Areal Data Overview

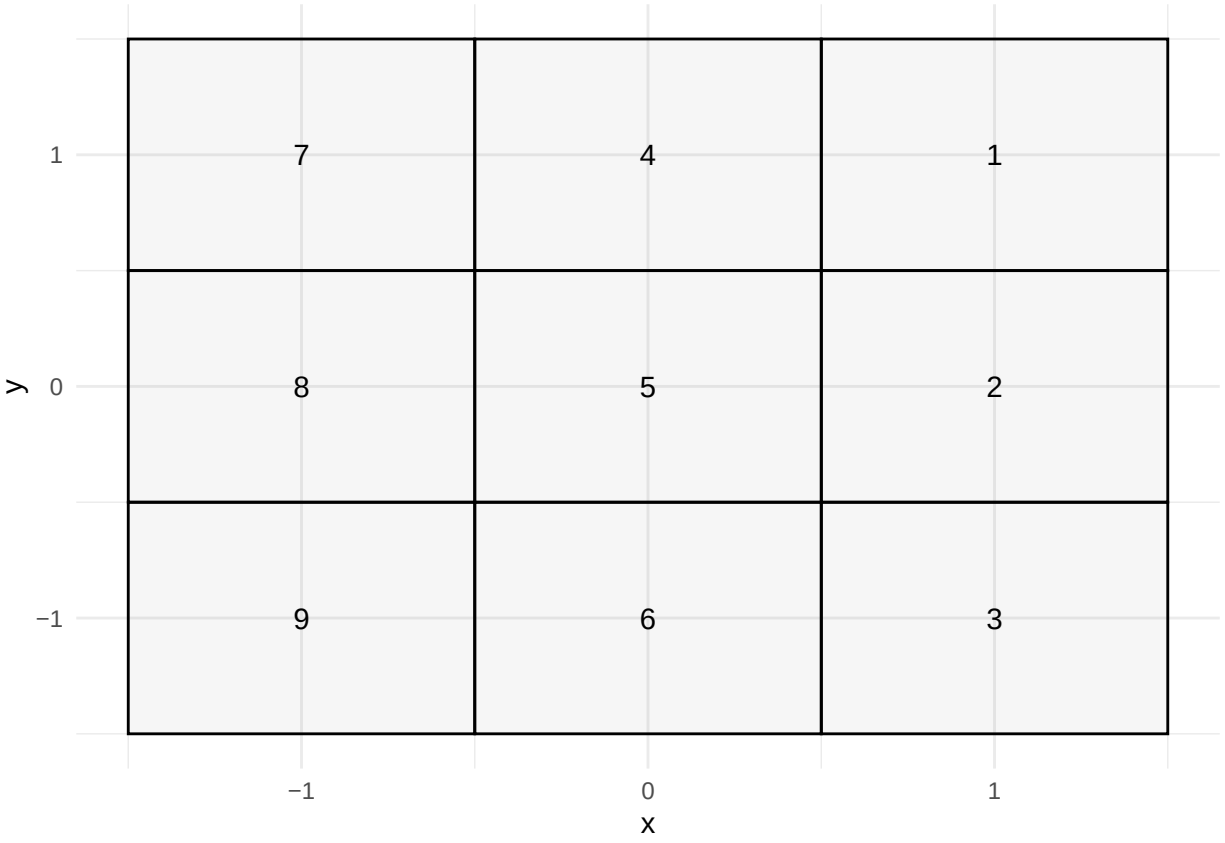
Proximity Matrix

Similar to the distance matrix with point-reference data, a proximity matrix W is used to model areal data.

Grid Example

Create an adjacency matrix with diagonal neigbors

Create an adjacency matrix without diagonal neigbors



Spatial Association

There are two common statistics used for assessing spatial association: Moran's I and Geary's C.

Moran's I

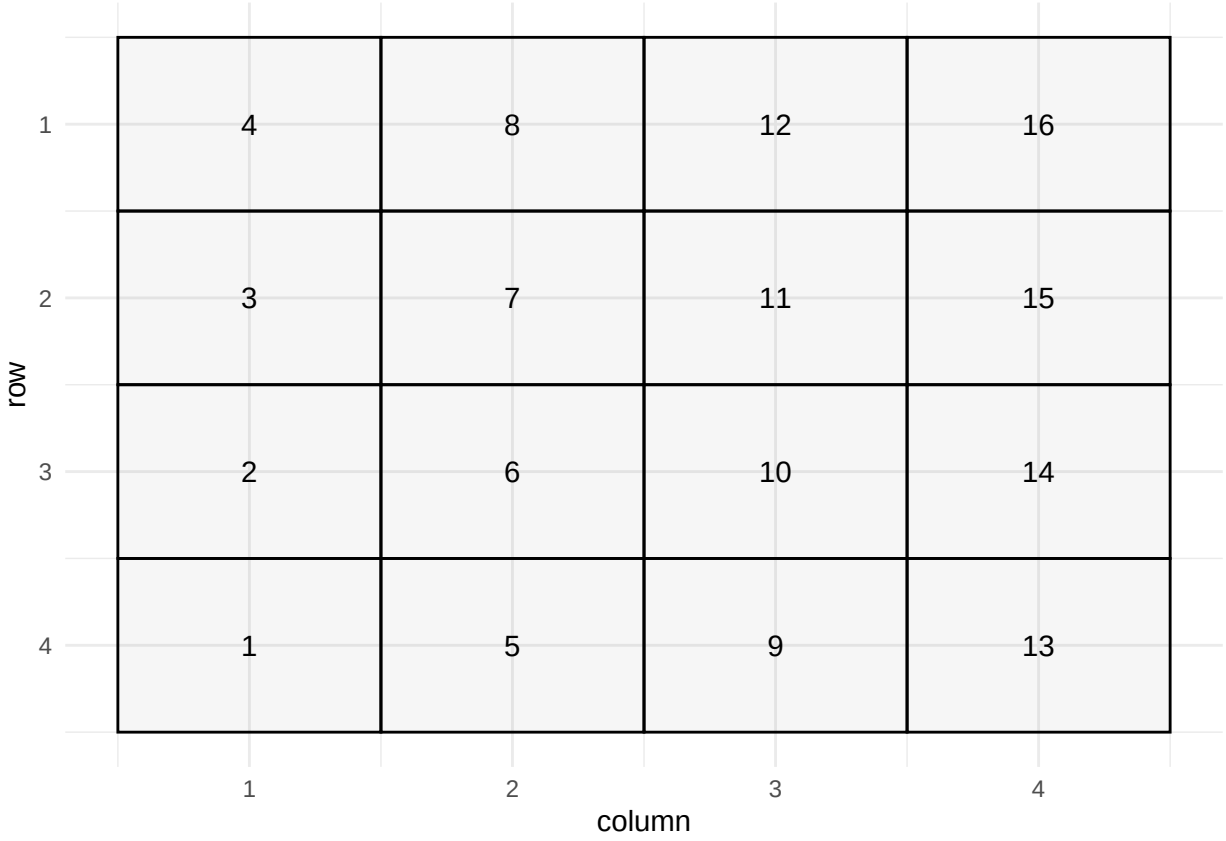
$$I = \frac{n \sum_i \sum_j w_{ij}(Y_i - \bar{Y})(Y_j - \bar{Y})}{(\sum_{i \neq j} w_{ij}) \sum_i (Y_i - \bar{Y})^2}$$

Geary's C

$$C = \frac{(n-1) \sum_i \sum_j w_{ij}(Y_i - Y_j)^2}{2(\sum_{i \neq j} w_{ij}) \sum_i (Y_i - \bar{Y})^2}$$

Spatial Association Exercise

Consider the following scenarios and use the following 4-by-4 grid



and proximity matrix

```
W <- matrix(0, 16, 16)
for (i in 1:16){
  W[i,] <- as.numeric((d4$rpos[i] == d4$rpos & (abs(d4$cpos[i] - d4$cpos) == 1)) |
                        (d4$cpos[i] == d4$cpos & (abs(d4$rpos[i] - d4$rpos) == 1)))
}
head(W)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13]
## [1,]    0    1    0    0    1    0    0    0    0     0     0     0     0
## [2,]    1    0    1    0    0    1    0    0    0     0     0     0     0
## [3,]    0    1    0    1    0    0    1    0    0     0     0     0     0
## [4,]    0    0    1    0    0    0    0    1    0     0     0     0     0
## [5,]    1    0    0    0    0    1    0    0    1     0     0     0     0
## [6,]    0    1    0    0    1    0    1    0    0     1     0     0     0
##      [,14] [,15] [,16]
## [1,]     0     0     0
## [2,]     0     0     0
## [3,]     0     0     0
## [4,]     0     0     0
## [5,]     0     0     0
## [6,]     0     0     0
```

for each scenario plot the grid, calculate I `spdep::moran.test` and G `spdep::geary.test`.

1. Simulate data where the responses are i.i.d. N(0,1).

2. Simulate data and calculate I and G for a 4-by-4 grid with a chess board approach, where "black squares" $\sim N(-2, 1)$ and "white squares" $\sim N(2, 1)$.

3. Simulate multivariate normal response on a 4-by-4 grid where $y \sim N(0, (I - \rho W)^{-1})$, where $\rho = .3$ is a correlation parameter and W is a proximity matrix.