STAT 534 - Lecture 24

Modeling Point Patterns using NHPPs

- Statistical modeling depends on a sampling model for the data and the associated likelihood function.

- *Conditional on the number of points, $Num(D) = n$, the location density can be specified as:*

- Then condidering the location and number of points simultaneously, we have

$$f(\boldsymbol{s}_1, \ldots, \boldsymbol{s}_n, Num(D) = n) = \prod_i \frac{\lambda(\boldsymbol{s}_i)}{(\lambda(D))^n} \times$$

- *Finally, the likelihood is*

- Alternatively, consider a fine partition for D, then using the Poisson assumption, the likelihood is the product of the counts across the partitions.

$$\prod_l \exp(-\lambda(A_l))(\lambda(A_l))^{Num(A_l)}$$

As the grid becomes finer, $Num(A_l) \in \{0, 1\}$.

- *The likelihood is a function of the entire intensity surface $\lambda(\boldsymbol{s})$.*

- **Discuss**: how could $\lambda(\boldsymbol{s})$ be obtained?

- *There are two general approaches for modeling $\lambda(\boldsymbol{s})$:*

- A simple, but "silly" example of a parametric form would be $\lambda(\boldsymbol{s}) = \sigma\lambda_0(\boldsymbol{s})$.

- Suppose remoted-sensed covariate information is available on a grid. How might this be used for constructing $\lambda(\boldsymbol{s})$?

- In general a parametric function $\lambda(\boldsymbol{s};\theta)$ could be specified, but it would required a richly specified class that would need to be non-negative. A common approach for this type of problem is to use a set of basis functions such that $\lambda(\boldsymbol{s};\theta) = \sum_k a_k g_k(\boldsymbol{s})$.

- Sketch a set of basis functions to create a multimodal curve in 1D.

- *Sometimes a trend surface*

- The most common approach is to specify $\log(\lambda(\boldsymbol{s})) = X^t(\boldsymbol{s})\gamma$.

- Specifying covariates still requires integrating $\int_D \exp(X^t(\boldsymbol{s})\gamma)d\boldsymbol{s}$, which is challenging as $X^t(\boldsymbol{s}$ is not often specified in a functional form.

- *For a fine resolution*

- Now, considering $\lambda(\boldsymbol{s})$ through the non-parametric lens,

- Suppose $\lambda(\boldsymbol{s}) = \exp(Z(\boldsymbol{s}))$ where $Z(\boldsymbol{s})$ is a realization from a spatial Gaussian process with mean $X^t(\boldsymbol{s})\gamma$ and covariance function $\sigma^s\rho()$.

- The LGCP provides a prior for $\lambda(\boldsymbol{s})$. This can be written as $X^t(\boldsymbol{s})\gamma + w(\boldsymbol{s})$ where $w(\boldsymbol{s}) = \log \lambda_0(\boldsymbol{s})$.

- **Discuss**: How to fit a model for a LGCP?

- In the parametric setting,

- In the non-parametric setting,

- *Thus priors are needed*

-In general algorithms for this type of data are computationally intensive.

- Due to the challenges in modeling LGCP, the data is sometimes considered on a grid. Then the intensity for grid i could be estimated as:

$$\log \lambda_i = X_i^t \beta + \phi_i.$$

However, the results can be sensitive to the grid specification.