

K function

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We previously looked at the $F(d)$ and $G(d)$ functions, which corresponded to

Another interesting feature of a point process is the number of points in a specified area. Consider $E(\text{Num}(\mathbf{s}, d, \mathbf{S}))$, the expected number of points in $\delta_d \mathbf{s}$, a circle of radius d centered at \mathbf{s} .

With CSR, $K(d) = \frac{\lambda \pi d^2}{\lambda} = \pi d^2$.

To estimate $K(d)$, we use

$$\hat{K}(d) = (\hat{\lambda})^{-1} \sum_i \sum_j 1(\|\mathbf{s}_i - \mathbf{s}_j\| \leq d)/n$$

The empirical K statistic is compared with πd^2 .

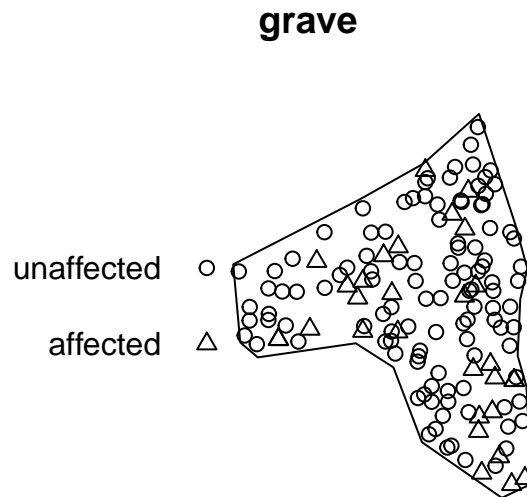
More spatstat

Consider a dataset with medieval grave site information.

```
data(grave)
summary(grave)
```

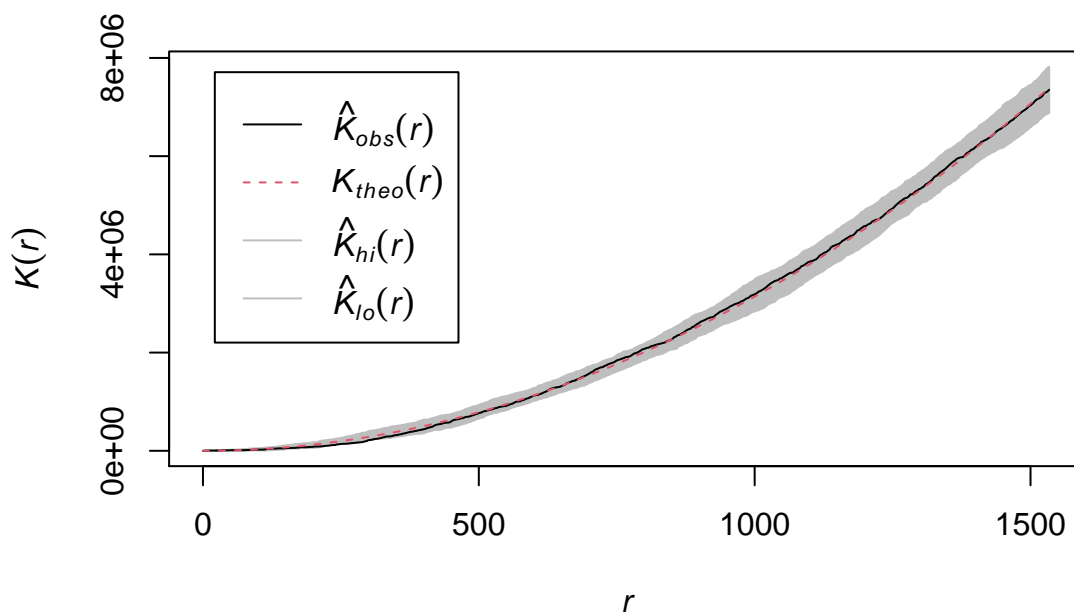
```
## Marked planar point pattern: 143 points
## Average intensity 5.70489e-06 points per square unit
##
## Coordinates are integers
## i.e. rounded to the nearest unit
##
## Multitype:
##      frequency proportion    intensity
## unaffected      113  0.7902098 4.50806e-06
## affected         30  0.2097902 1.19683e-06
##
## Window: polygonal boundary
## single connected closed polygon with 16 vertices
## enclosing rectangle: [4376.579, 10511.88] x [2809.612, 10702.971] units
##                      (6135 x 7893 units)
## Window area = 25066200 square units
## Fraction of frame area: 0.518
```

```
plot(grave)
```



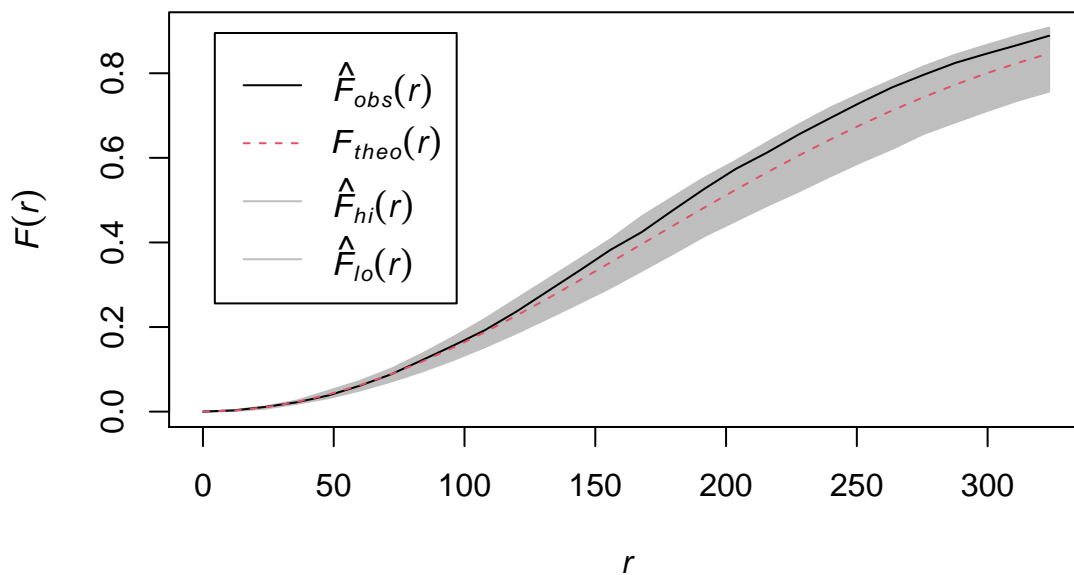
```
plot(envelope(grave, Kest, verbose = F))
```

envelope(grave, Kest, verbose = F)



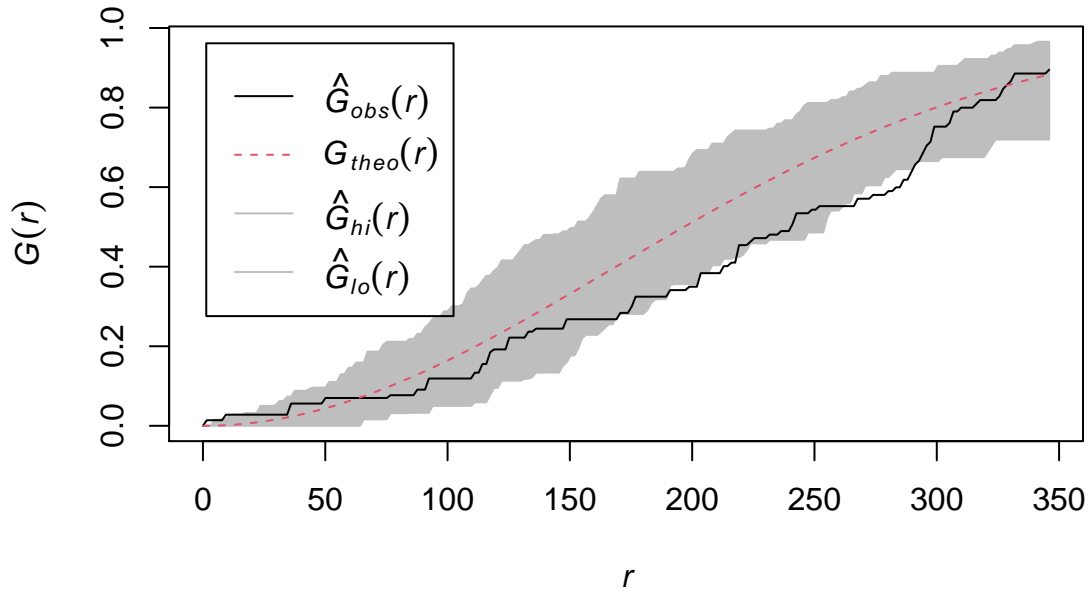
```
plot(envelope(grave, Fest, verbose = F))
```

envelope(grave, Fest, verbose = F)



```
plot(envelope(grave, Gest, verbose = F))
```

envelope(grave, Gest, verbose = F)



Estimating the intensity Function

- With CSR, the intensity function is trivial
- **Discuss:** given a realization of a point process, how could an intensity function be estimated?

Now using the `plot(density(.))` function, plot and interpret the empirical intensity for the grave dataset along with the four synthetic examples.

density(grave)

