STAT 534 - Lecture 21

Point Process Data

Point Process Motivation

With point process data, the defining characteristic is that the location is random. Sketch two hypothetical point processes: one with a pattern and one without any spatial pattern.

How did you select the number of points to add to your figure?

- *With*

- Both

There are many interesting questions related to point process data. Consider, for example, locations of Ponderosa Pines and Aspen Trees. With location data for these trees, we can answer the following questions:

- *What*
- *Do*
- *Can*

Point process data is also prevalent in public health settings; in this context the goal is to understand disease patterns.

- *Are*

- *Do*

- *Is*

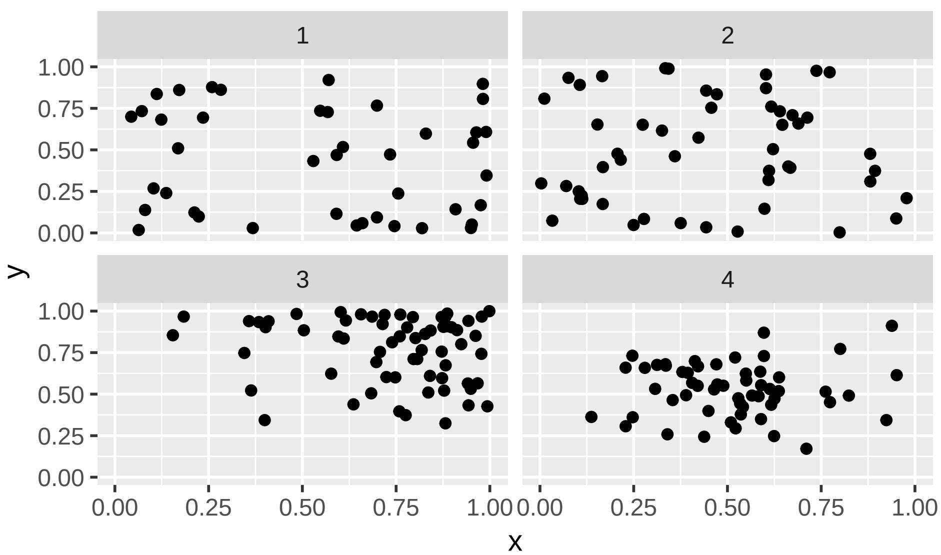
If there is additional meta-information associated with a point pattern, this can be used in what is known as a *marked* point process. For instance, the location of a tree could be considered the point process with the marks associated with tree species.

Discuss the interpretation of marked point process data set with continuous data. For instance, suppose that rather than tree species that the height of the tree was collected. What questions would be of interest?

Exploratory Point Process Analysis

With point process data, there are a few things to assess.

1.



2.

3.

4.

Theoretical Details

- Our focus will be largely on $\mathcal{R}^2$, but the same principles apply for other dimensions.

- A random realization of a point pattern is denoted $\mathcal{S}$ with elements $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_n$. Hence, $\mathcal{S}$ is random and inference focuses on a probabilistic model for $\mathcal{S} \in \mathcal{D}$.

To specify a probabilistic model for $\mathcal{S}$, there are two essential elements.

1.

2.

- Then the likelihood of $\mathcal{S}$ can be written as

$$\mathcal{L}(\mathcal{S}) = Pr(\mathcal{N}(d) = n) n! f(\boldsymbol{s}_1, \ldots, \boldsymbol{s}_n)$$

- Given the likelihood,

Counting Measure

- The first part of a point process is the number of points.

- *Let*

- One common approach for the counting measure is a Poisson Process.

- *Thus*

- If $\mathcal{B}_1$ and $\mathcal{B}_2$ are disjoint, then $N(\mathcal{B})_1$ and $N(\mathcal{B})_2$ are independent.

- Sketch $\mathcal{D}$ such that $\mathcal{B}_1$ and $\mathcal{B}_2$ are disjoint.

- What would $\lambda(\mathcal{B}_1)$ and $\lambda(\mathcal{B}_2)$ be if we had spatial homogeneity?

- When $\lambda(\boldsymbol{s}) = \lambda$, this is referred to as a homogenous Poisson process (HPP).

- *Then*

- What is the expected number of points for a HPP in $\mathcal{B}$?

- Note $\lambda(\mathcal{D}) = \int_D \lambda \boldsymbol{s} d\boldsymbol{s}$ is an intensity measure, not a probability distribution. What, in general, is necessary to normalize the location density?

Nonhomogenous Poisson process

- A HPP is the simplest form of a point process, sketch the intensity function or a nonhomogenous Poisson process (NHPP). Label the intensity $\lambda(\boldsymbol{s})$ on your figure.

- *When*

- *With*

- *The*