

Invariance Proof of Algorithm Consensus

Inv is an invariant of algorithm Consensus.

1. $Init \Rightarrow Inv$

PROOF: By the definitions of *Init* and *Inv*, since *Init* implies *chosen* = \emptyset .

- $$2. \textit{Inv} \wedge \textit{Next} \Rightarrow \textit{Inv}'$$

PROOF: By the definitions of *Next* and *Inv*, since *Next* implies *chosen* will contain one element iff *chosen* = \emptyset .

- ### 3. Q.E.D

PROOF: By 1, 2.

	MODULE <i>Consensus</i>	
CONSTANT	<i>Value</i>	
VARIABLE	<i>chosen</i>	
<i>Init</i>	$\triangleq \text{chosen} = \{\}$	
<i>Next</i>	\triangleq	
	$\wedge \text{chosen} = \{\}$	
	$\wedge \exists v \in \text{Value} : \text{chosen}' = \{v\}$	
<i>Spec</i>	$\triangleq \text{Init} \wedge \Box[\text{Next}]_{\text{chosen}}$	
<i>Inv</i>	$\triangleq \text{Cardinality}(\text{chosen}) \leq 1$	