STAT 534 - Lecture 23: Key

K function

- We previously looked at the $F(d)$ and $G(d)$ functions, which corresponded to *CDFs, as a function of distance, for open space and distance between points.*

- Another interesting feature of a point process is the number of points in a specified area.

- Consider $E(Num(\boldsymbol{s}, d, \boldsymbol{S}))$, the expected number of points in $\delta_d \boldsymbol{s}$, a circle of radius d centered at $\boldsymbol{s}$.

- *Ripley's K or just the K function, considers the expected number of points within a distance d of an arbitrary point. Formally this is defined for CSR as*

$$K(d) = \frac{E(\text{number of points within d})}{\lambda}$$

  . In other words, this is scaled by λ

- With CSR, $K(d) = \frac{\lambda \pi d^2}{\lambda} = \lambda d^2$.

- To estimate $K(d)$, *we use*

$$\hat{K}(d) = (\hat{\lambda})^{-1} \sum_i \sum_j 1(||\boldsymbol{s}_i - \boldsymbol{s}_j|| \leq d)/n$$

  where $\hat{\lambda} = n/|\mathcal{D}|$, note the typo in the book.

- The empirical K statistic is compared with πd^2. *For $K > \pi d^2$, the series exhibits clustering, for $K < \pi d^2$ the process exhibits inhibition.*

- Similar to the G and F functions, edge correction is typically applied.

`spatstat` Intro

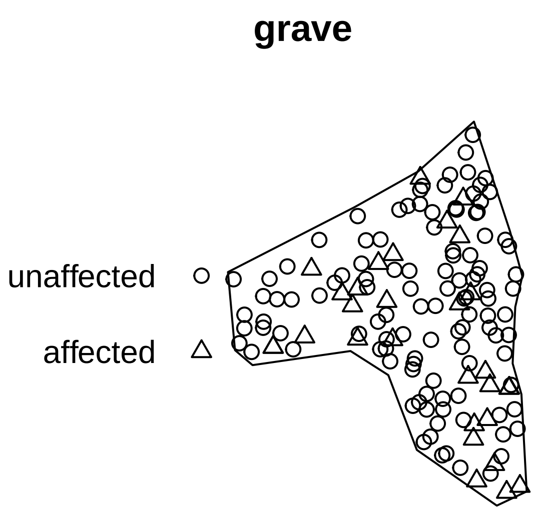
- The `spatstat` package is a comprehensive R package for point process data. It has a website and a nice vignette.

- Consider a dataset with medieval grave site information.

```
data(grave)
summary(grave)
```

```
## Marked planar point pattern:  143 points
## Average intensity 5.70489e-06 points per square unit
##
## Coordinates are integers
## i.e. rounded to the nearest unit
##
## Multitype:
##            frequency proportion   intensity
## unaffected       113  0.7902098 4.50806e-06
## affected          30  0.2097902 1.19683e-06
##
## Window: polygonal boundary
## single connected closed polygon with 16 vertices
## enclosing rectangle: [4376.579, 10511.88] x [2809.612, 10702.971] units
## Window area = 25066200 square units
## Fraction of frame area: 0.518
```

```
plot(grave)
```



- Explore the `Fest()`, `Gest()`, and `Kest()` functions in `spatstat` and summarize the results for the `grave` dataset. (Also look at the `envelope` function for plots.)

- Next convert the point processes from early classes to `ppp` objects and explore `Fest()/Kest()`.

```
set.seed(04082019)
n <- rpois(4, 50)
x <- c(rbeta(n[1], 1, 1), rbeta(n[2], 1, 1), rbeta(n[3], 3, 1),rbeta(n[4], 3, 3))
y <- c(rbeta(n[1], 1, 1), rbeta(n[2], 1, 1), rbeta(n[3], 3, 1),rbeta(n[4], 3, 3))

comb.df <- data.frame(group = c(rep(1, n[1]), rep(2, n[2]), rep(3, n[3]), rep(4, n[4])), x = x, y = y)

df1 <- comb.df %>% filter(group ==1)
df2 <- comb.df %>% filter(group ==2)
df3 <- comb.df %>% filter(group ==3)
df4 <- comb.df %>% filter(group ==4)
```

Estimating the intensity Function

- With CSR, the intensity function is trivial - *Just uniform with intensity* λ.

- **Discuss:** given a realization of a point process, how could an intensity function be estimated?

- *One option would be to discretized histogram. Consider a fine grid, then let* $\lambda(\delta \boldsymbol{s}) = \int_{\delta \boldsymbol{s}} \lambda(\boldsymbol{s}) d\boldsymbol{s} \approx \lambda(\boldsymbol{s})|\delta \boldsymbol{s}|$*, where* $\lambda(\boldsymbol{s})$ *is constant over the grid square. Thus* $\lambda(grid) = \frac{Num(grid)}{Area(grid)}$

- *An alternative is to use kernel density estimates. These are basically a smoothed version of a histogram using some sort of symmetric PDF.*

- Now using the `plot(density(.))` function, plot and interpret the empirical intensity for the grave dataset along with the four synthetic examples.