HW 6

Question 1

This question continues the exercise at the end of activity 8 / start of activity 9. Consider adding another term to our working exponential covariance function such that

$$\sigma_{i,j} = \sigma^2 \exp\left(-d_{ij}/\phi\right) + \tau^2$$

1. (2 points)

Create a plot that simulates a surface with $\sigma^2 = 1$ and $\tau^2 = 0$. Use a high resolution grid to visualize this surface.

2. (2 points)

Create a plot that simulates a surface with $\sigma^2 = 0$ and $\tau^2 = 1$. Use a high resolution grid to visualize this surface.

3. (2 points)

Create a plot that simulates a surface with $\phi = 0$ and $\tau^2 = 1$. Use a high resolution grid to visualize this surface.

4. (3 points)

Discuss the differences in the plots in parts 1.1, 1.2, and 1.3. How do the parameters contribute to those differences?

5. (3 points)

Broadly interpret how σ^2, τ^2, and ϕ impact the covariance structure in the spatial surface.

Now we have introduced two new parameters into this function. What do you suppose that they do?

- σ^2: controls the magnitude of the covariance.

- ϕ: controls the range of the spatial correlation

Question 2

Building on the model framework from the previous question, simulate data from the following model

$$y \sim N(\mu, \Sigma)$$
$$\sigma_{i,j} = \sigma^2 \exp\left(-d_{ij}/\phi\right) + \tau^2$$

where $\sigma^2 = 5$, $\tau^2 = 1$, you can choose ϕ.

1. (4 points)

Visualize a draw from this spatial random surface with $n = 50$ points.

2. (4 points)

Use STAN code to estimate the model parameters. How do your results match your expectations?