been carried out by several researchers [8], [9], [10].

This paper makes a contribution by proposing an improved detailed model of the considered 120-degree BLDC motor-drive systems by properly including the trapezoidal back-EMF harmonics into the model. It is also shown that an accurate model may be only obtained using simulation packages that allow making proper changes in the model such that the effect of back-EMF harmonics is appropriately included in the current and torque relationships.

CCECE 2011 Niagara Falls, Canada
978-1-4244-9789-8/11/$26

2. DETAILED MODELING

A diagram of the considered BLDC motor-inverter system is shown in Fig. 1 where the logical signals from the Hall sensors are used to control the inverter switches– transistors S_1–S_6. Without loss of generality, the advance in firing angle is assumed to be $\phi = 30^\circ$ [1], [8], [9]. The electrical dynamics of stator may be described by the well-known voltage equations:

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + \frac{d\boldsymbol{\lambda}_{abcs}}{dt}. \tag{1}$$

Here, the variables are arranged in vectors such that $\mathbf{f}_{abcs} = \begin{bmatrix} f_{as} & f_{bs} & f_{cs} \end{bmatrix}^T$, and f represents voltage, current or flux linkage. The stator resistance matrix is

$$\mathbf{r}_s = diag\left[r_s, r_s, r_s \right]. \tag{2}$$

The flux linkage equation is

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \boldsymbol{\lambda}'_m \tag{3}$$

where the inductance matrix is defined by

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_m & -0.5L_m & -0.5L_m \\ -0.5L_m & L_{ls} + L_m & -0.5L_m \\ -0.5L_m & -0.5L_m & L_{ls} + L_m \end{bmatrix}. \tag{4}$$

Equations (1)–(4) hold true for either sinusoidal or trapezoidal back-EMF. Considering the trapezoidal back-EMF, the vector of flux linkages can be expressed as [11]

$$\boldsymbol{\lambda}'_m = \lambda'_m \sum_{n=1}^{\infty} K_{2n-1} \begin{bmatrix} \sin\left((2n-1)\theta_r\right) \\ \sin\left((2n-1)\left(\theta_r - \frac{2\pi}{3}\right)\right) \\ \sin\left((2n-1)\left(\theta_r + \frac{2\pi}{3}\right)\right) \end{bmatrix} \tag{5}$$

where λ'_m is the magnitude of the fundamental component of the permanent magnet flux linkage. The coefficients K_n denote the normalized magnitudes of the n^{th} flux harmonic relative to the fundamental, i.e. $K_1 = 1$. The index $2n-1$ tells that only odd harmonics (3rd, 5th, 7th, etc.) can be present due to the rotor symmetry. The developed electromagnetic torque is then given by [11], [12]

$$T_e = \left(\frac{P}{2}\right)\lambda'_m \times \sum_{n=1}^{\infty} (2n-1) K_{2n-1} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}^T \begin{bmatrix} \cos\left((2n-1)\theta_r\right) \\ \cos\left((2n-1)\left(\theta_r - \frac{2\pi}{3}\right)\right) \\ \cos\left((2n-1)\left(\theta_r + \frac{2\pi}{3}\right)\right) \end{bmatrix}. \tag{6}$$

The back-EMF voltages can be measured at the stator terminals when the machine is open-circuited and rotated by a prime mover. Based on (1)–(5), the back-EMF phase voltages can be calculated as [12]

$$\mathbf{e}_{abcs} = \omega_r \lambda'_m \times \sum_{n=1}^{\infty} (2n-1) K_{2n-1} \begin{bmatrix} \cos\left((2n-1)\theta_r\right) \\ \cos\left((2n-1)\left(\theta_r - \frac{2\pi}{3}\right)\right) \\ \cos\left((2n-1)\left(\theta_r + \frac{2\pi}{3}\right)\right) \end{bmatrix}. \tag{7}$$

The mechanical subsystem is assumed to be a single rigid body, for which the dynamics can be expressed as

$$\frac{d\omega_r}{dt} = \left(\frac{P}{2}\right)\frac{1}{J}\left(T_e - T_m\right) \tag{8}$$

where ω_r is the rotor electrical angular speed; J is the combined moment of inertia of the load and the rotor; and P is the number of magnetic poles. The combined mechanical torque is denoted by T_m.

To verify the accuracy of the proposed detailed model and show the effect of harmonics, a typical industrial BLDC machine with trapezoidal back-EMF and parameters summarized in the Appendix is considered in this paper. As can be seen in Appendix, the back-EMF waveform possesses significant amounts of 3rd, 5th and 7th harmonics. The detailed switching model has been implemented in Matlab/ASMG toolbox [5] according to the methodology defined by (1)–(8). The measured and simulated waveforms of the phase back-EMF, stator phase voltage and current are superimposed in Fig. 2. These steady-state waveforms correspond to an operating point defined by 330W mechanical load at 2140 rpm when the inverter is supplied with $V_{dc} = 26V$. An excellent match of the detailed model with the experimental prototype and measurements confirms the model accuracy.

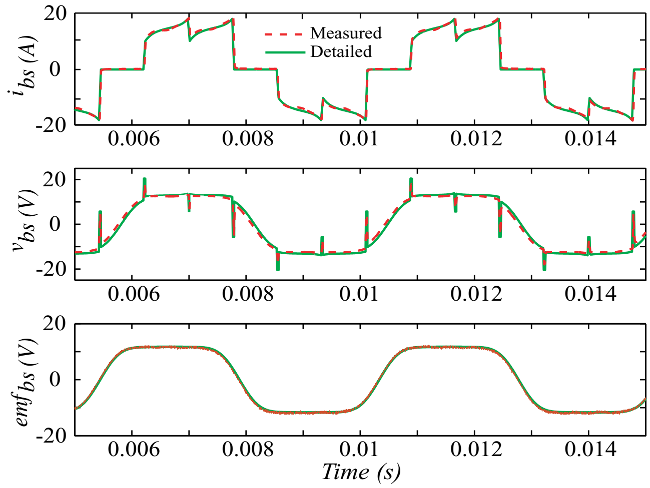

Figure 2. Measured and simulated phase current, phase voltage and back-EMF waveforms.

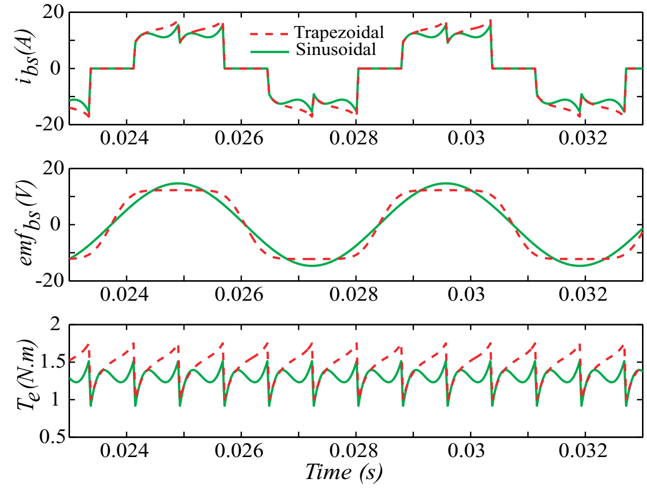

Figure 3. Steady-state waveforms of phase current, back-EMF, and developed electromagnetic torque as predicted by various models.

3. CASE STUDIES

The same industrial BLDC prototype with parameters summarized in the Appendix is used here to show the improvement of the proposed model when implemented in Matlab/ASMG [5] against (I) the previously developed models that consider sinusoidal back-EMF [1], [8], [9]; and (II) models for the BLDC motor with trapezoidal back-EMF that are implemented in other simulation packages such as SimPowerSystems [3]. The proposed model is used as a reference in the following comparisons with other detailed models. For evaluating the waveforms predicted by various models, the same steady-state operating point, defined by 330W mechanical load at 2140 rpm when the inverter is supplied with $V_{dc} = 26V$.

3.1. Trapezoidal vs. Sinusoidal Model

The proposed model implemented in ASMG [5] is compared here to the previous typical BLDC model with sinusoidal back-EMF [1], [8], [9]. The simulated waveforms of the phase current, phase back-EMF, and electromagnetic torque predicted by both models for the same steady-state operating point are superimposed in Fig. 3. According to (6), the electromagnetic torque is expected to have a larger average value due to the added harmonics terms. As a result, for the considered BLDC machine with considerable amount of 3rd, 5th, and 7th harmonics, the sinusoidal model predicts different waveform of the torque ripple as well as lower average value of 1.3 $N.m$, whereas the trapezoidal detailed model predicts 1.5 $N.m$. The difference seen in Fig. 3 confirms the error that will be introduced by using sinusoidal model for simulation of brushless dc motors with trapezoidal back-EMF.

3.2. ASMG vs. SIMPOWER Model

The BLDC motor-inverter system modeling may be carried out using various simulation packages [3], [4], [5]. Experimenting with these packages, it has been found that implementing the BLDC motor that considers only the sinusoidal back-EMF will result in the same output waveforms of the currents and electromagnetic torque that are consistent among all the mentioned tools.

However, when the model includes the back-EMF harmonics, the simulation results of some packages might not be consistent anymore. To demonstrate this point, the same prototype BLDC machine with trapezoidal back-EMF (see Appendix) has been also implemented in SimPowerSystems [3], wherein the user has a choice of using either sinusoidal or trapezoidal back-EMF. For comparison, the simulated waveforms of the phase current, phase back-EMF, and electromagnetic torque predicted by ASMG [5] and SimPowerSystems [3] for the same operating point are shown in Fig. 4. As can be seen in Fig. 4, SimPowerSystems uses an ideal trapezoid for representing the back-EMF, which can be well matched with the measured/simulated waveform of the back-EMF shown in Fig. 2 with the specified 3rd, 5th and 7th harmonics. However, as it can be seen in Fig. 4, when the trapezoid parameters are selected to match the back-EMF waveform (see middle subplot), quite noticeable error will appear in the phase current (see top subplot) and electromagnetic torque (see bottom subplot).

To further show the impact of accurately including the back-EMF harmonics on the system performance, the steady-state torque-speed characteristic of the prototype machine predicted by various models is shown in Fig. 5. As expected, the implemented model in ASMG [5] that includes the effect of back-EMF harmonics represents an improvement and predicts higher torque that also confirms the results shown in Fig. 3 and 4. Hence, it can be again implied that neither the model that considers sinusoidal back-EMF nor the implemented model in SimPowerSystems can be used as a reference detailed model for analysis of the BLDC systems with trapezoidal back-EMF.

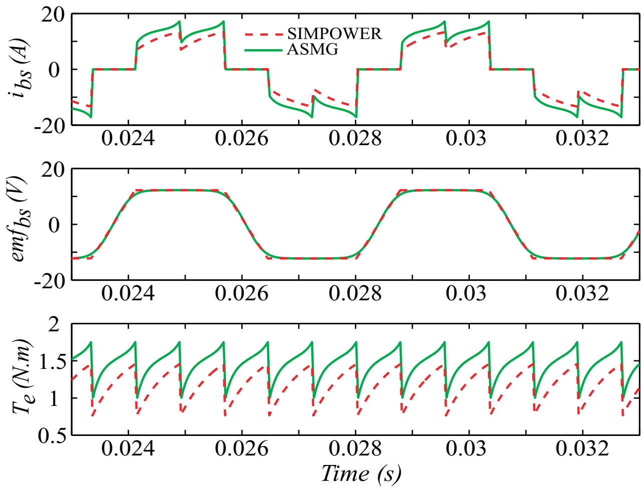

Figure 4. Steady-state waveforms of phase current, back-EMF, and electromagnetic torque as predicted by models with trapezoidal back-EMF implemented in different simulation packages.

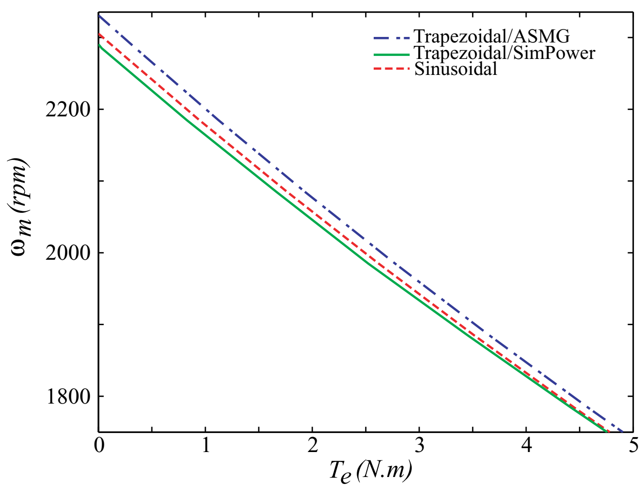

Figure 5. Steady-state torque-speed characteristic predicted by various models and simulation packages.

4. CONCLUSION

This paper discusses the differences between the VSI-driven BLDC motor models with trapezoidal and sinusoidal back-EMF. The proposed model properly takes into account the effect of back-EMF harmonics in establishing relationships for the stator phase current and electromagnetic torque. For a considered typical industrial BLDC machine with trapezoidal back-EMF, the accuracy of the proposed model implemented in ASMG is verified against the measurements. It is also shown that other simulation packages i.e. SimPowerSystems, although allow trapezoidal representation of the back-EMF waveform, may not be very accurate when considering the trapezoidal BLDC machines. The presented detailed model shows an appreciable improvement in accuracy when compared to the typically assumed sinusoidal back-EMF or the commonly-used trapezoidal model of SimPowerSystems. Therefore, the proposed detailed model can be used as a reference for further studies on BLDC motors such as average-value modeling that will be subject to the future research.

5. APPENDIX

BLDC parameters: combined stator phase and cable resistance $r_s = 0.0522\Omega$; $\lambda'_m = 10.9 mV.s$; $P = 12$ poles; back-EMF harmonics $K_1 = 1$, $3K_3 = -0.20$, $5K_5 = 0.047$, $7K_7 = -0.0067$.

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