

# Write formulas

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Definitions:

- $x_{i,j}^r$ : raw counts of cells in cluster  $i$  at time  $j$  (obtained as in the usual single cell analysis pipelines, i.e., from per cell normalization after quality control, and clustering of the umap landscape).
- $T_j^o$ : measured real total number of cells at time  $j$  (cells in the dish, measured by FACS).
- $\tilde{x}_{i,j}^o$ : measured fraction of cells in cluster  $i$  at time  $j$ . This can be obtained as:  $\tilde{x}_{i,j}^o = \frac{x_{i,j}^r}{\sum_{i=1,\dots,11} x_{i,j}^r}$ .
- $x_{i,j}^o$ : measured real number of cells in cluster  $i$  at time  $j$ . This can be obtained upon scaling the fractions by the total number of cells in a dish at time point  $j$ :  $x_{i,j}^o = \tilde{x}_{i,j}^o T_j^o$ .
- $x_{i,j}$ : model prediction for the real number of cells in cluster  $i$  at time  $j$  (output of the ODE system).
- $T_j$ : model prediction for the real number of cells at time  $j$ .  $T_j = \sum_{i=1,\dots,11} x_{i,j}$ .
- $\tilde{x}_{i,j}$ : model prediction for the fraction of cells in cluster  $i$  at time  $j$ . This can be obtained as:  $\tilde{x}_{i,j} = \frac{x_{i,j}}{T_j}$ .

We can write the cost function  $L$  in two ways:

- $L = \sum_{i=1}^{11} \sum_{j=1}^3 \frac{(x_{i,j} - x_{i,j}^o)^2}{\sigma_i}$ , or
- $L = \sum_{i=1}^{11} \sum_{j=1}^3 \frac{(\tilde{x}_{i,j} - \tilde{x}_{i,j}^o)^2}{\sigma_i} + \sum_{j=1}^3 \frac{(T_j - T_j^o)^2}{\sigma}$