Areal Data Model Fitting, Part 2

Continuing with the Tester election dataset.

Now consider some covariates to explain the response

Consider a linear model with county population, using both `lm` and `S.glm`

```
lm_dat <- Tester %>%
  left_join(usmap::countypop %>% rename("county_fips" = fips), by = 'county_fips') %>%
  mutate(scale_pop = scale(pop_2015))

pop_model <- lm(Tester_Prop ~  scale_pop, data = lm_dat)
display(pop_model)
```

```
## lm(formula = Tester_Prop ~ scale_pop, data = lm_dat)
##             coef.est coef.se
## (Intercept) 0.40     0.02
## scale_pop   0.06     0.02
## ---
## n = 56, k = 2
## residual sd = 0.13, R-Squared = 0.18
```

```
S.glm(Tester_Prop ~ scale_pop, data = lm_dat, family = 'gaussian',
      burnin = 100, n.sample = 1000, verbose = F)
```

```
##
## #################
## #### Model fitted
## #################
## Likelihood model - Gaussian (identity link function)
## Random effects model - None
## Regression equation - Tester_Prop ~ scale_pop
## Number of missing observations - 0
##
## ############
## #### Results
## ############
## Posterior quantities and DIC
##
##             Median   2.5%  97.5% n.effective Geweke.diag
## (Intercept) 0.3982 0.3650 0.4348         900        -1.5
## scale_pop   0.0621 0.0241 0.0967        1053         1.4
## nu2         0.0182 0.0125 0.0262         900        -1.1
##
## DIC =  -62.58489       p.d =  2.901435       LMPL =  30.64
```

We previously extract the residuals create a choropleth and test for spatial association. Now we will directly run the CAR model using `CARBayes`. See vignette for more info and options on model fitting, including GLM models.

This model can be expressed as

$Y_k \sim N(\mu_k, \nu^2)$, where $\mu_k = x_k\beta + \psi_k$.

A CAR prior is placed on ψ_k such that the full conditional is expressed as

$$\psi_k|- \sim N\left(\frac{\rho\sum_i w_{ki}\psi_i}{\rho\sum_i w_{ki} + 1 - \rho}, \frac{\tau^2}{\rho\sum_i w_{ki} + 1 - \rho}\right)$$

If $\rho = 0$ there is no spatial structure present, if $\rho \to 1$ this is the intrinsic CAR model.

```
S.CARleroux(Tester_Prop ~ scale_pop, data = lm_dat, family = 'gaussian', burnin = 10000,
            n.sample = 100000, W = mt.adj.mat, thin = 5, verbose = F)
```

```
##
## #################
## #### Model fitted
## #################
## Likelihood model - Gaussian (identity link function)
## Random effects model - Leroux CAR
## Regression equation - Tester_Prop ~ scale_pop
## Number of missing observations - 0
##
## ############
## #### Results
## ############
## Posterior quantities and DIC
##
##             Median   2.5%  97.5% n.effective Geweke.diag
## (Intercept) 0.3977 0.3698 0.4258     18000.0         0.9
## scale_pop   0.0538 0.0208 0.0881     15840.8         0.0
## nu2         0.0108 0.0034 0.0201      3285.2         2.1
## tau2        0.0123 0.0029 0.0373      3096.0        -1.6
## rho         0.5307 0.0646 0.9454      4799.9         1.9
##
## DIC =  -81.44965       p.d =  17.83747       LMPL =  34.33
```