Modeling Point Patterns using NHPPs

Statistical modeling depends on a sampling model for the data and the associated likelihood function. Conditional on the number of points, $Num(D) = n$, the location density can be specified as:

Then considering the location and number of points simultaneously, we have

Alternatively, consider a fine partition for D, then using the Poisson assumption, the likelihood is the product of the counts across the partitions.

The likelihood is a function of the entire intensity surface $\lambda(\boldsymbol{s})$. Hence, a functional description of $\lambda(\boldsymbol{s})$ is necessary.

There are two general approaches for modeling $\lambda(\boldsymbol{s})$: parametric and non-parametric.

A simple, but "silly" example of a parametric form would be

Suppose remote-sensed covariate information is available on a grid. How might this be used for constructing $\lambda(\boldsymbol{s})$?

In general a parametric function $\lambda(\boldsymbol{s};\theta)$ could be specified, but it would required a richly specified class that would need to be non-negative. A common approach for this type of problem is to use a set of basis functions such that $\lambda(\boldsymbol{s};\theta) = \sum_k a_k g_k(\boldsymbol{s})$.

- Sketch a set of basis functions to create a multimodal curve in 1D.

Sometimes

The most common approach is to specify $\log(\lambda(\boldsymbol{s})) = X^t(\boldsymbol{s})\gamma$.

Specifying covariates still requires integrating $\int_D \exp(X^t(\boldsymbol{s})\gamma)d\boldsymbol{s}$, which is challenging as $X^t(\boldsymbol{s}$ is not often specified in a functional form.

Now, considering $\lambda(\boldsymbol{s})$ through the non-parametric lens, *let*

Suppose $\lambda(\boldsymbol{s}) = \exp(Z(\boldsymbol{s}))$ where $Z(\boldsymbol{s})$ is a realization from a spatial Gaussian process with mean $X^t(\boldsymbol{s})\gamma$ and covariance function $\sigma^s\rho()$.

The LGCP provides a prior for $\lambda(\boldsymbol{s})$. This can be written as $X^t(\boldsymbol{s})\gamma + w(\boldsymbol{s})$ where $w(\boldsymbol{s}) = \log \lambda_0(\boldsymbol{s})$.