Skew-T & Group-T Copula

I-Lun Tu

4/26/2022

Content

- Research Motivation and Possible Contribution
- Skew-T Copula and Group-T Copula
- Copula Estimation Process
- Application to Financial Data
- Unconditional Coverage of VaR
- Possible Improvement

Research Motivation and Possible Contribution

- Straightforward way to generalize copula to higher dimensions
  - More parsimonious model than Vine based Copula
  - Easier Interpretation - shape, dispersion, and skewness
  - Bivariate Student-T is effective for pairs of stocks
- Captures stylized facts of financial markets
  - Asymptotic tail dependence
  - Asymmetry - joint lower-tailed events
- Empirical results to support skew-T Copula in market risk
  - One-stage estimation usually does not select skew-T over standard T
  - Modern unconditional tests for tail risk model
  - High computational costs and accuracy of skewness parameter

Skew-T Copula and Group-T Copula

- Skew-T Distribution:

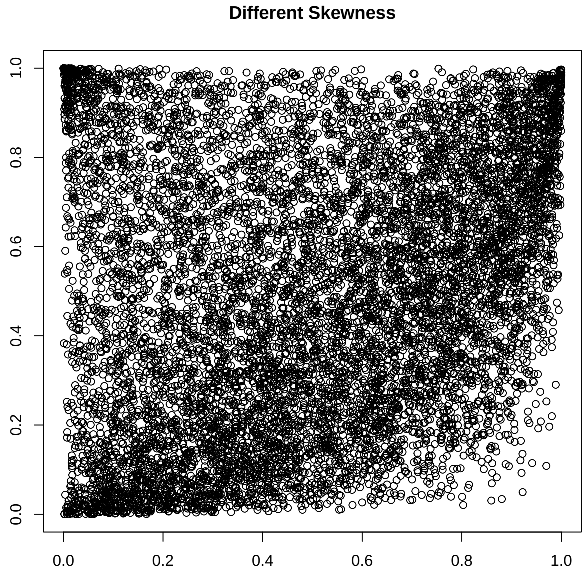
$$X = \gamma V^{-1} + V^{-\frac{1}{2}}\mathbf{Z}$$

  where V is $G(\frac{\nu}{2}, \frac{\nu}{2})$, γ is skewness parameter vector.
  - General Hyperbolic Distribution - Normal mean-variance mixture distribution
  - When $\gamma = 0$, it reduces to Student-T distribution
  - When $\nu \to \infty$, it becomes Normal distribution (not skew Normal)
  - $\nu > 4$ to have finite covariance - difficulty in application
- Skew-T Copula:

$$C^{t}_{\nu,P,\gamma}$$

  where P is the correlation matrix.
  - Same copula for different dispersion and location
  - Different ν, γ to form skew and group copula (generalized T)

Skew-T Copula and Group-T Copula



Copula Estimation Process

- Two Stage Estimation:
  - Forming pseudo observations from the copula
    - Parametric estimation
    - Non-parametric estimation (empirical distribution function)
  - Maximum likelihood estimation for the copula parameters
- Difficulty:
  - When maximizing copula density, marginal quantile functions have to be calculated $n * d$ times
    - No close form quantile function for univariate skew-T
    - Empirical quantile functions has to be simulated for a large number
  - Positive semi-definiteness of the correlation matrix is not guaranteed
    - Empirical correlation using Kendall's tau might not work
- Recent Advancement (Toshinao Yoshiba 2018):
  - Monotone interpolator (100 interpolating quantiles)
  - Reparameterize the Cholesky decomposed triangular matrix with trigonometric functions

Application to Financial Data

- Establish VaR for stock portfolio on financial institution
  - Consumer finance, commercial banking, brokerage and investment management
  - Dependence modelling for 15 stocks (5 each) equal weight portfolio
  - Simulate VaR to set up threshold for loss distribution
  - Measure the dependence structure

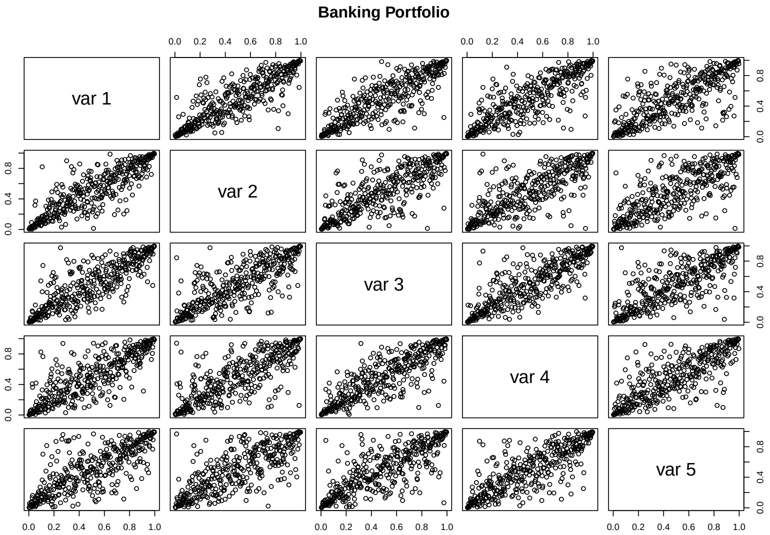
Application to Financial Data

Copula Estimation Process

- Data Preprocess: unfiltered 5 years weekly log-returns for stocks (serial uncorrelated)
- Pseudo copula observations: nonparametric estimation $\frac{1}{n+1}\sum_{t=1}^{n} I_{(X_{t,i<=x})}$ (McNeil 2015)
- Copula estimation: use the recently proposed method to estimate ν, γ, P (equal-skewness)

Application to Financial Data

Pseudo Copula



Application to Financial Data

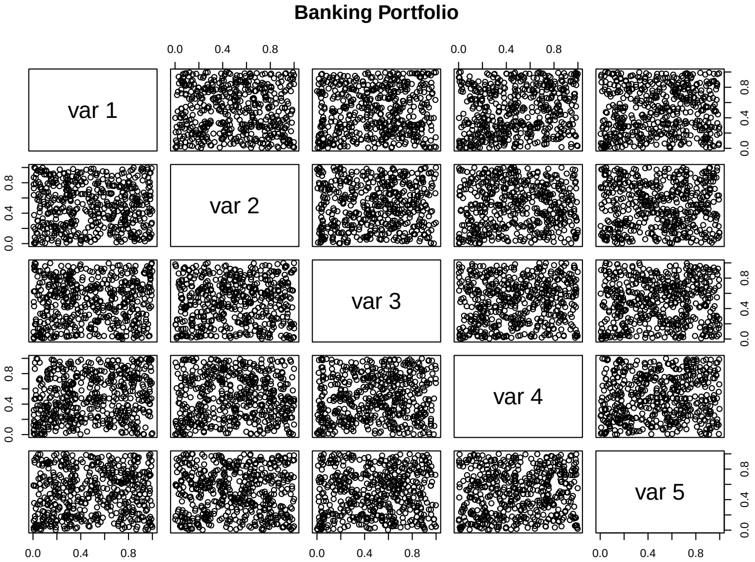
Copula Estimation

- Substantial improvement in log-likelihood
- Skewness parameter is warranted

	T-copula	Skew-T copula
nu	5.658878	5.9137642
gamma	NA	-0.2259792
log_lik	734.429860	851.7734165
AIC	-1256.859720	-1489.5468330
BIC	-843.470601	-1072.2578162

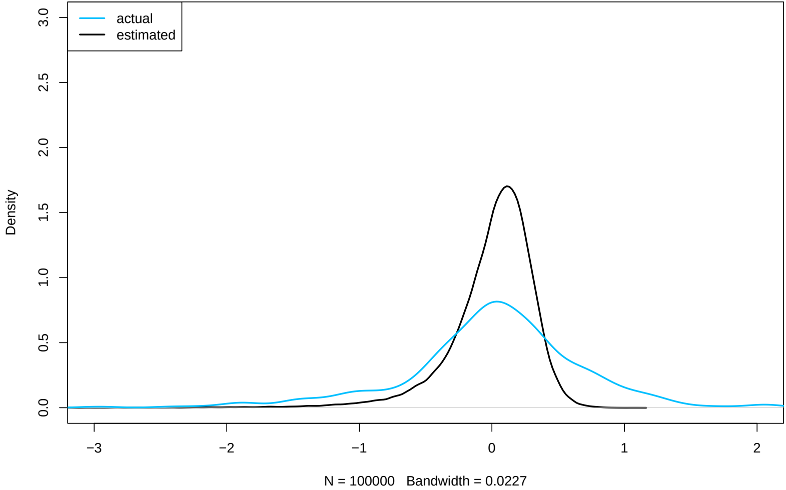
Application to Financial Data

Copula Estimation - copula



Application to Financial Data

Copula Simulation - Aggregate Loss



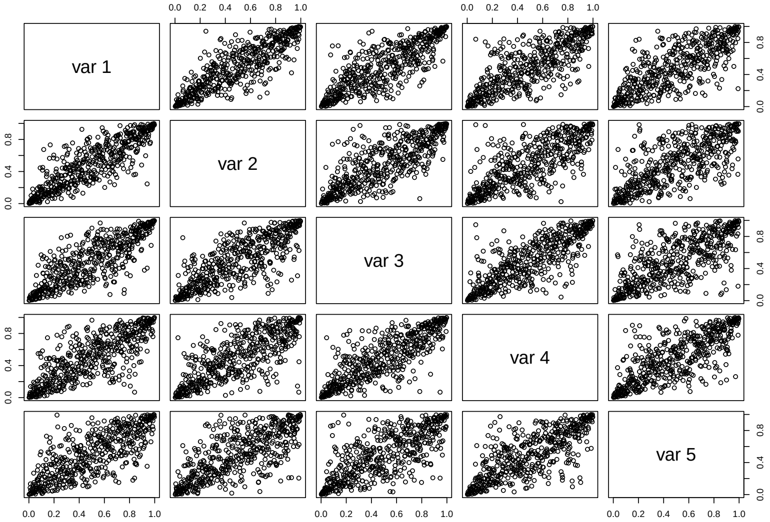
Application to Financial Data

In-sample Testing for Aggregate Weekly Loss

	99%	95%	90%	85%
% VaR	-1.19	-0.58	-0.37	-0.26
Empirical Violation Percentage	0.06	0.14	0.21	0.28

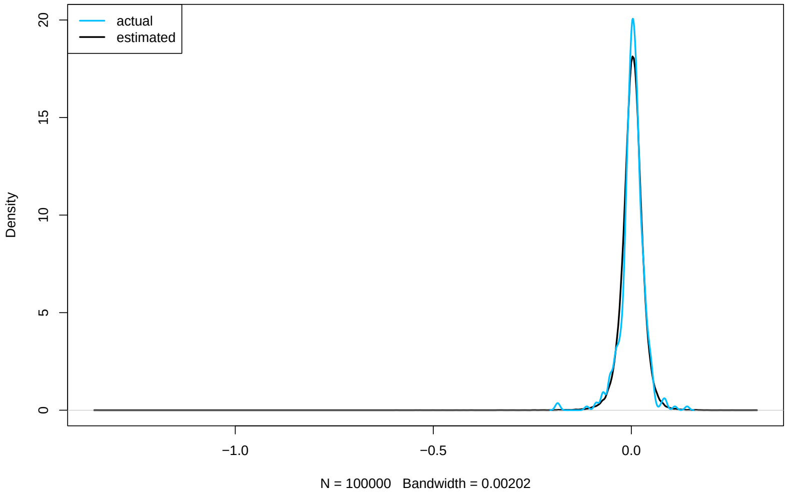
Application to Financial Data

One Stage EM Estimation



Application to Financial Data

One Stage EM Estimation



Application to Financial Data

In-sample Testing for Aggregate Weekly Loss

	99%	95%	90%	85%
% VaR	-0.08	-0.04	-0.03	-0.02
Empirical Violation Percentage	0.01	0.06	0.10	0.14

Possible Improvement

- Omission of skewness information on pseudo copula
- ν is close to boundary - group-T copula construction
- Semi-parametric estimation on the marginals
- Dynamic P&L and VaR modeling through t-Garch models
- More comprehensive backtesting methods - Risk Map

Reference

- McNeil, A. J., R. Frey, and P. Embrechts (2015) Quantitative Risk Management: Concepts, Techniques, and Tools, Princeton University Press, revised ed
- Demarta, S. and A. J. McNeil (2005) “The t copula and related copulas,” International Statistical Review, 73(1), 111–129.
- Toshinao Yoshiba Maximum likelihood estimation of skew- t copulas with its applications to stock returns May 2018 Journal of Statistical Computation and Simulation 88(2):1-18
- Colletaz, G., Hurlin, C. and Perignon, C. (2013). The risk map: A new tool for validating risk models. Journal of Banking and Finance, 37, 3843-3854