

Spatiotemporal Modeling of Voting in North Carolina

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Project Goal

The 13 voting districts in North Carolina have repeatedly been struck down by the courts for (illegal) racial gerrymandering; and the state has received national attention for its highly (legally) politically gerrymandered districts – which have been accused of guarding the majority party (Republican) seats, despite voting fluctuations.

This project examines the publically available highly local precinct-level voting and demographic data which feed into the district voting data, using a spatial areal-data model.

The Data

- Examining voting data for US House of Representatives races, for years 2002, 2004, . . . , 2018.
- Currently 2,704 precincts. Due to boundary changes over time, and missing data, we model a subset of 2,045 of these precincts.
- The publically available election data includes: precinct-level vote counts (the response); as well as precinct-level demographic data on age, race, and sex (the predictors).

•covariate plots?

- Because this project relied of many different publically available datasets, a substantial amount of work was put into cleaning and standardizing the data. As a result of data irregularities, there are a number of caveats that we include with out data:
 - A substantial number of precincts were not reported for each year – either due to the precincts having changed over time, changing of precinct naming schemes, or missing data. All such precincts were excluded from our model.
 - Because absentee and early voting was not often reported at the individual precinct level, we excluded all absentee and early voting voters counts (usually ~ 3% of the votes) which could introduce bias.
 - Within each precinct, there were often covariates that had missing data. Our estimates simply summed over the available covariates.
 - One particular modeling nuisance was the occurrence of several elections with *unopposed* candidates (resulting in θ_{kt} at exactly 0 or 1.)
- Lastly, we used the full Binomial model, rather than the common normal approximation, because the normal approximation is ineffective for probabilities close to 0 or 1 – which is the case with a substantial number of our observed precincts.

Modeling Decisions

- 1 Type of data
 - I think binomial is the correct choice now
 - We are still modeling the logit of the proportion, but it is within the Binomial model
- 2 Type of Spatiotemporal CAR model
 - Many options in the package

Spatiotemporal Model

Variables Definitions

- Y_{kt} : Number of Democratic votes in precinct k for year t
- n_{kt} : Sum of Democratic and Republican votes in precinct k for year t
- θ_{kt} : Relative proportion of Democratic votes in precinct k for year t
- \mathbf{x}_{kt} : Vector of covarites in precinct k for year t
- ψ_{kt} : Latent component incorporating spatiotemporal effects in precinct k for year t

Model form

$$\begin{aligned} Y_{kt} &\sim \text{Binomial}(n_{kt}, \theta_{kt}) \\ \log(\theta_{kt}/(1 - \theta_{kt})) &= \mathbf{x}_{kt}^\top \beta + \psi_{kt} \\ \beta &\sim \text{Normal}(\mu_\beta, \Sigma_\beta) \\ \psi_{kt} &= \beta_1 + \phi_k + (\alpha + \delta_k) \frac{t - \bar{t}}{N} \\ \phi_k | \boldsymbol{\phi}_{-k}, \mathbf{W} &\sim \text{Normal} \left(\frac{\rho_{int} \sum_{j=1}^K w_{kj} \phi_j}{\rho_{int} \sum_{j=1}^K w_{kj} + 1 - \rho_{int}}, \frac{\tau_{int}^2}{\rho_{int} \sum_{j=1}^K w_{kj} + 1 - \rho_{int}} \right) \\ \delta_k | \boldsymbol{\delta}_{-k}, \mathbf{W} &\sim \text{Normal} \left(\frac{\rho_{slo} \sum_{j=1}^K w_{kj} \delta_j}{\rho_{slo} \sum_{j=1}^K w_{kj} + 1 - \rho_{slo}}, \frac{\tau_{int}^2}{\rho_{slo} \sum_{j=1}^K w_{kj} + 1 - \rho_{slo}} \right) \\ \tau_{int}^2, \tau_{slo}^2 &\sim \text{Inverse Gamma}(1, .01) \\ \rho_{int}, \rho_{slo} &\sim \text{Uniform}(0, 1) \\ \alpha &\sim \text{Normal}(0, 1000) \end{aligned}$$

where $\bar{t} = N^{-1} \sum_{t=1}^N t$ and the linear trend $(t - \bar{t})/N$ runs over $[-\frac{1}{2}, \frac{1}{2}]$; β_1 is the first element of β ; $k = 1, \dots, K$, where K is the number of precincts; $t = 1, \dots, T = 9$; $\sum_{j=1}^K \phi_j = \sum_{j=1}^K \delta_j = 0$;

- The random effect ψ_{kt} incorporates both the spatial effect ϕ_k and a linear-trend time component $(\alpha + \delta_k \frac{t - \bar{t}}{N})$ for that individual location k . (*int* and *slo* refer to “intercept” and “slope”.)
- The terms ϕ_k and δ_k are modeled using their full conditional distributions, as determined by their neighbors (specified in adjacency matrix \mathbf{W} .)
- The hyper-parameters were chosen to be uninformative.

Parameter Estimates

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-307.3524	102.7686	-2.99	0.0042
log(GDP)	80.3790	10.1575	7.91	0.0000
locWestern	416.3502	139.9841	2.97	0.0044
locOther	640.5218	128.5153	4.98	0.0000
log(GDP):locWestern	-43.3499	13.6999	-3.16	0.0026
log(GDP):locOther	-73.3333	12.8909	-5.69	0.0000

$$R^2 = 0.853$$

Results

- Fitted* is based off our model fit, for the precincts included.
- Data* is based off the true data, for the precincts included.
- Reality* shows the actual observed election counts.

Year	Fitted	Data	Reality
2002	6	6	6
2004	7	6	6
2006	6	7	7
2008	6	8	8
2010	6	8	7
2012	3	3	4
2014	3	3	3
2016	4	3	3
2018	3	4	2 or 3

Figure: Total Democratic District Winners

Conclusions