

SSM-*tutorial*

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Organized in collaboration with Institut Pasteur

Some context first



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examples	update README	3 days ago
json-schema	file structure of JS code	3 days ago
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src	test_Algorithms adapted to v 0.8.0	2 days ago
tests	tests for Cmodel, Ccoder and clar all pass	3 days ago
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SSM is free



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Code

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The core of SSM in C...



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... wrapped up in so that you don't have to write C



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SSM is still alpha



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The hope is to get **more reliable code** by coding collectively

State
Space
Model ?

Finite
dimensional
representation

$$X_t = \begin{bmatrix} X^{(1)} \\ \vdots \\ X^{(d)} \end{bmatrix}$$

$$\forall t, p(X_{t+dt} | X_{0:t}) = p(X_{t+dt} | X_t)$$

Markovian
dynamic

For this
tutorial

Bayesian inference
for compartmental
models in
epidemiology

Program

1. Your very first step
2. Random walk in a deterministic world
3. Model creation and model selection
4. Stochasticity and statistical inference
5. Going further

First steps

Terminal terminology

cd: change directory

ls: list files

open <file>: open file

| : pipe

cat: read the content

4 SSM objects:

ssm.json

data.csv

theta.json

prior.json

SSM: data.csv

```
"date","incidence_obs"  
"2009-06-01",39  
"2009-06-08",113  
"2009-06-15",317  
"2009-06-22",1233  
"2009-06-29",2075  
"2009-07-06",3739  
"2009-07-13",11520  
"2009-07-20",11120  
"2009-07-27",6480  
"2009-08-03",4000  
"2009-08-10",1840  
"2009-08-17",960  
"2009-08-24",640
```

data.csv

"date","incidence_obs"
"2009-06-01",39
"2009-06-08",113
"2009-06-15",317
"2009-06-22", ISO 8601 format
"2009-06-29",2075
"2009-07-06",3739
"2009-07-13",11520
"2009-07-20",11120
"2009-07-27",6480
"2009-08-03",4000
"2009-08-10",1840
"2009-08-17",960
"2009-08-24",640

```
"date","incidence_obs"  
"2009-06-01",39  
"2009-06-08",113  
"2009-06-15",317  
"2009-06-22",1233  
"2009-06-29",2075  
"2009-07-06",3739  
"2009-07-13",11520  
"2009-07-20",11120  
"2009-07-27",6480  
"2009-08-03",4000  
"2009-08-10",1840  
"2009-08-17",960  
"2009-08-24",640
```

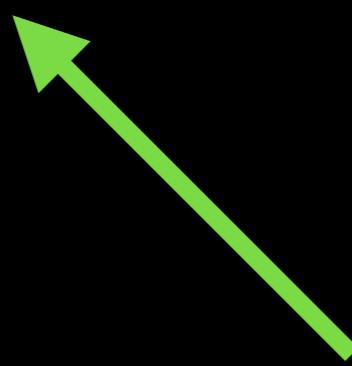
no spaces

prior.json

```
{  
  "name": "uniform",  
  "distributionParameter" : [  
    { "name" : "lower", "value" : 1e-07 },  
    { "name" : "upper", "value" : 1e-02 }  
  ]  
}
```

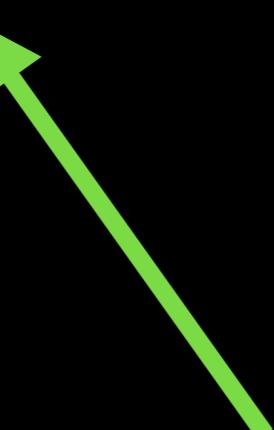
JSON: JavaScript Object Notation

```
{  
  "name": "uniform",  
  "distributionParameter" : [  
    { "name" : "lower", "value" : 1e-07 },  
    { "name" : "upper", "value" : 1e-02 }  
  ]  
}
```



Lists, between ['s:
ex: ['a',2,[],'jiji']

```
{  
  "name": "uniform",  
  "distributionParameter" : [  
    { "name" : "lower", "value" : 1e-07 },  
    { "name" : "upper", "value" : 1e-02 }  
  ]  
}
```



key-value objects, between {'s:
ex: { “key1”: “value1”, “key2”: 2,
 “key3”: [3],
 “key4”: { “four”: quatre }
 }

```
{  
  "name": "uniform"  
  "distributionParameter": [  
    { "name" : "lower", "value" : 1e-07 },  
    { "name" : "upper", "value" : 1e-02 }  
  ]  
}
```

Name of the distribution

Supported distributions: uniform, normal, dirac

Parameters of the distribution

```
{  
  "name": "uniform",  
  "distributionParameter": [  
    { "name" : "lower", "value" : 1e-07 }  
    { "name" : "upper", "value" : 1e-02 }  
  ]  
}
```



Supported distributions: uniform, normal, dirac
Supported parameters: lower, upper, mean, sd, value

```
{  
  "name": "normal",  
  "distributionParameter" : [  
    { "name" : "mean", "value" : 2 },  
    { "name" : "sd", "value" : 1 }  
  ]  
}
```

Supported distributions: uniform, normal, dirac

Supported parameters: lower, upper, mean, sd, value

```
{  
  "name": "dirac",  
  "distributionParameter" : [  
    { "value" : 10 }  
  ]  
}
```

Supported distributions: uniform, normal, dirac
Supported parameters: lower, upper, mean, sd, value

```
{  
  "name": "normal",  
  "distributionParameter" : [  
    { "name" : "mean", "value" : 2 },  
    { "name" : "sd", "value" : 1 },  
    { "name" : "lower", "value" : 0 }  
  ]  
}
```

Supported distributions: uniform, normal, dirac

Supported parameters: lower, upper, mean, sd, value

theta.json

part 1

```
{  
  "name": "values",  
  "data": {  
    "prop_S_london": 0.835,  
    "prop_E_london": 0.00057,  
    "prop_I_london": 0.000685705,  
    "beta_t": 1.113,  
    "v_inv": 1.139,  
    "k_inv": 1.609  
  }  
}
```

Parameter
vector
 θ

part 2

```
{  
  "name": "covariance",  
  "data": {  
    "prop_S_london": {"prop_S_london": 0.02} ,  
    "prop_E_london": {"prop_E_london": 0.02} ,  
    "prop_I_london": {"prop_I_london": 0.02} ,  
    "beta_t": {"beta_t": 0.02},  
    "v_inv": {"v_inv": 0.02},  
    "k_inv": {"k_inv": 0.02}  
  }  
}
```



Covariance
matrix

you shouldn't have to worry about that one

ssm.json

5 components

1. Load data
2. Load priors
3. Define population structure
4. Define reactions
5. Define observations

5 components

$$(y_i)$$

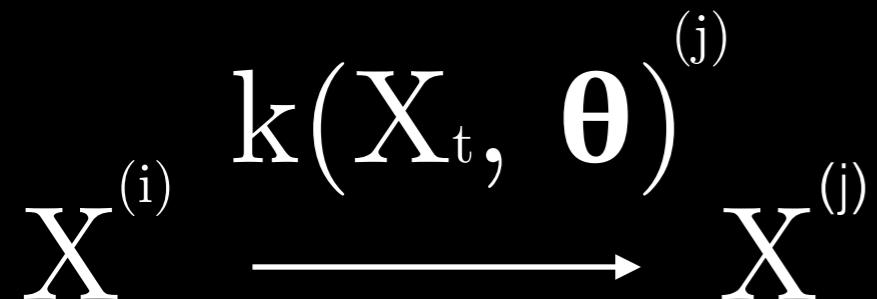
1. Load data

$$p(\theta)$$

2. Load priors

$$S_t + I_t + R_t = N_t$$

3. Define population structure



4. Define reactions

$$p(y_i | X_t)$$

5. Define observations

1. Load data

$$(y_i)$$

```
"data": [
  {
    "name": "incidence_obs",
    "require": {
      "path": "../data/data-H1N1-1wave.csv",
      "fields": ["date", "incidence_obs"]
    }
  }
],
```

2. Load priors

$$p(\theta)$$

Quantity used
in the model

```
"inputs": [  
  {  
    "name": "S_london",  
    "transformation": "prop_S_london*N_london",  
    "to_resource": "S_london/N_london",  
    "description": "Number of susceptible individuals",  
    "require": {  
      "path": "../priors/prop_S_london.json",  
      "name": "prop_S_london"  
    }  
  },
```

Quantity on which
prior is defined

2. Load priors

$$p(\theta)$$

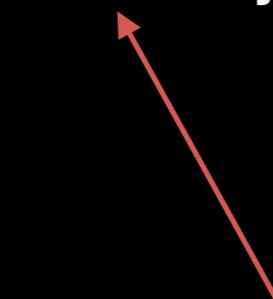
Transformation

```
"inputs": [  
  {  
    "name": "S_london",  
    "transformation": "prop_S_london*N_london",  
    "to_resource": "S_london/N_london",  
    "description": "Number of susceptible individuals",  
    "require": {  
      "path": "../priors/prop_S_london.json",  
      "name": "prop_S_london"  
    }  
  },
```

2. Load priors

$$p(\boldsymbol{\theta})$$

```
"inputs": [
  {
    "name": "S_london",
    "transformation": "prop_S_london*N_london",
    "to_resource": "S_london/N_london",
    "description": "Number of susceptible individuals",
    "require": {
      "path": "../priors/prop_S_london.json",
      "name": "prop_S_london"
    }
  },
]
```



Relative path

3. Define population structure

```
"populations": [  
    {  
        "name": "london",  
        "composition": ["S_london", "I_london"],  
        "remainder": {  
            "name": "R_london",  
            "pop_size": "N_london"  
        }  
    }  
],
```



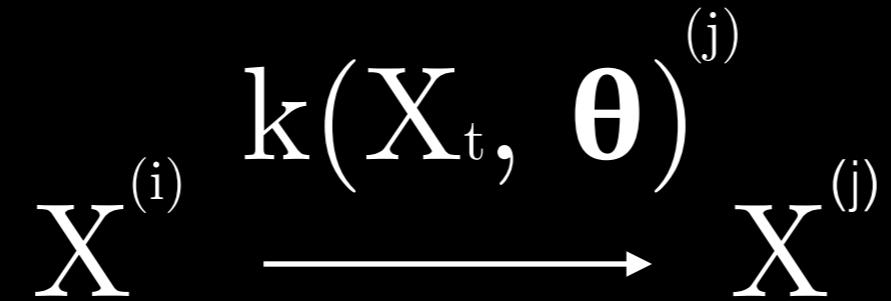
Constant population

(R becomes a dummy variable)

3. Define population structure

```
"populations": [  
    {  
        "name": "london",  
        "composition": ["S_london", "I_london", "R_london"]  
    }  
,
```

4. Define reactions



"reactions": [

{

 "from": "S_london", "to": "I_london",

 "rate": "beta_t/N_london*I_london",

 "description": "infection", "keywords": ["transmission"],

 "accumulators": ["incidence"]

},

{

 "from": "I_london", "to": "R_london", "rate": "v",

 "description": "recovery"

}

$$\text{incidence} = \int k(X_t, \theta) X^{(i)} dt$$

],

5. Define observations

$$p(y_i | X_t) = \mathcal{N}\{h(X_t, \theta); \mu, \sigma\}$$

```
"observations": [
  {
    "name": "incidence_obs",
    "start": "2009-05-01",
    "distribution": "discretized_normal",
    "mean": "rep * incidence",
    "sd": "sqrt(rep * ( 1.0 - rep ) * incidence )"
  }
]
```

The diagram illustrates the mapping of variables from the mathematical formula to the JSON configuration. A red arrow points from the variable y to the key "name": "incidence_obs". Another red arrow points from the function $h(X_t)$ to the key "mean": "rep * incidence".

You've made it

ssm.json

data.csv

theta.json

prior.json

Your first SSM command

Compile: `ssm`

Enter bin: `cd bin`

Look around: `ls`

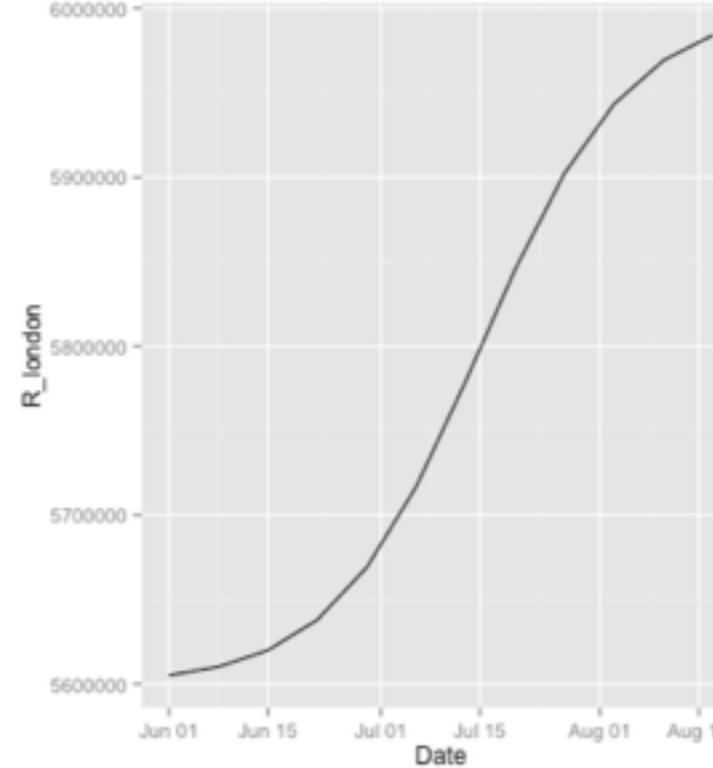
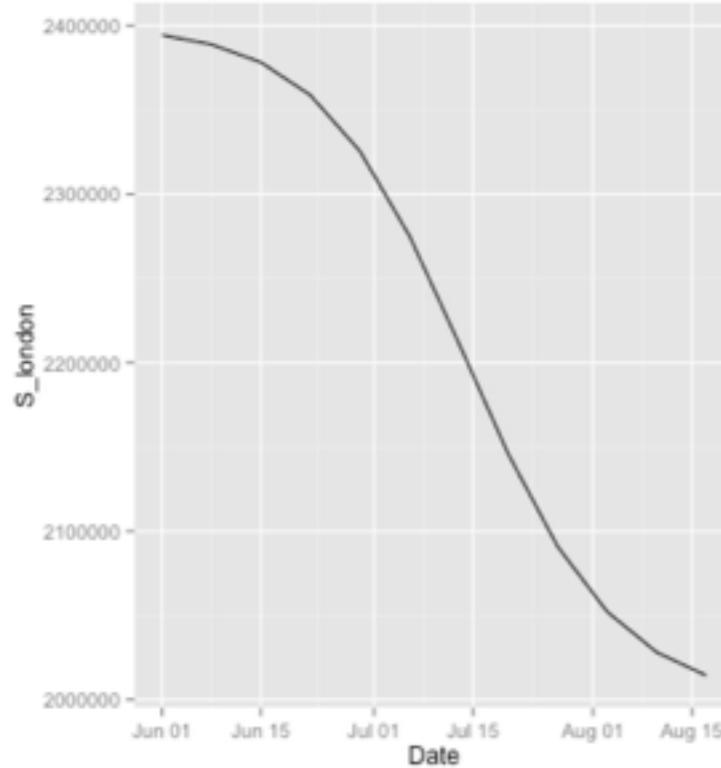
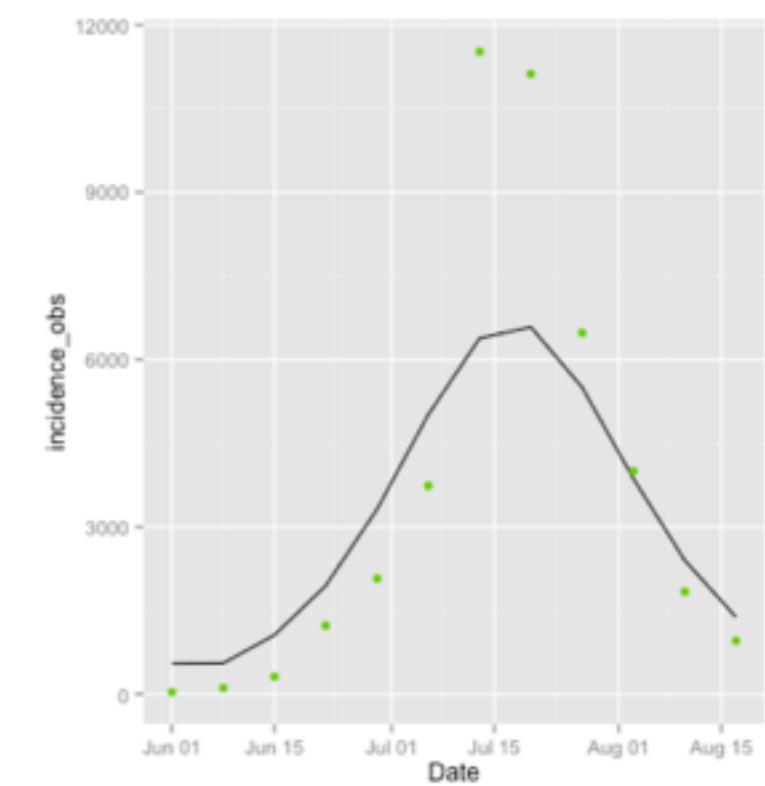
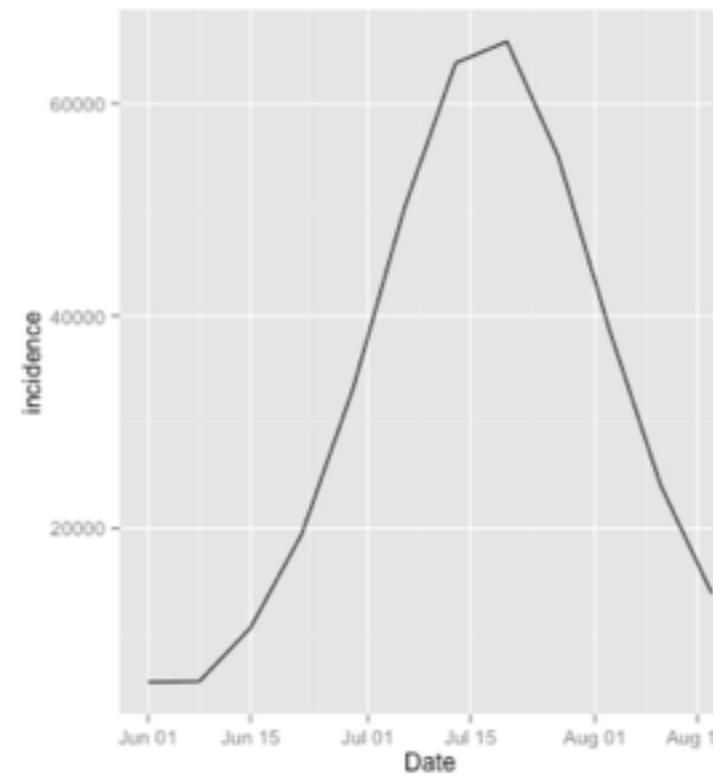
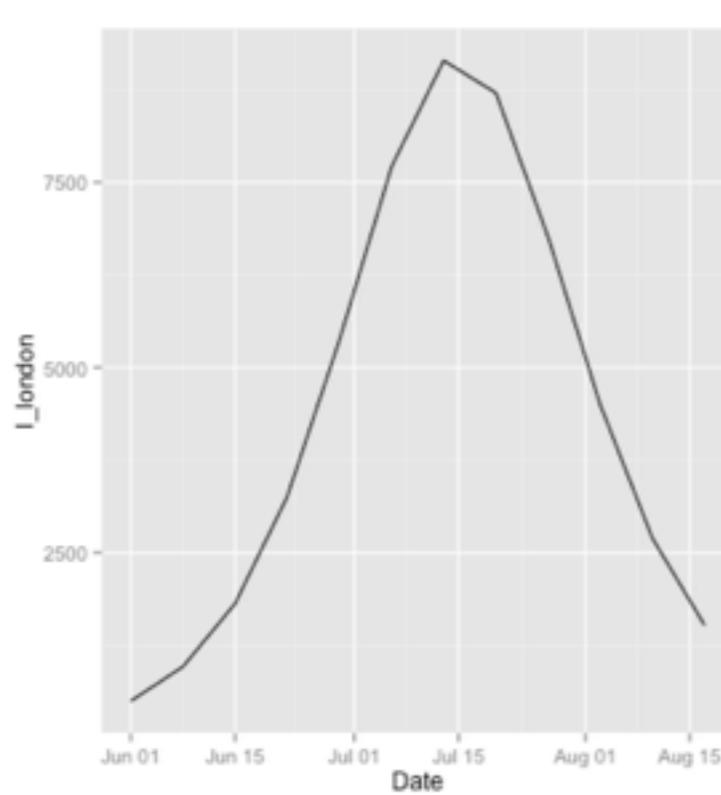
In doubt: `./simul -h`

Simulate:

```
cat ./theta.json | ./simul --traj
```

Check out the output: `open X_0.csv`

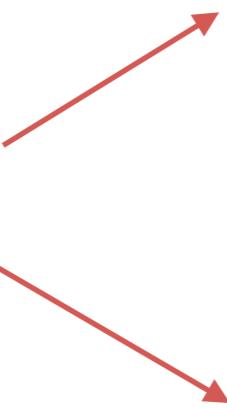
Plot the output: open `Tutorial.R` with R or RStudio



Let's talk
about inference!

Random walk
in a
deterministic
world

Explore $p(X_t, \theta|y)$



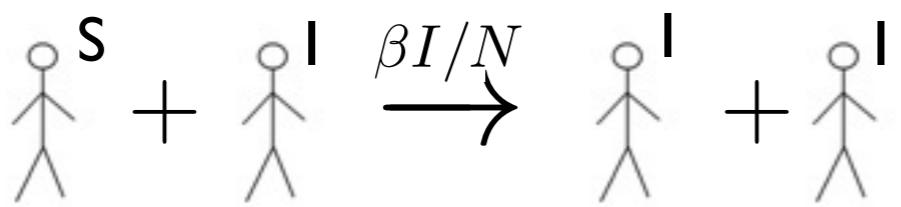
Estimate parameters
 $p(\theta|y) = \int p(X_t, \theta|y) dX_t$

Reconstruct past scenarios
 $p(X_t|y) = \int p(X_t, \theta|y) d\theta$

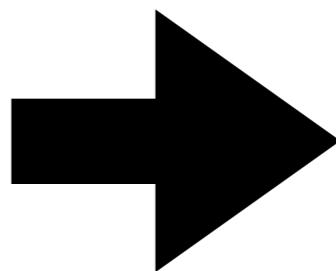
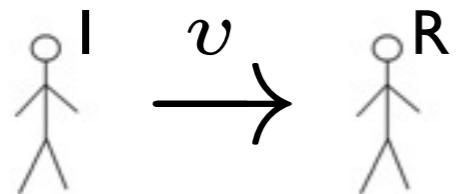
Derive model choice indicators: $p(M1|y) > p(M2|y)$

Objectives of Bayesian inference

infection:



recovery:



$$\frac{dS}{dt} = -\beta I/N \times S$$

$$\frac{dI}{dt} = \beta I/N \times S - vI$$

$$\frac{dR}{dt} = vI$$

In a
nice deterministic world....

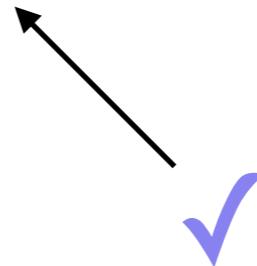
Good approximation using
Euler or Runge-Kutta methods

Likelihood

$$p(y|\theta) = \prod p(y_i|X_{t_i}, \theta)p(X_{t_i}|\theta)$$



Model



In a
nice deterministic world....

Objective



$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

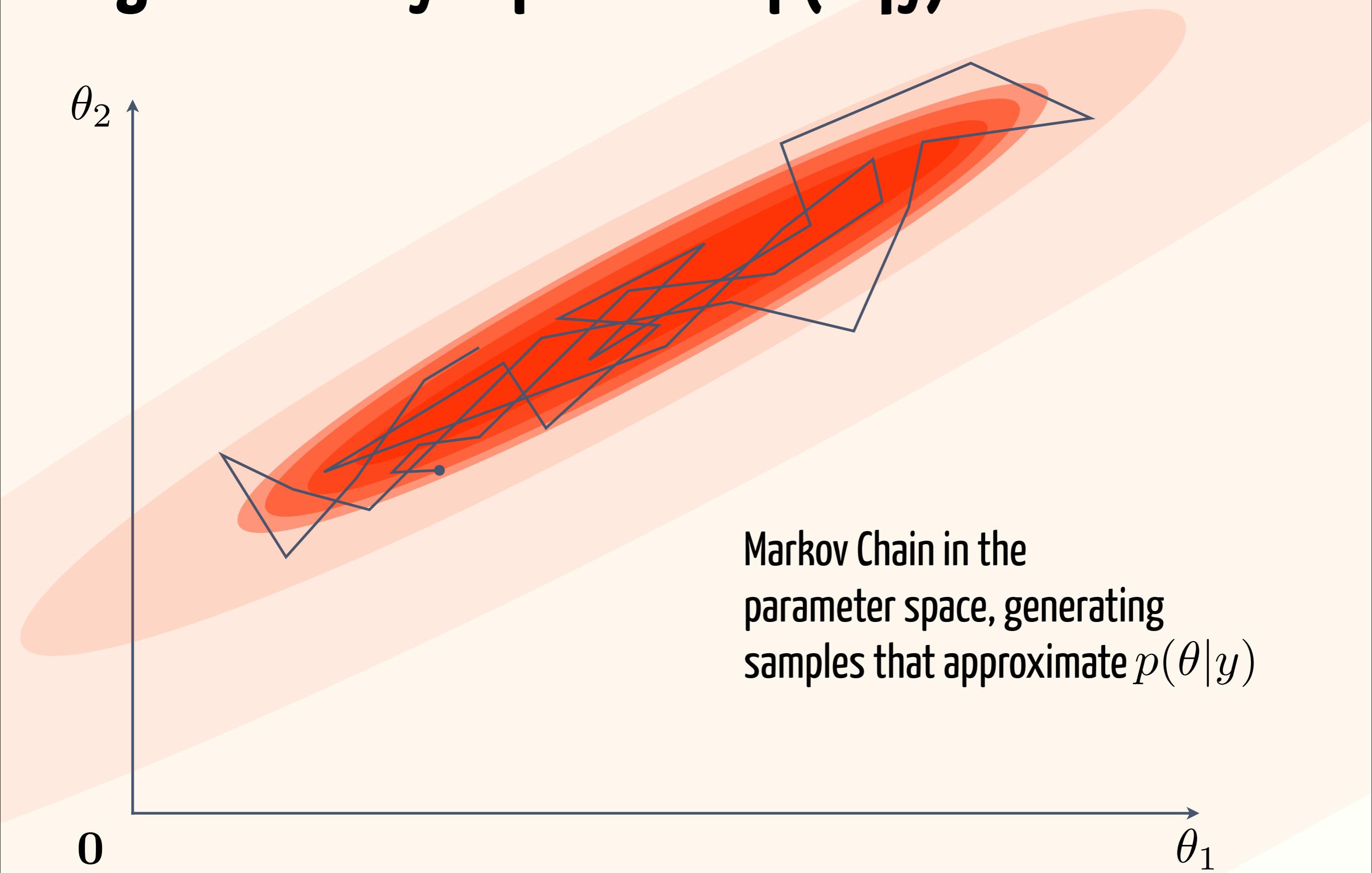
Can be computed *



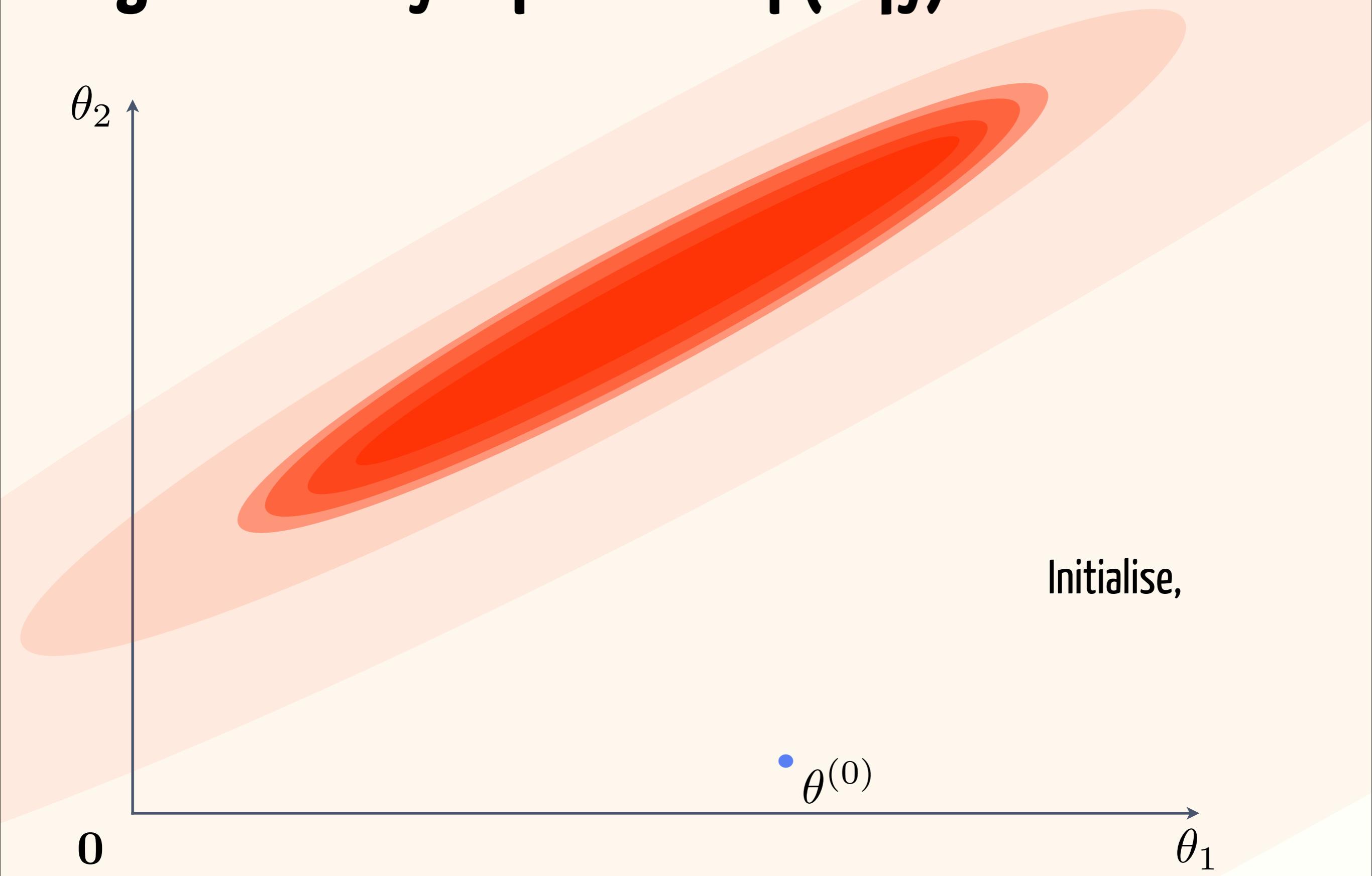
A typical
MCMC application problem

* with arbitrary
precision

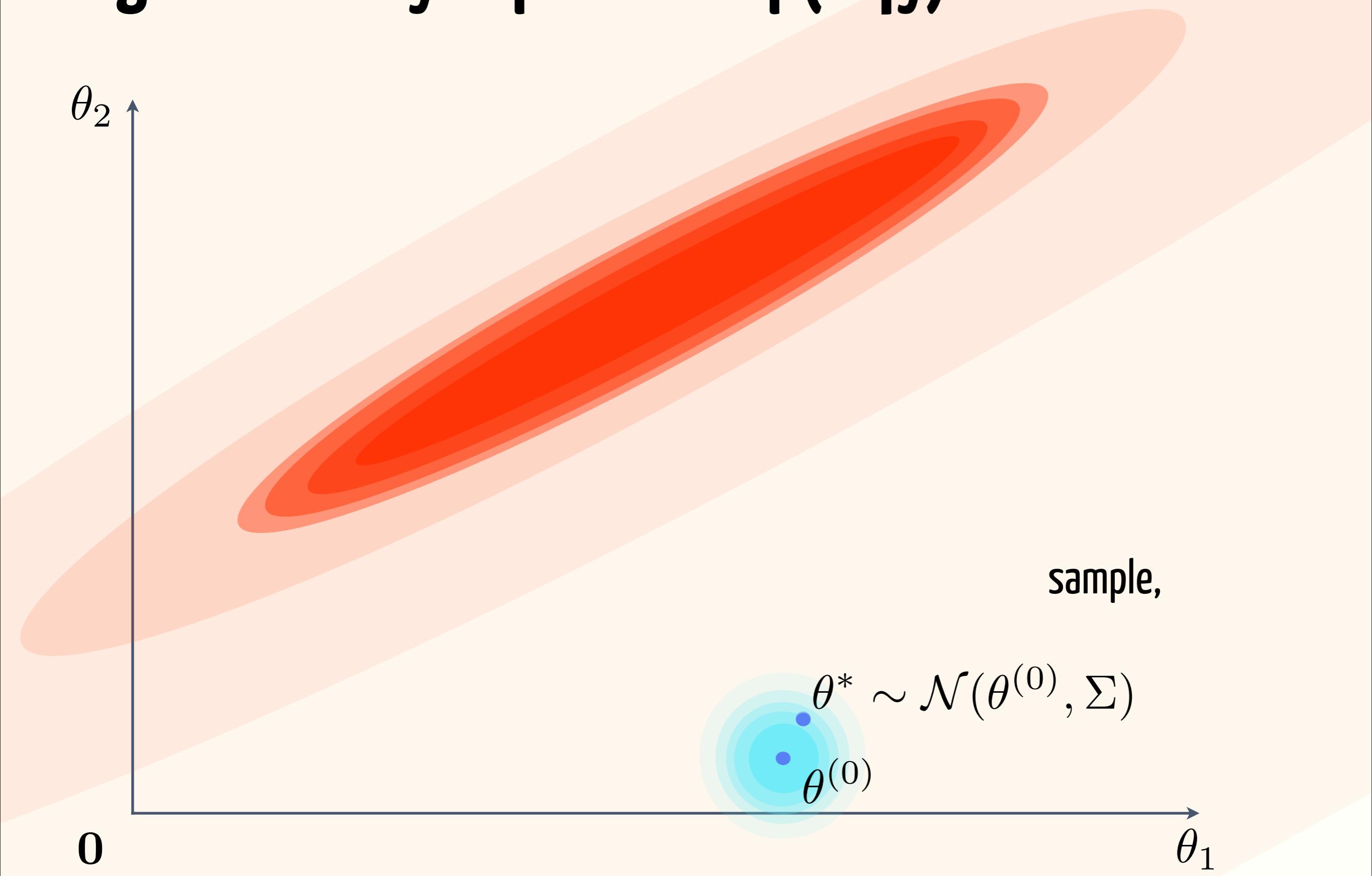
Marginal density exploration: $p(\theta|y)$



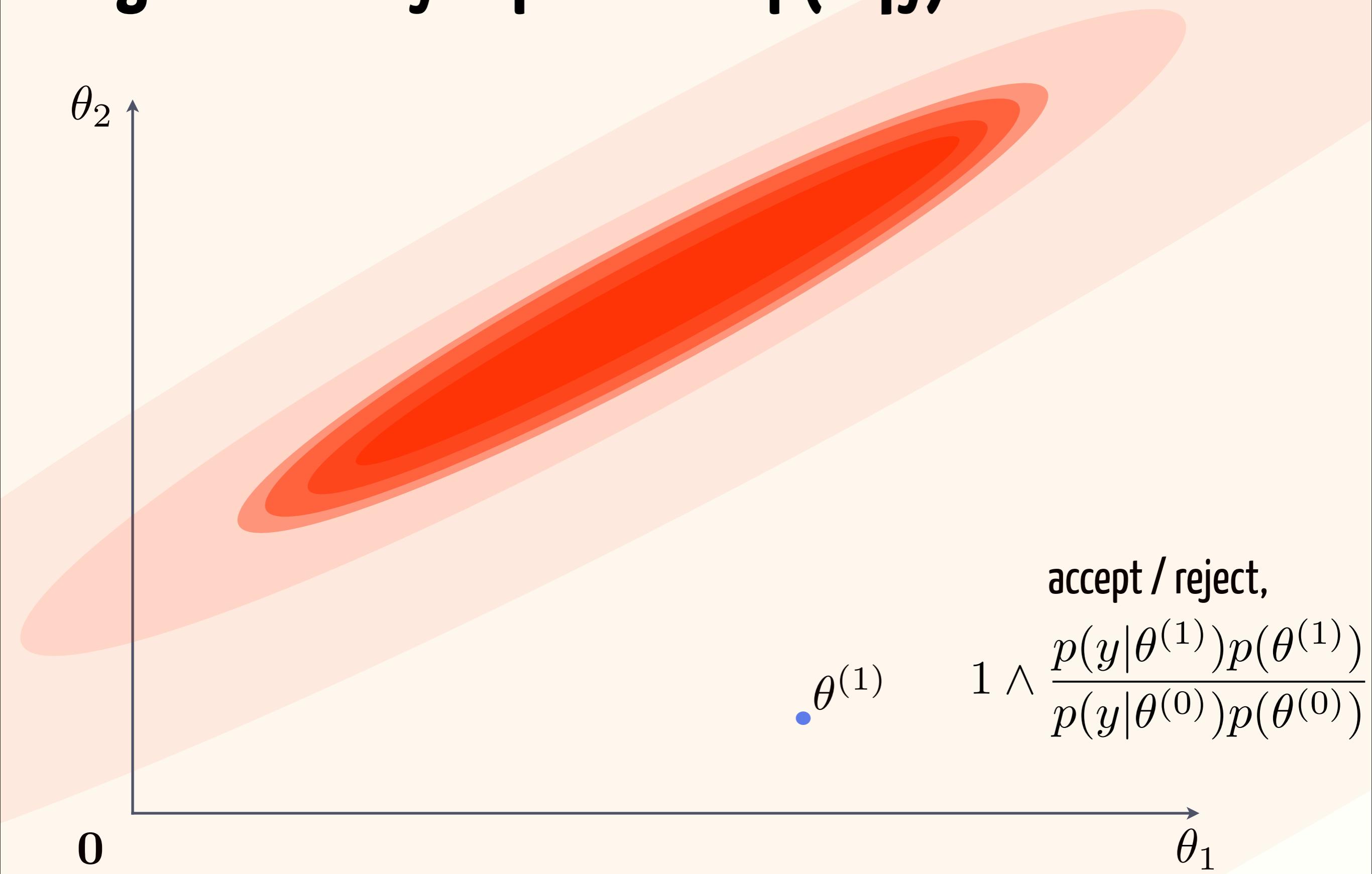
Marginal density exploration: $p(\theta|y)$



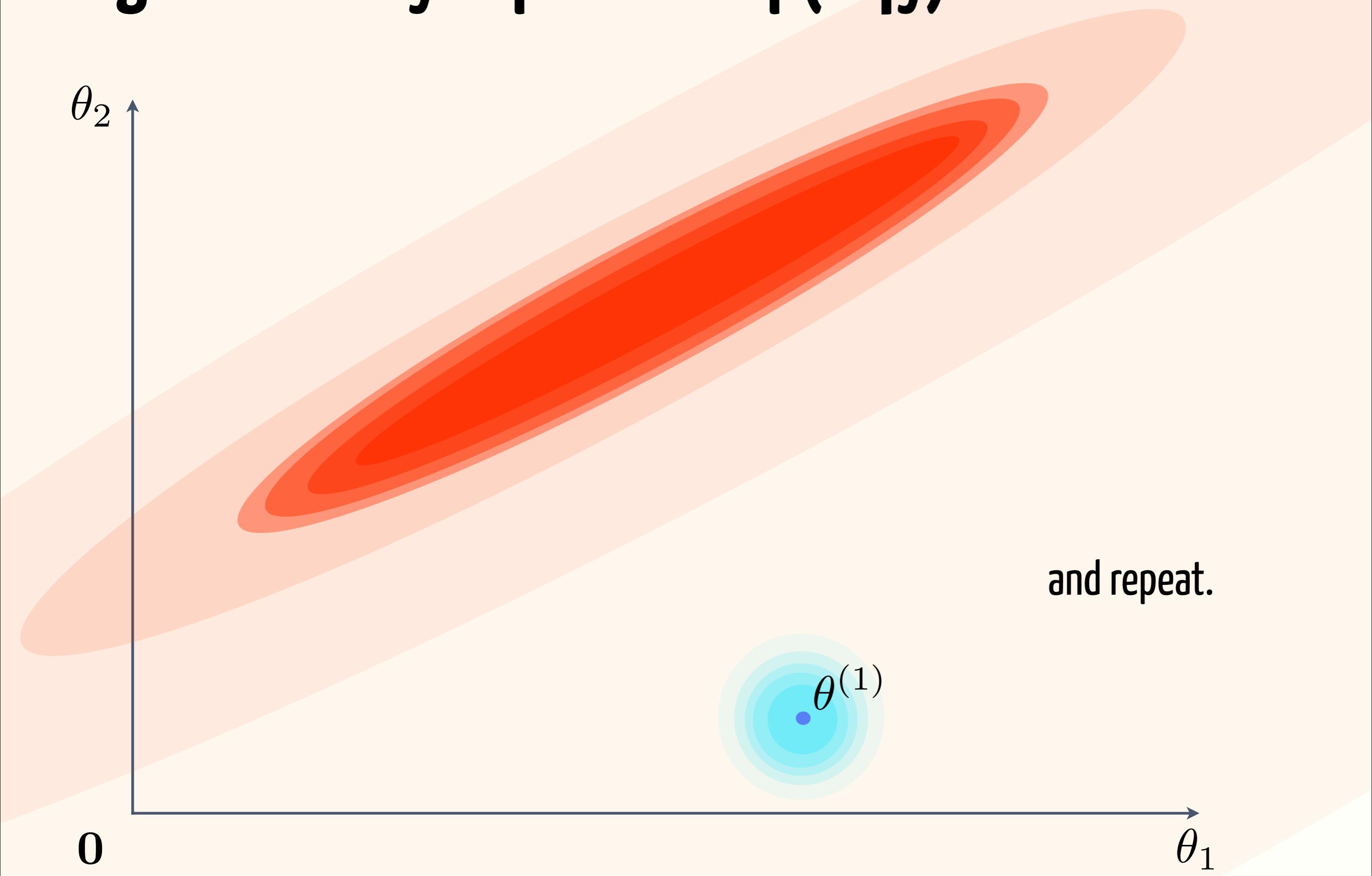
Marginal density exploration: $p(\theta|y)$

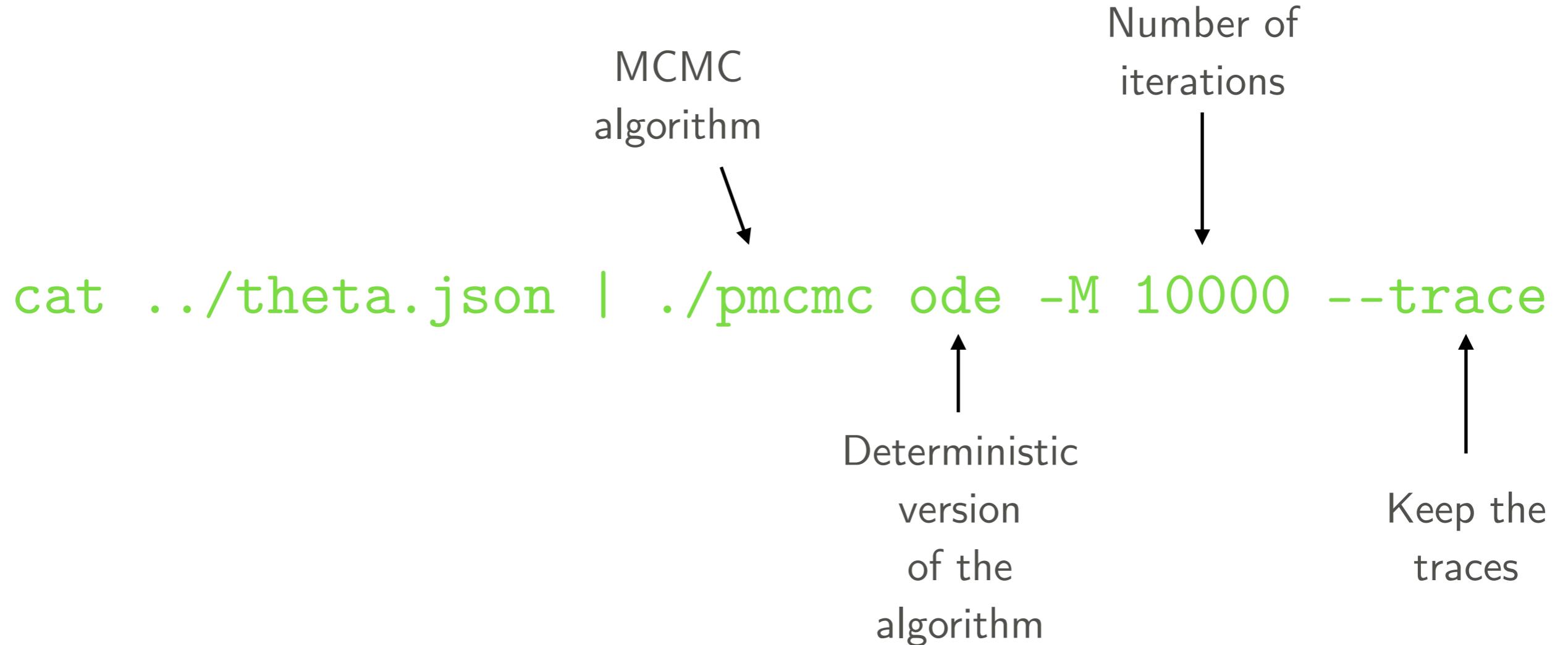


Marginal density exploration: $p(\theta|y)$



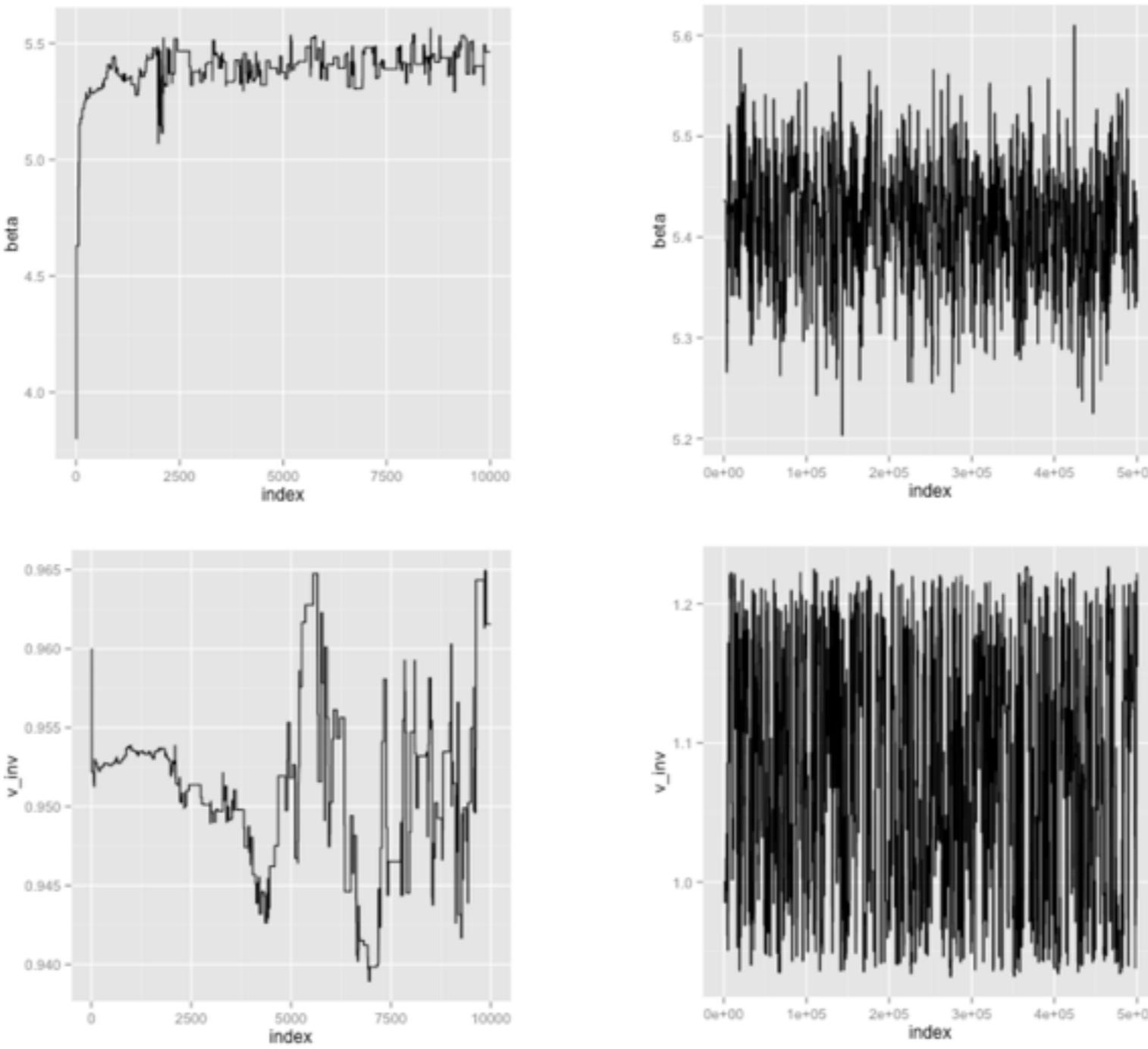
Marginal density exploration: $p(\theta|y)$





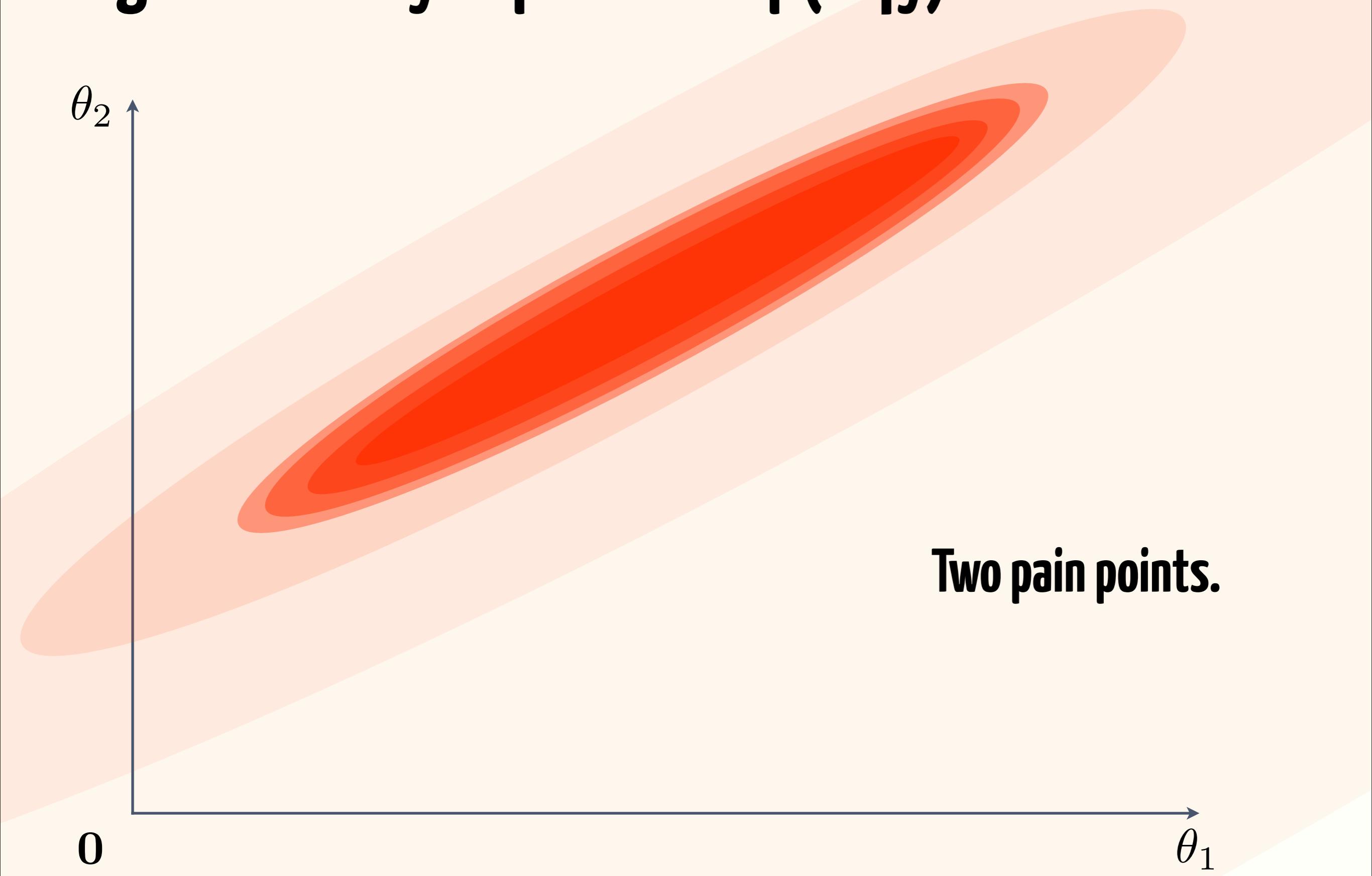
Run 10000 iterations

And plot the traces of the MCMC

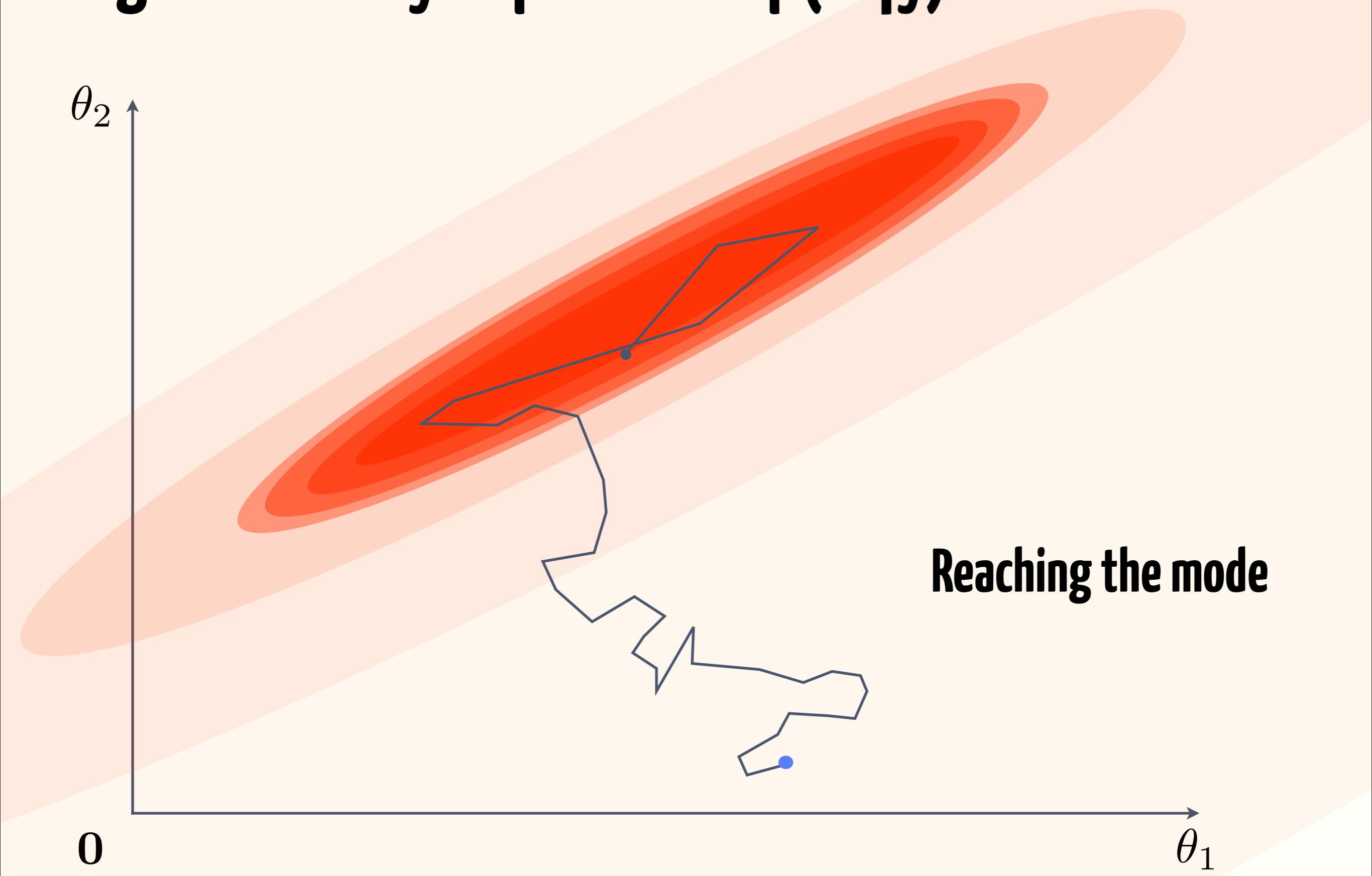


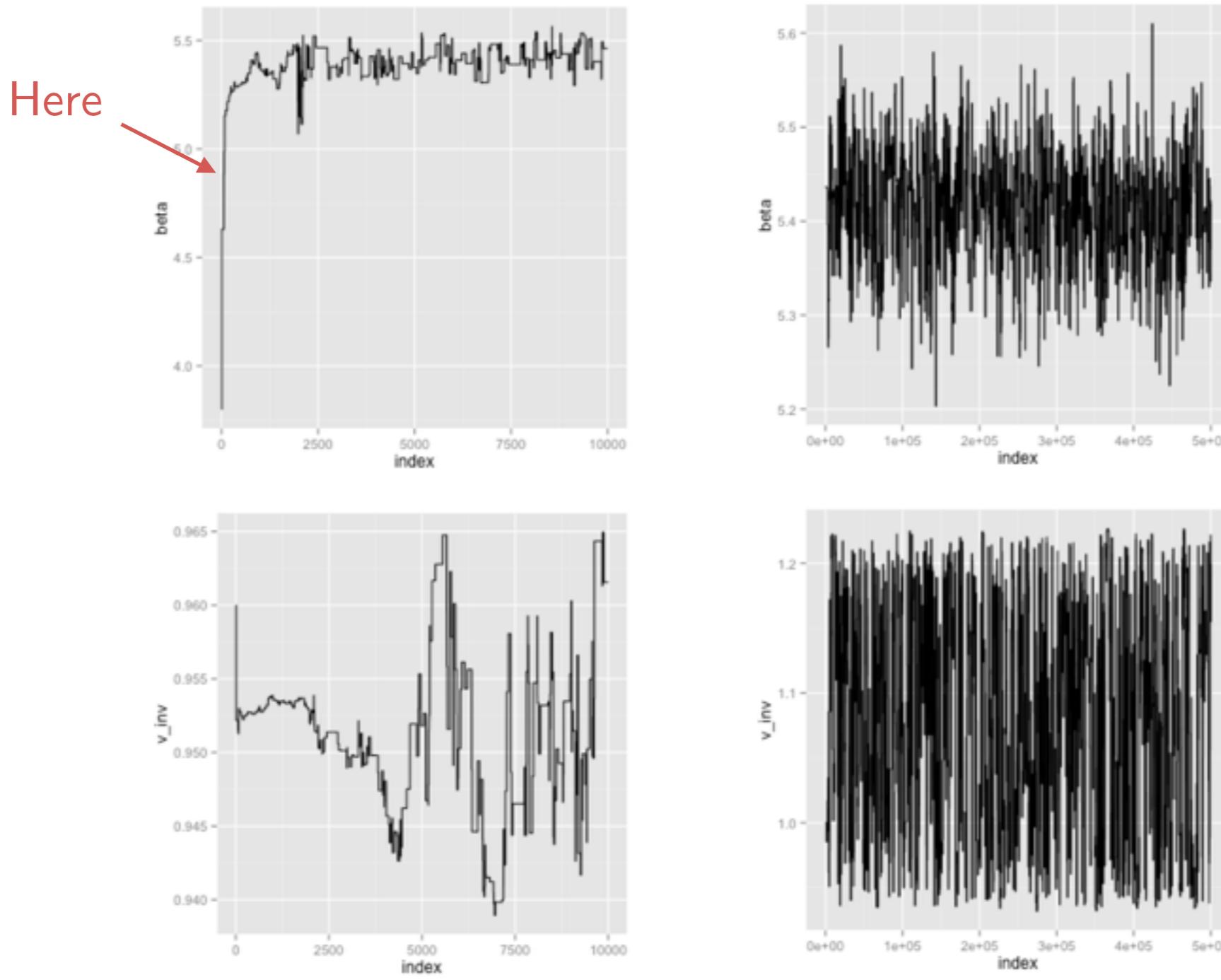
**Ideally:
Independent and identically distributed samples**

Marginal density exploration: $p(\theta|y)$



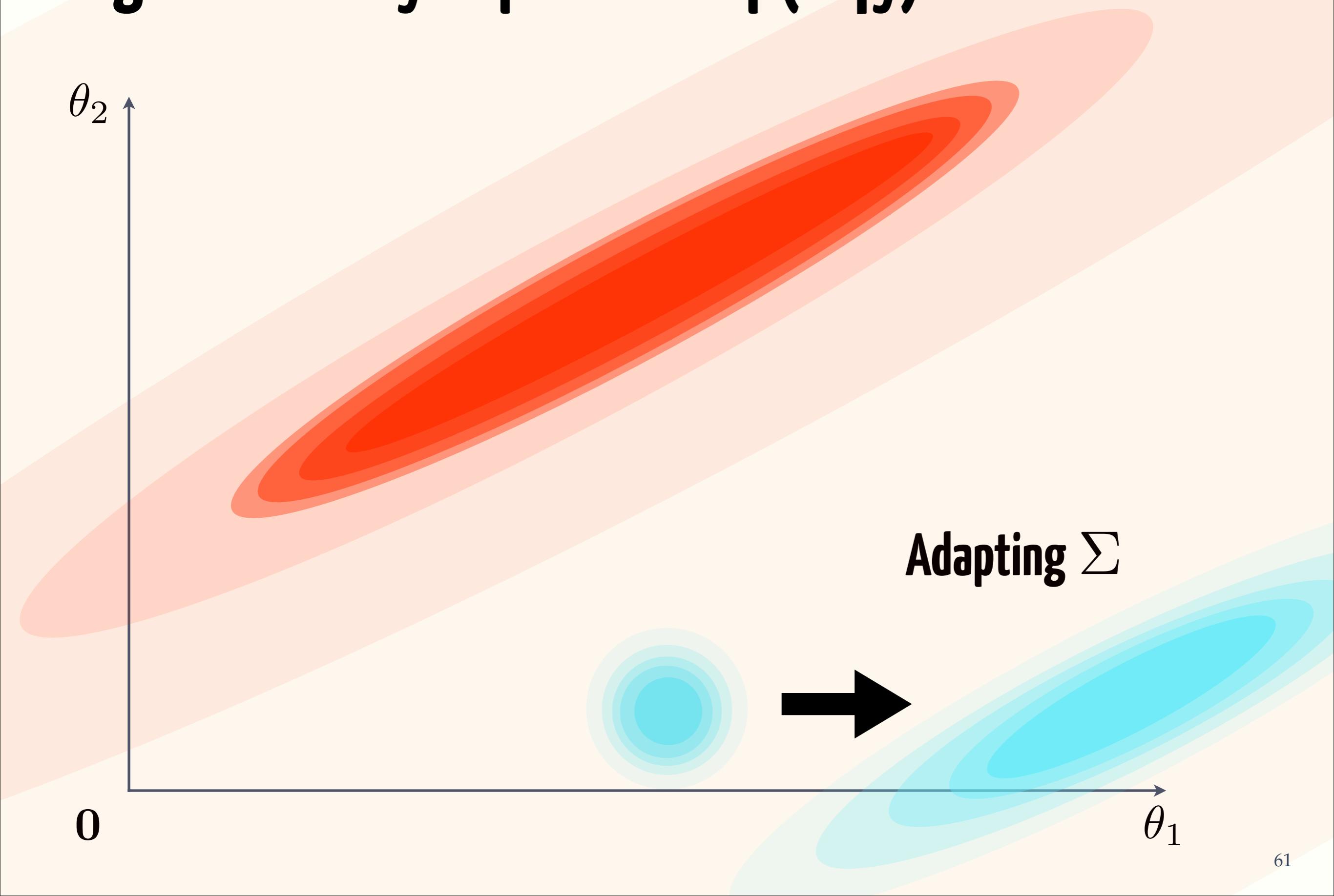
Marginal density exploration: $p(\theta|y)$





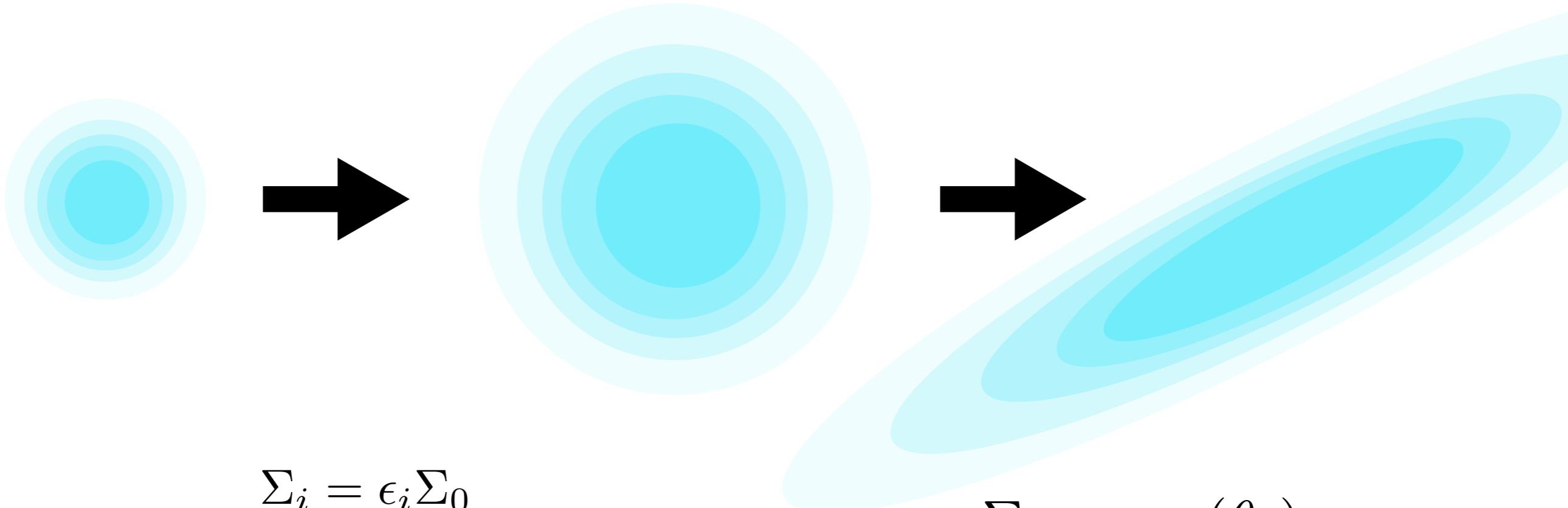
Ideally:
Independent and identically distributed samples

Marginal density exploration: $p(\theta|y)$



Size

Shape

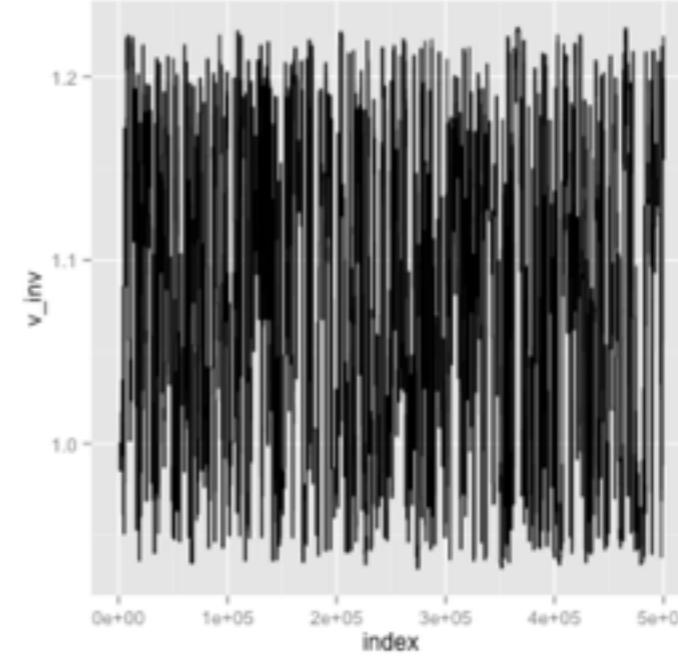
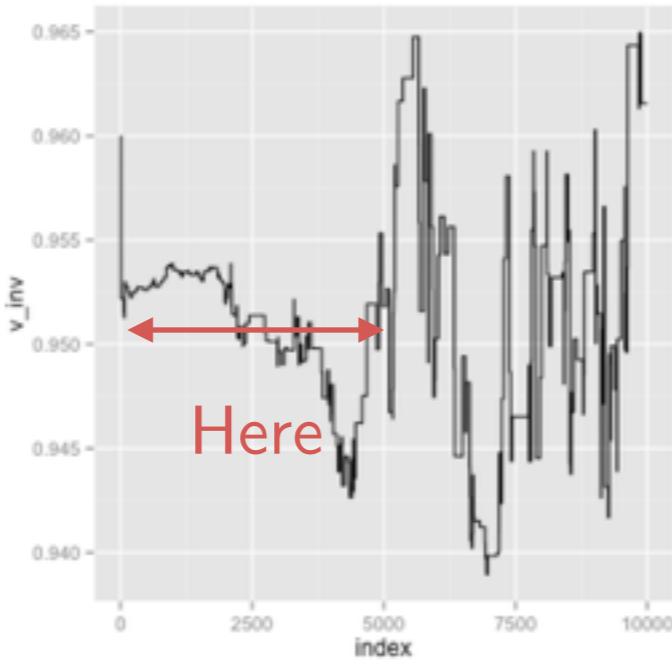
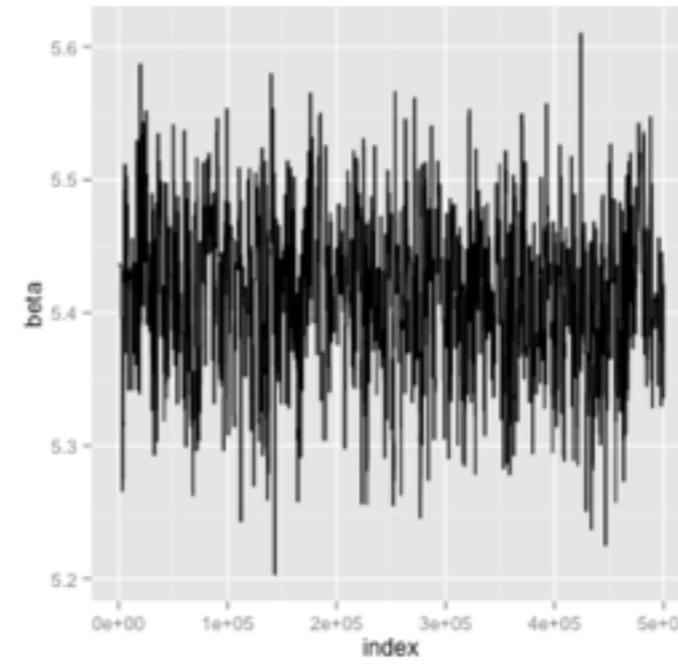
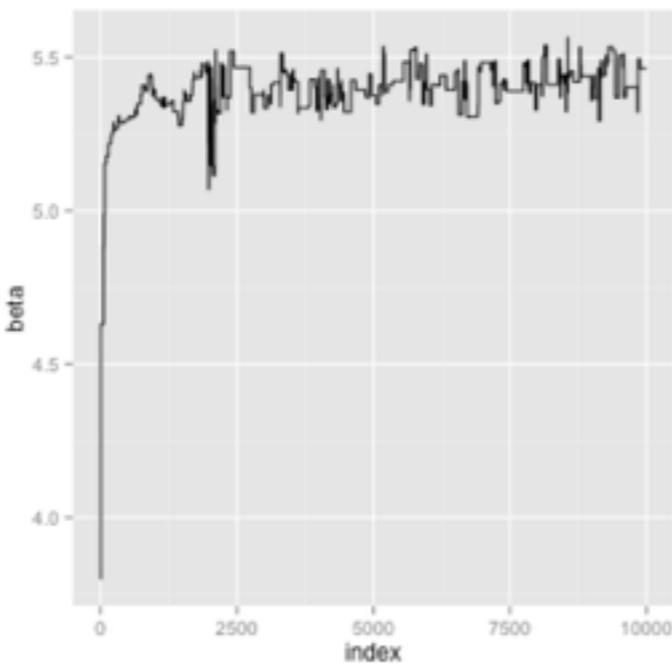


$$\Sigma_i = \epsilon_i \Sigma_0$$

$$\log(\epsilon_{i+1}) = \log(\epsilon_i) + (\text{AccRate} - 0.23)^i$$

$$\Sigma_i = \text{cov}(\theta_j)_{j \leq i}$$

Covariance adaptation in 2 phases



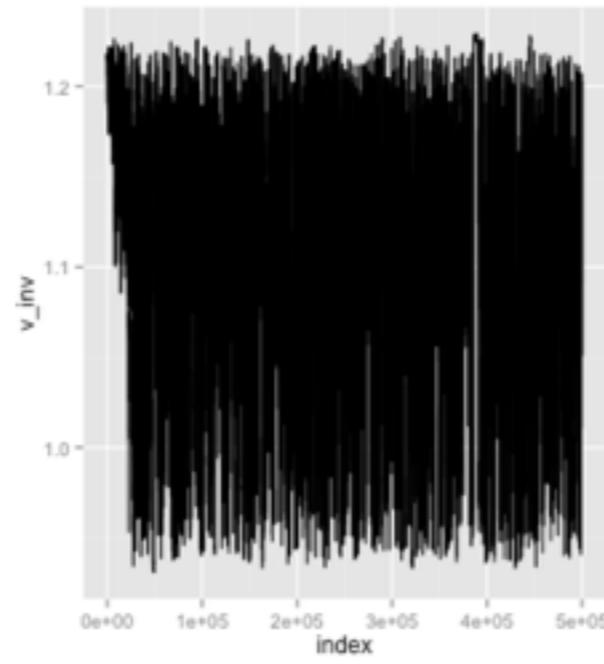
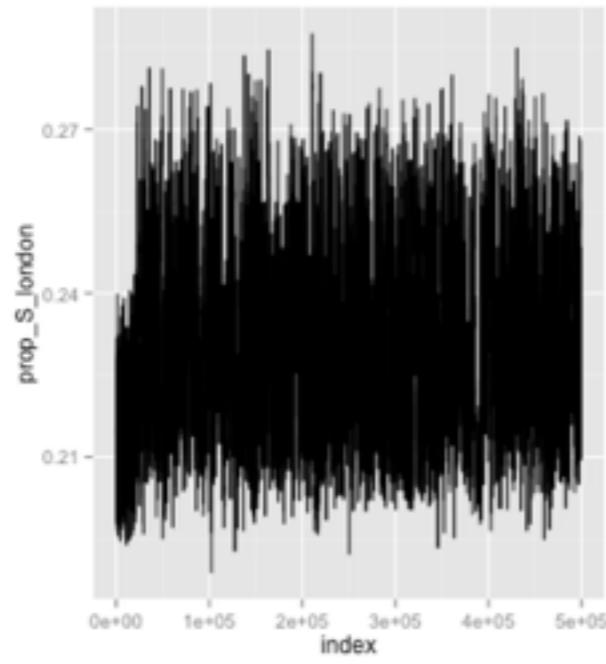
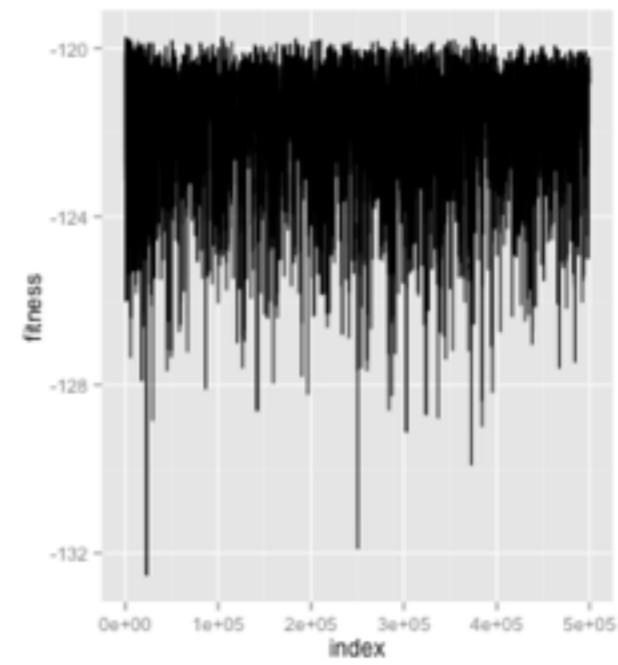
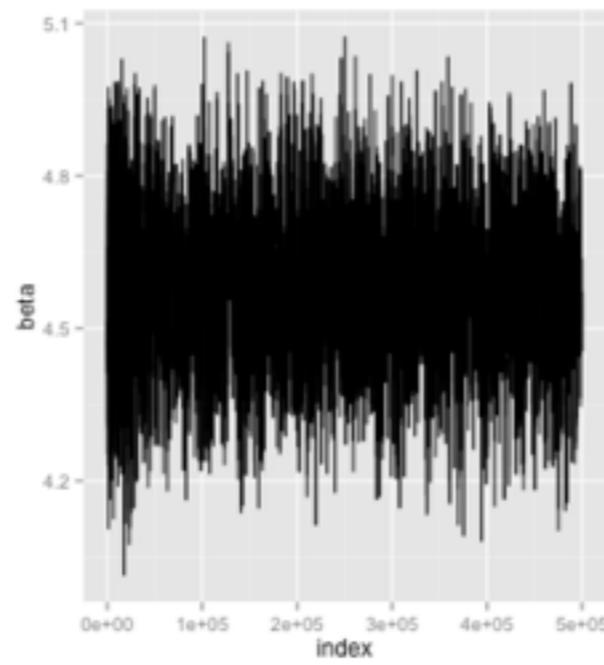
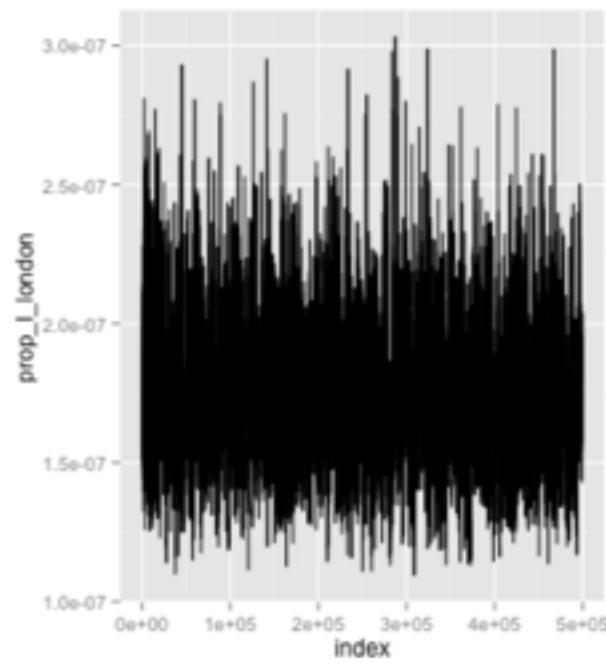
Ideally:
Independent and identically distributed samples

The output
theta is *piped*
into the following
algorithm

```
cat ./theta.json | ./pmcmc ode -M 500000 | ./pmcmc  
ode -M 500000 -trace
```



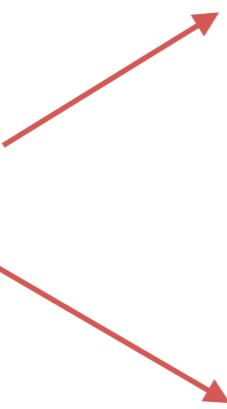
**Chaining
algorithms like playing with duplo blocks**



**Ideally:
Independent and identically distributed samples**

Model creation
and
model selection

Explore $p(X_t, \theta|y)$

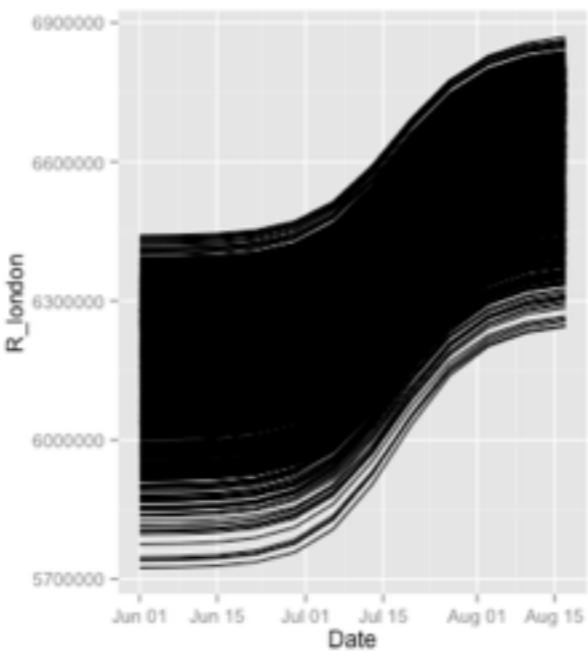
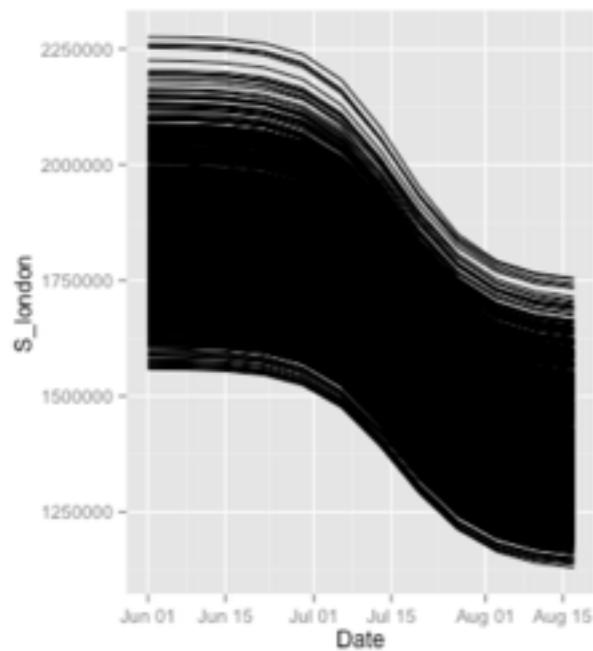
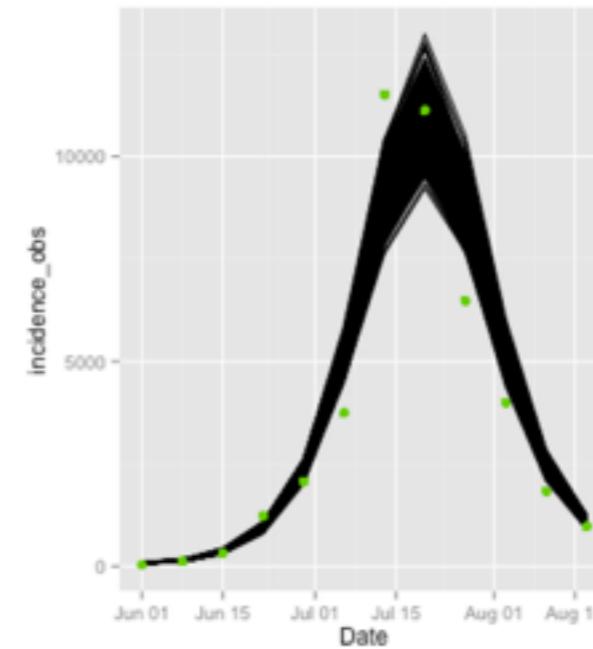
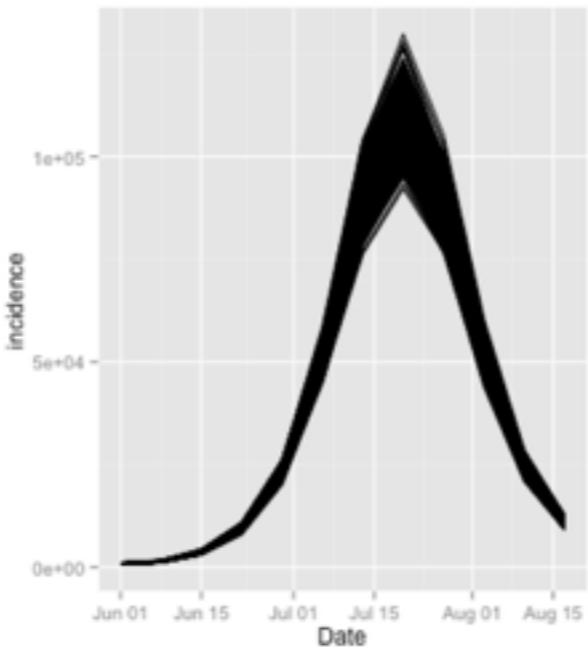
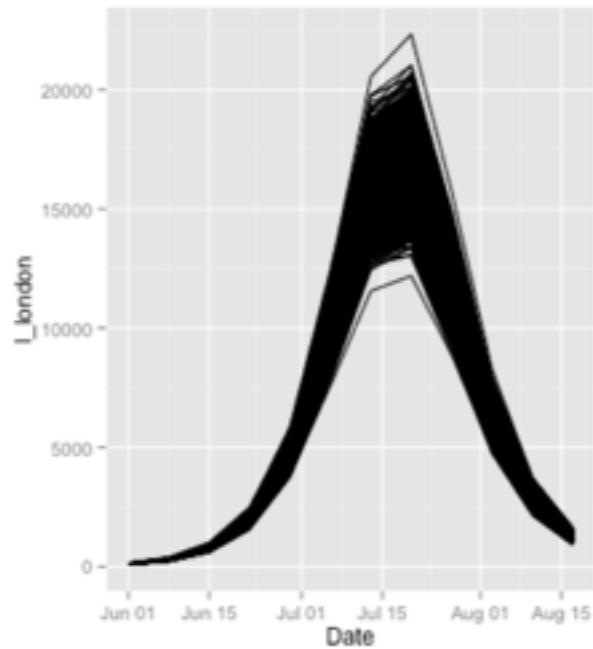


Estimate parameters
 $p(\theta|y) = \int p(X_t, \theta|y) dX_t$

Reconstruct past scenarios
 $p(X_t|y) = \int p(X_t, \theta|y) d\theta$

Derive model choice indicators: $p(M1|y) > p(M2|y)$

Objectives of Bayesian inference



Can we improve the SIR fit?

What about...

...holidays?

...incubation period?

Deviance Information Criterion

Deviance

$$D(\theta) = -2\log(p(y|\theta)) + C$$

$$p_D = \frac{1}{2}\widehat{\text{var}}(D(\theta))$$

Proxy of the number
of independent parameters

$$DIC = D(\bar{\theta}) + 2p_D$$

A penalized likelihood
criterion

Comparing models

	SIR	SEIR	SIR-holidays
DIC	237		

Build your own SEIR model

TODO:

- **load prior**
- **define additional reaction**
- **modify theta**
- **compile**
- **run**

```
cat theta.json | ./pmcmc ode -M 500000 | ./pmcmc ode -M 500000 --trace
```

What about...

	SIR	SEIR	SIR-holidays
DIC	237	241	

SIR-holidays

TODO:

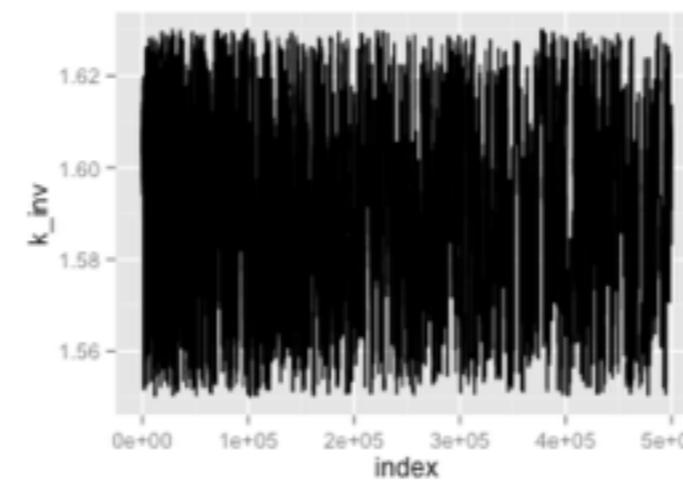
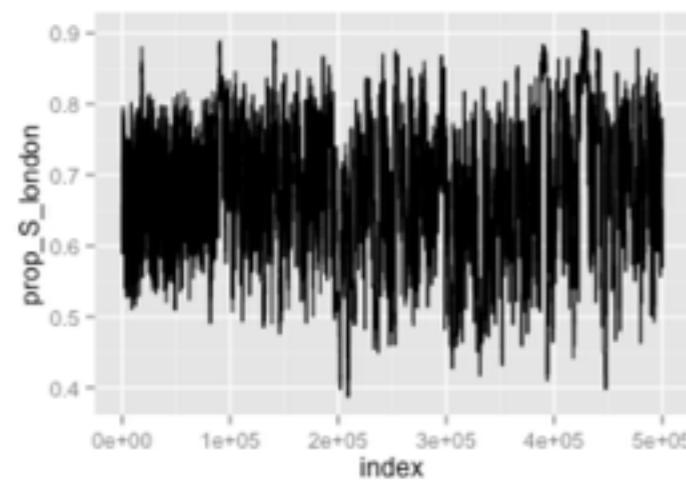
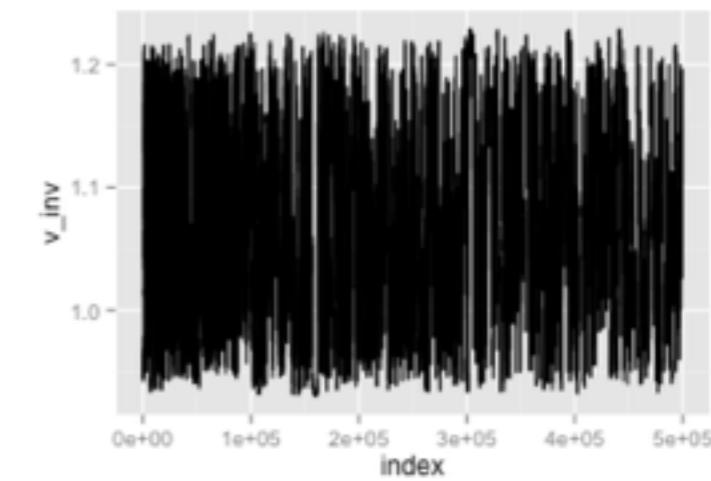
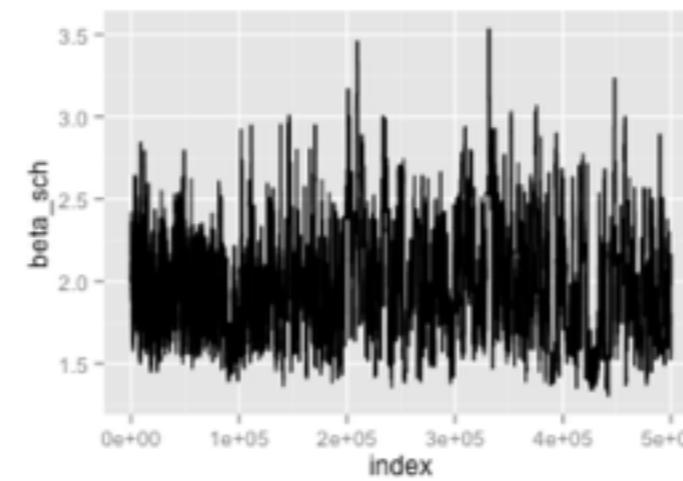
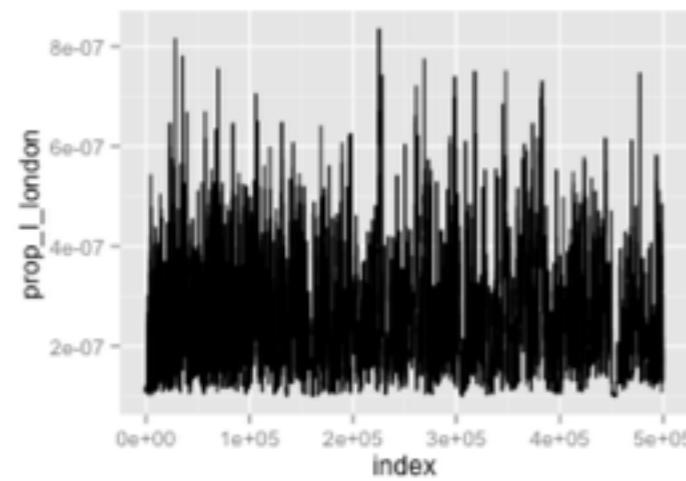
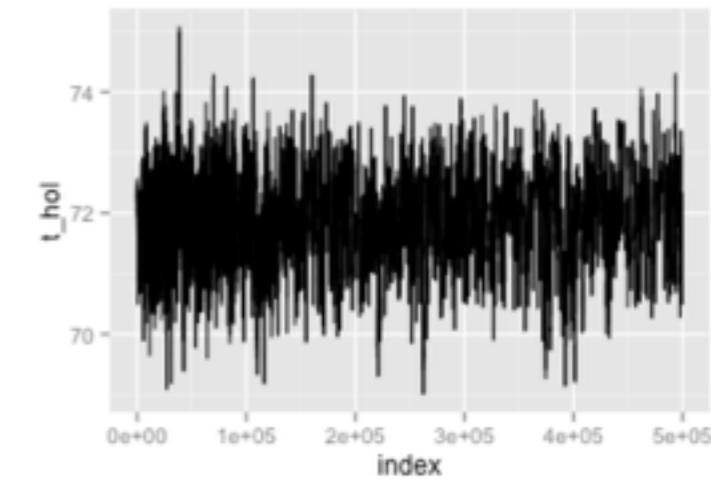
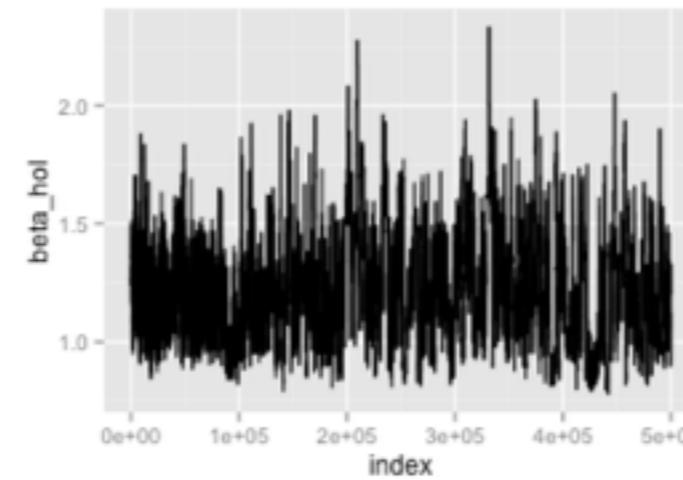
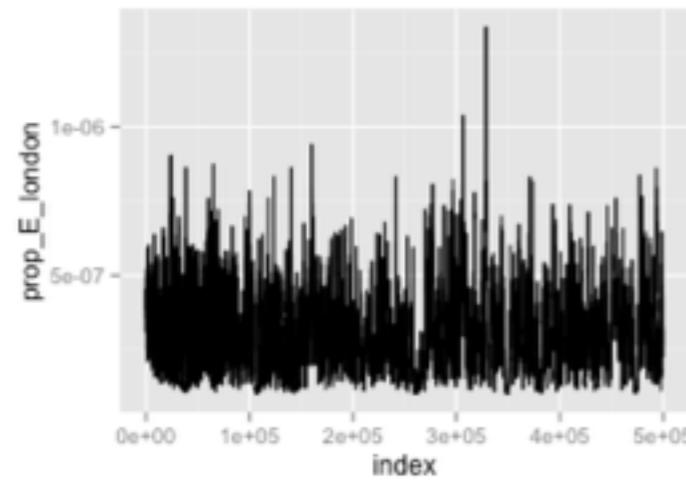
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- **define additional reaction**
- **modify theta**
- **compile**
- **run**

```
cat theta.json | ./pmcmc ode -M 500000 | ./pmcmc ode -M 500000 --trace
```

What about...

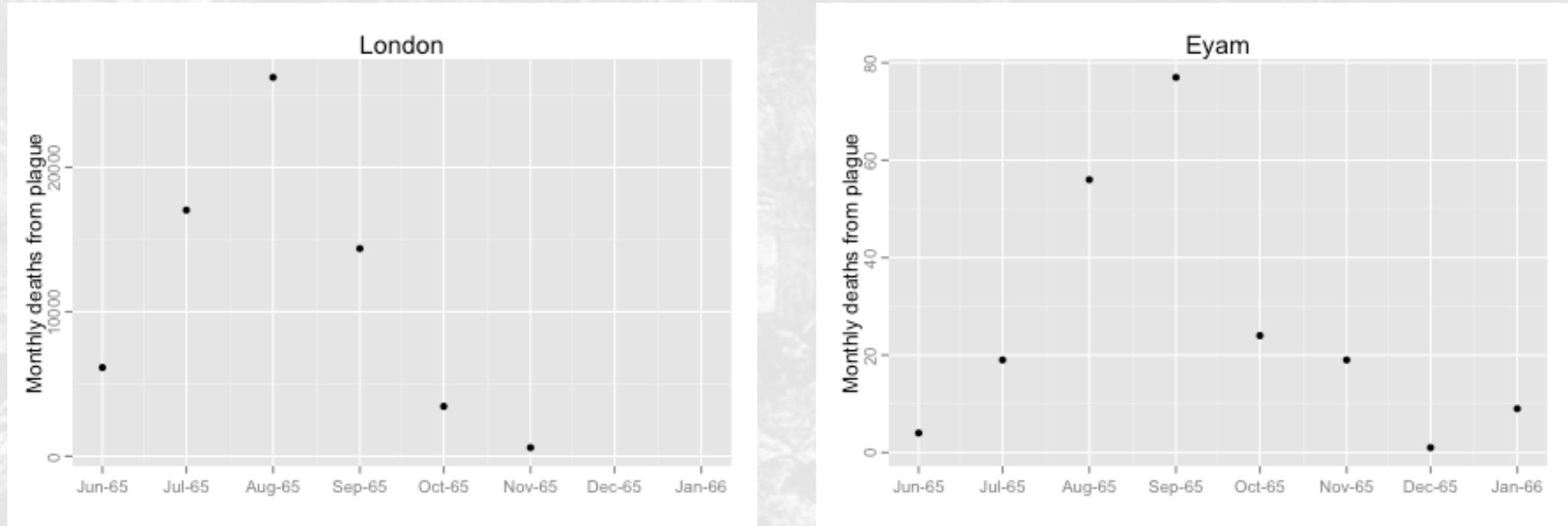
	SIR	SEIR	SIR-holidays
DIC	237	241	211

SIR-holidays



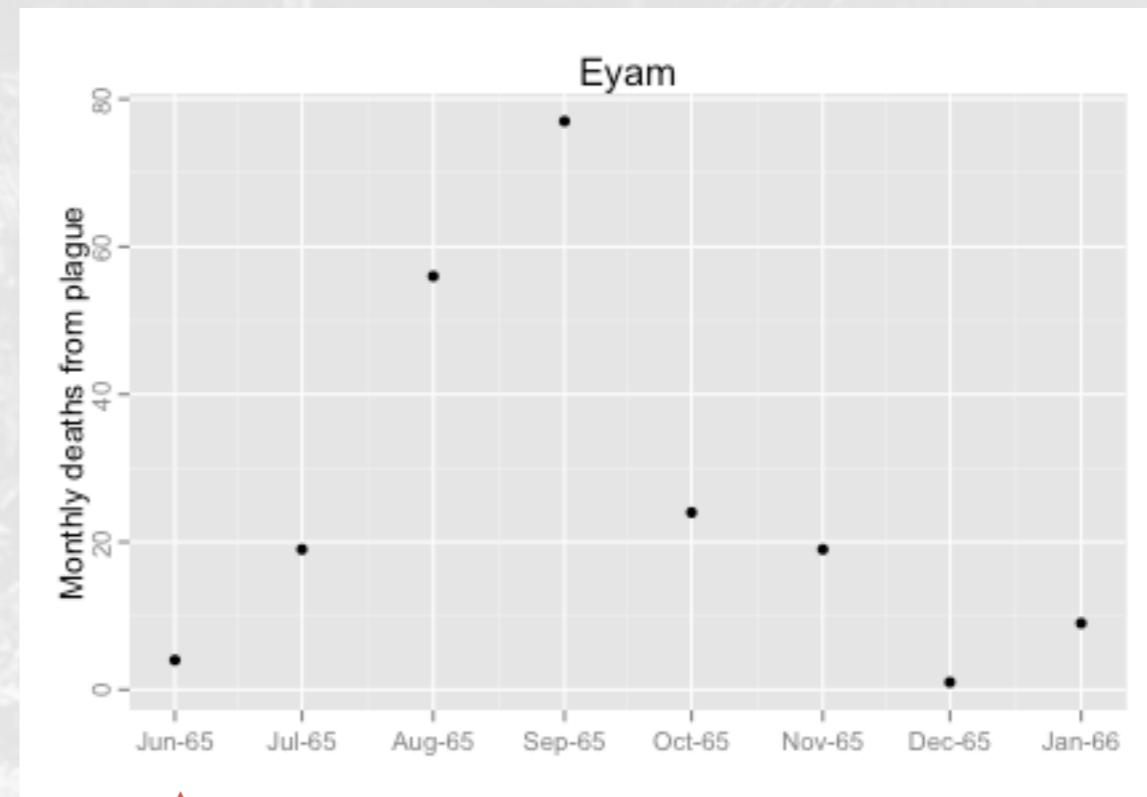
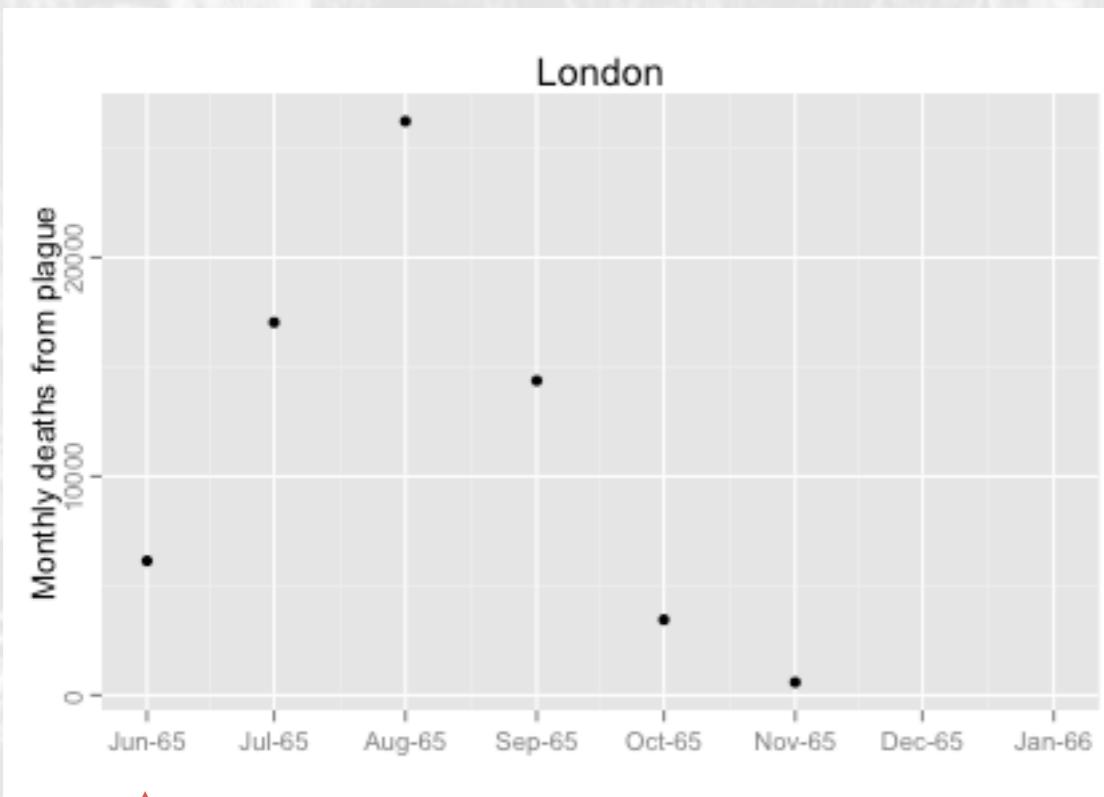
Stochasticity
and statistical
inference

1665 plague in England



What was life expectancy with plague?
Were infection patterns similar in
London and Eyam?

life_exp? r0_London? r0_Eyam?



460 000 inhabitants

350 inhabitants

Not really infinite populations....

ode: ordinary differential equation

sde: stochastic differential equation

psr: poisson with stochastic rates

Different mathematical formalisms

```
cat ../theta.json | ./simul --traj --hat
```

```
cat ../theta.json | ./simul ode --traj --hat
```

```
cat ../theta.json | ./simul sde --traj --hat
```

```
cat ../theta.json | ./simul sde -J 2 --traj --hat
```

```
cat ../theta.json | ./simul sde -J 100 --traj --hat
```

```
cat ../theta.json | ./simul psr -J 100 --traj --hat
```

Different mathematical formalisms

Compartmental description of the epidemic dynamic

S_t : number of individuals susceptible to be infected

I_t : number of individuals infected

Model:

Independently of the past, S individual transfers to I at

rate $\beta_t \frac{I_t}{N}$, where $\beta_t = R_0 \mu_D [1 + e \sin(t + \phi)]$

In other words:

$$P([S_{t+dt}, I_{t+dt}] = [S_t - 1, I_t + 1] | [S_t, I_t]) = \beta_t S_t \frac{I_t}{N} + o(dt)$$

$$P([S_{t+dt}, I_{t+dt}] = [S_t, I_t] | [S_t, I_t]) = 1 - \beta_t S_t \frac{I_t}{N} + o(dt)$$

(Markovian jump process)

Compartmental description of the epidemic dynamic

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$$P([S_{t+dt}, I_{t+dt}] = [S_t, I_t] | [S_t, I_t]) = 1 - \beta_t S_t \frac{I_t}{N} + o(dt)$$

Intractable for other than small populations

Alternative formulations

Multinomial approximation:

If n is the number of reaction occurrences during $[t, t + dt[$,

$$p = 1 - \exp(-\beta_t S_t \frac{I_t}{N} dt)$$

$$P(n|z_t) = \binom{S_t}{n} (1-p)^{S_t-n} p^n + o(dt)$$

SDE approximation:

Using the diffusion approximation (large populations),

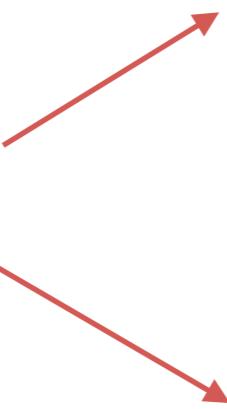
$$dS_t = -\beta_t S_t \frac{I_t}{N} - \sqrt{\beta_t S_t \frac{I_t}{N}} dB_t$$

$$dI_t = \beta_t S_t \frac{I_t}{N} + \sqrt{\beta_t S_t \frac{I_t}{N}} dB_t$$

Breto et al. (2009). Time series analysis via mechanistic models.

Ethier, S. and Kurtz, T. (1986). Markov processes: characterization and convergence.

Explore $p(X_t, \theta|y)$



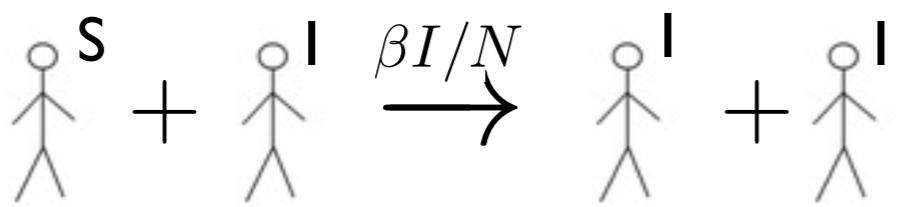
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 $p(X_t|y) = \int p(X_t, \theta|y) d\theta$

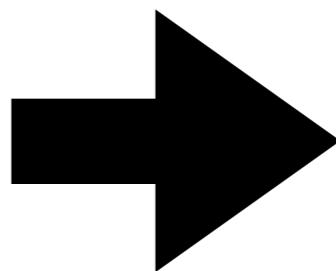
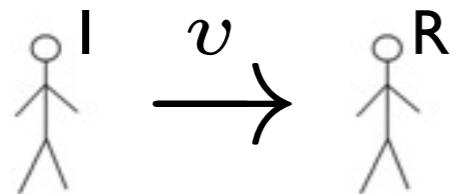
Derive model choice indicators: $p(M1|y) > p(M2|y)$

Objectives of Bayesian inference

infection:



recovery:



$$\frac{dS}{dt} = -\beta I/N \times S$$

$$\frac{dI}{dt} = \beta I/N \times S - vI$$

$$\frac{dR}{dt} = vI$$

In a
nice deterministic world....

Good approximation using
Euler or Runge-Kutta methods

Likelihood

$$p(y|\theta) = \prod p(y_i|X_{t_i}, \theta)p(X_{t_i}|\theta)$$



Model



In a
nice deterministic world....

Good approximation using
Euler or Runge-Kutta methods

Likelihood

$$p(y|\theta) = \prod p(y_i | X_{t_i}^{euler, \theta}, \theta) \times 1$$

In a
nice deterministic world....

No longer a dirac

Likelihood

$$p(y|\theta) = \prod p(y_i|X_{t_i}, \theta)p(X_{t_i}|\theta)$$



✓
Model

Not that simple
in the stochastic world....

Likelihood

Discrete approximation
with *particles*

$$p(y|\theta) = \prod_i \left(\sum_j p(y_i|X_{t_i}^{(j)}, \theta) p(X_{t_i}^{(j)}|\theta) \right)$$

Not that simple
in the stochastic world....

Objectives:

estimation of $p(y|\theta)$

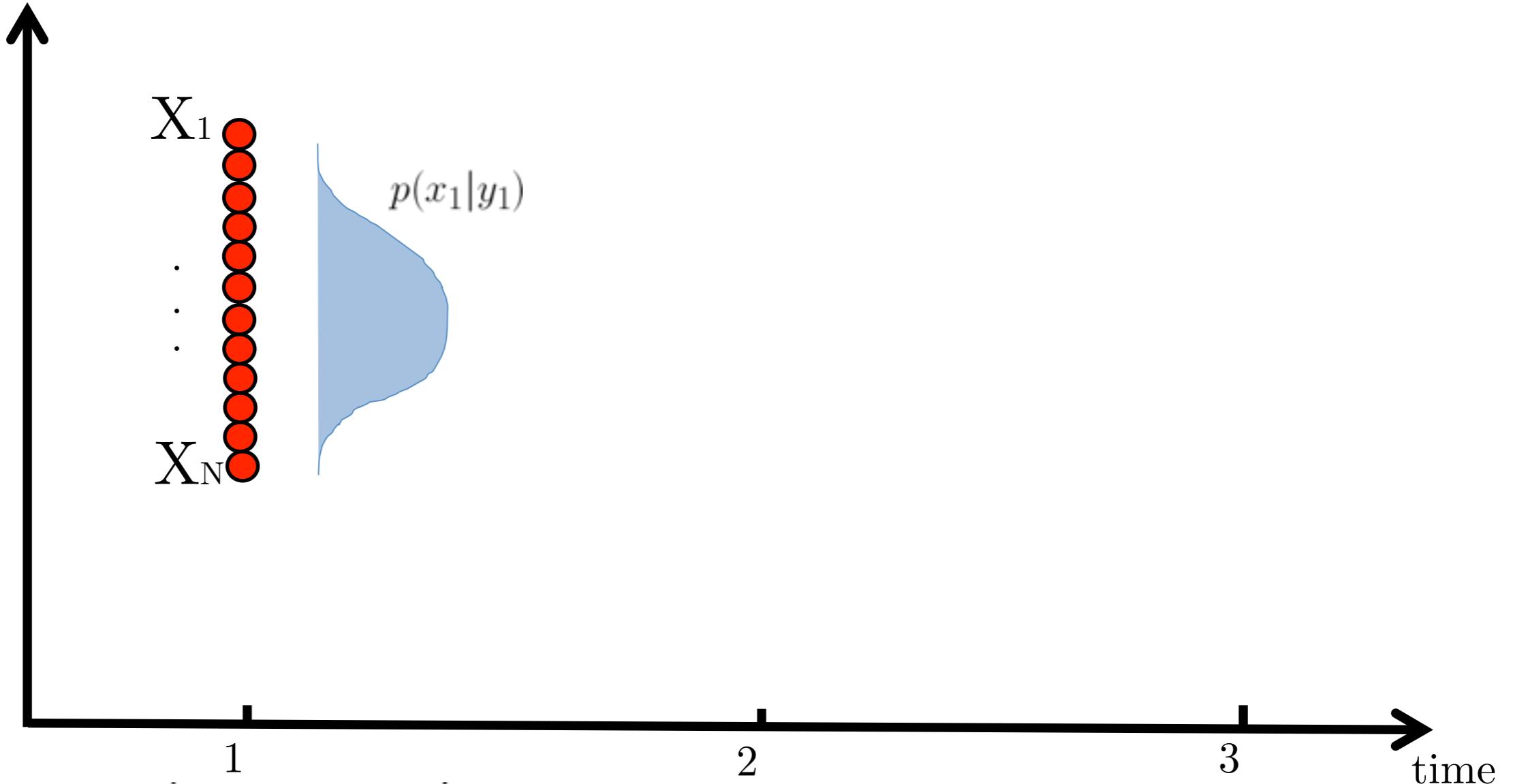
sample from $p(X_{0:n}|y, \theta)$

Introduction to particle filtering

Measured
quantity

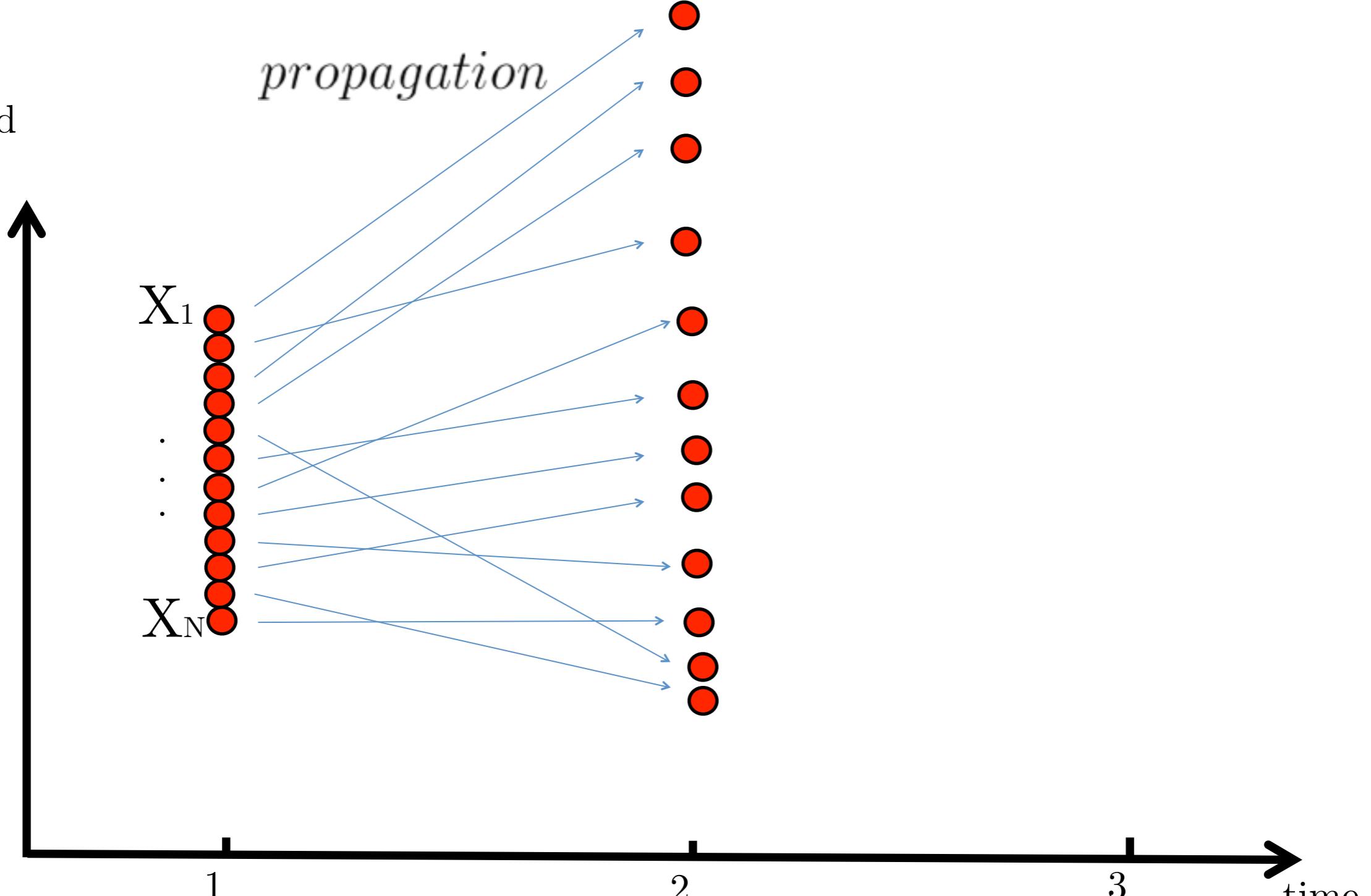


Measured
quantity



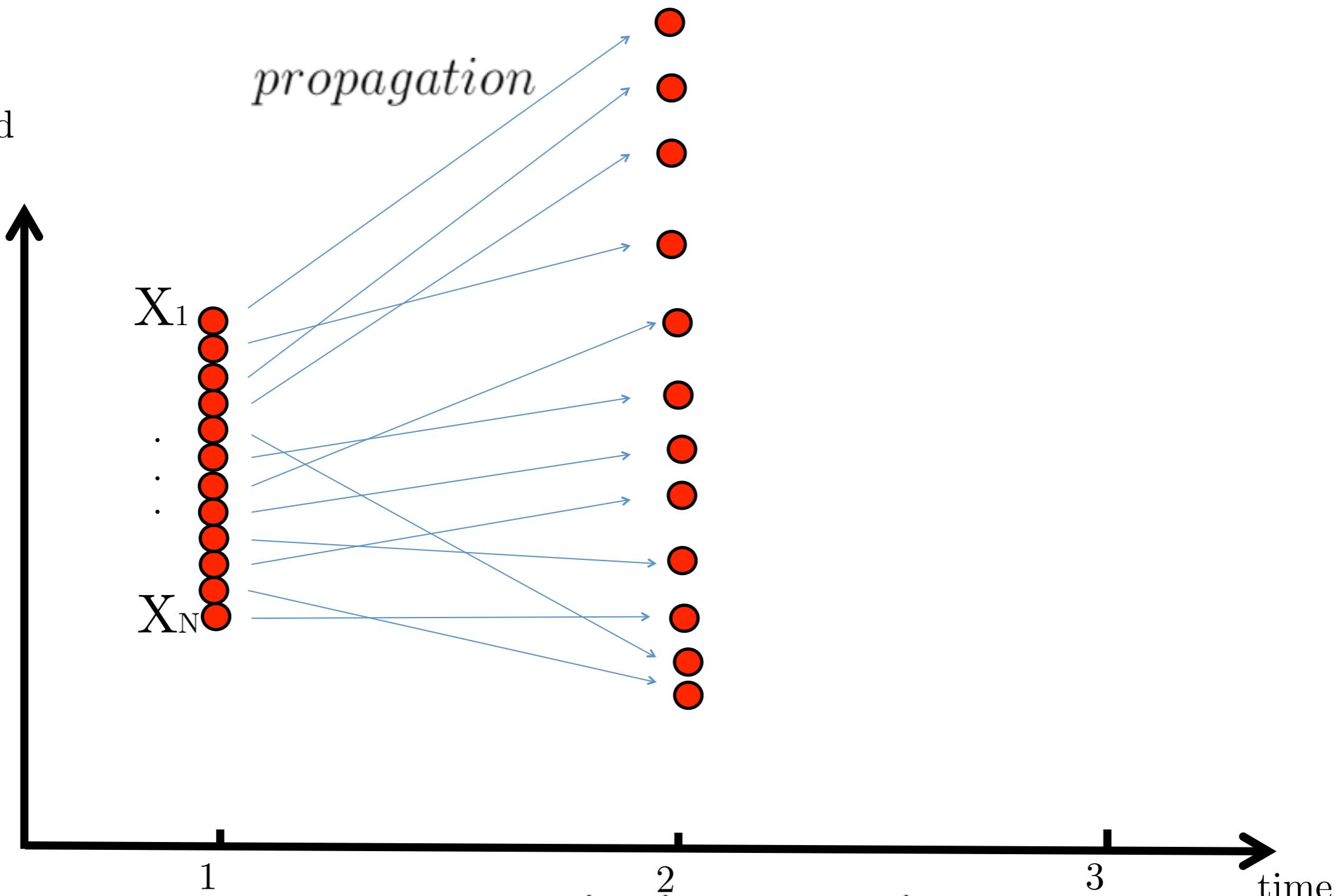
$$W_1^i = p(y_1|X_1^i)$$
$$(X_1^i, W_1^i) \sim p(x_1|y_1)$$

Measured
quantity



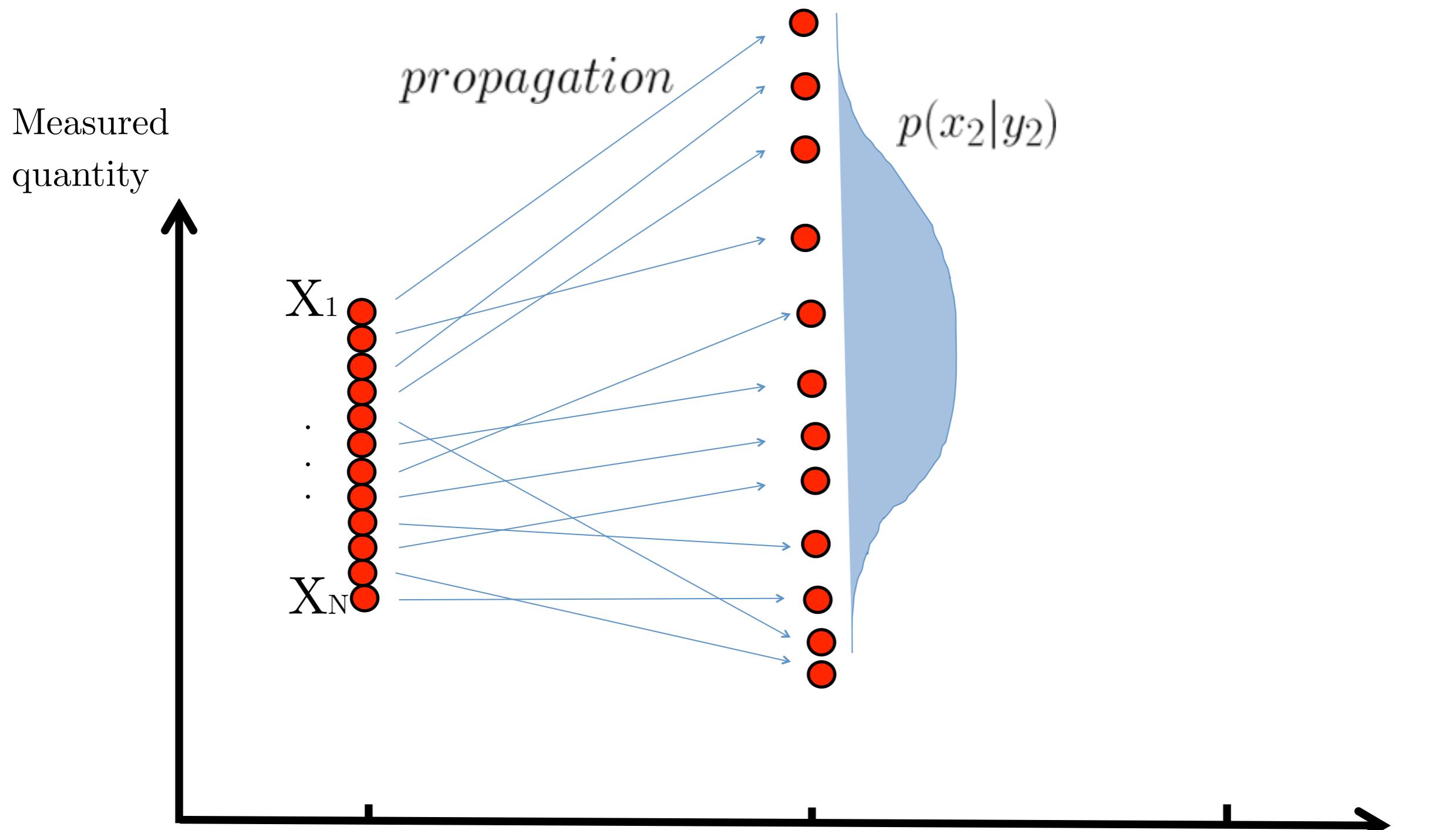
$$W_1^i = p(y_1|X_1^i)$$
$$(X_1^i, W_1^i) \sim p(x_1|y_1)$$
$$X_2^i | X_1^i \sim p(x_2|X_1^i)$$

Measured
quantity



$$W_1^i = p(y_1|X_1^i)$$
$$(X_1^i, W_1^i) \sim p(x_1|y_1)$$

$$X_2^i | X_1^i \sim p(x_2|X_1^i)$$
$$([X_1^i, X_2^i], W_1^i) \sim p(x_{1:2}|y_1)$$

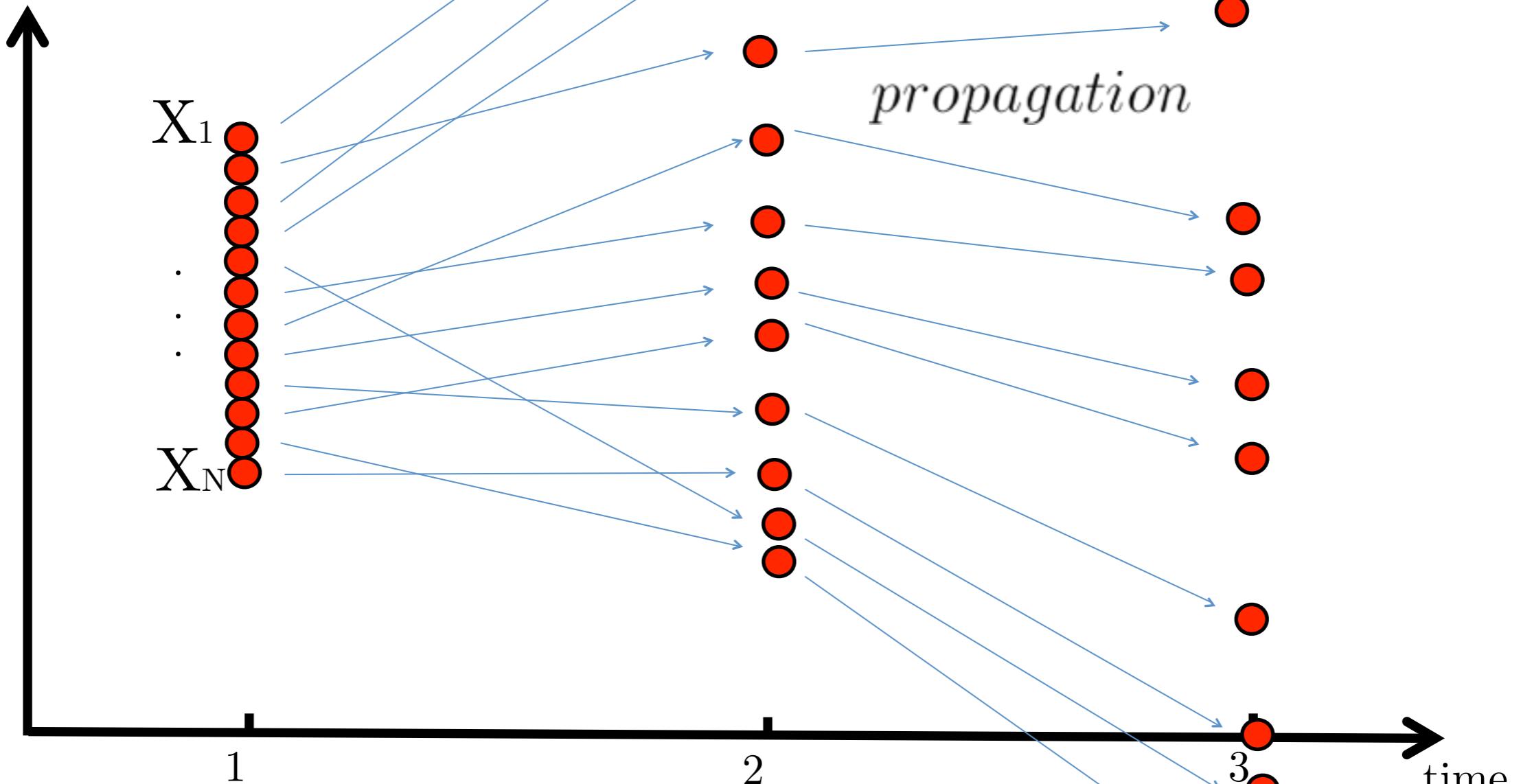


$$W_1^i = p(y_1|X_1^i)$$
$$(X_1^i, W_1^i) \sim p(x_1|y_1)$$

$$X_2^i | X_1^i \sim p(x_2|X_1^i)$$
$$([X_1^i, X_2^i], W_1^i) \sim p(x_{1:2}|y_1)$$

$$W_2^i = W_1^i \times p(y_2|X_2^i)$$
$$([X_1^i, X_2^i], W_2^i) \sim p(x_{1:2}|y_{1:2})$$

Measured
quantity



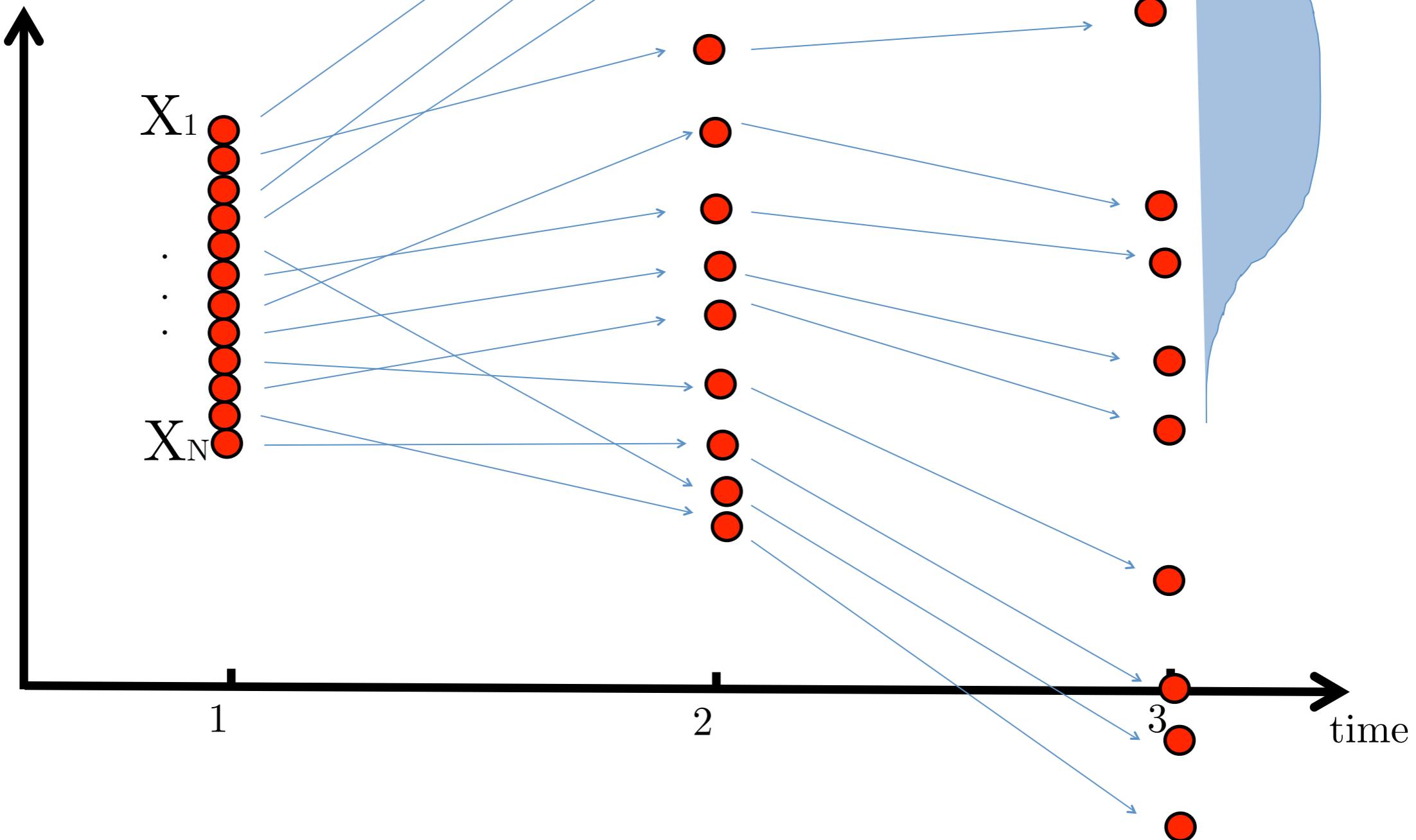
$$W_1^i = p(y_1|X_1^i)$$
$$(X_1^i, W_1^i) \sim p(x_1|y_1)$$

$$X_2^i | X_1^i \sim p(x_2|X_1^i)$$
$$([X_1^i, X_2^i], W_1^i) \sim p(x_{1:2}|y_1)$$

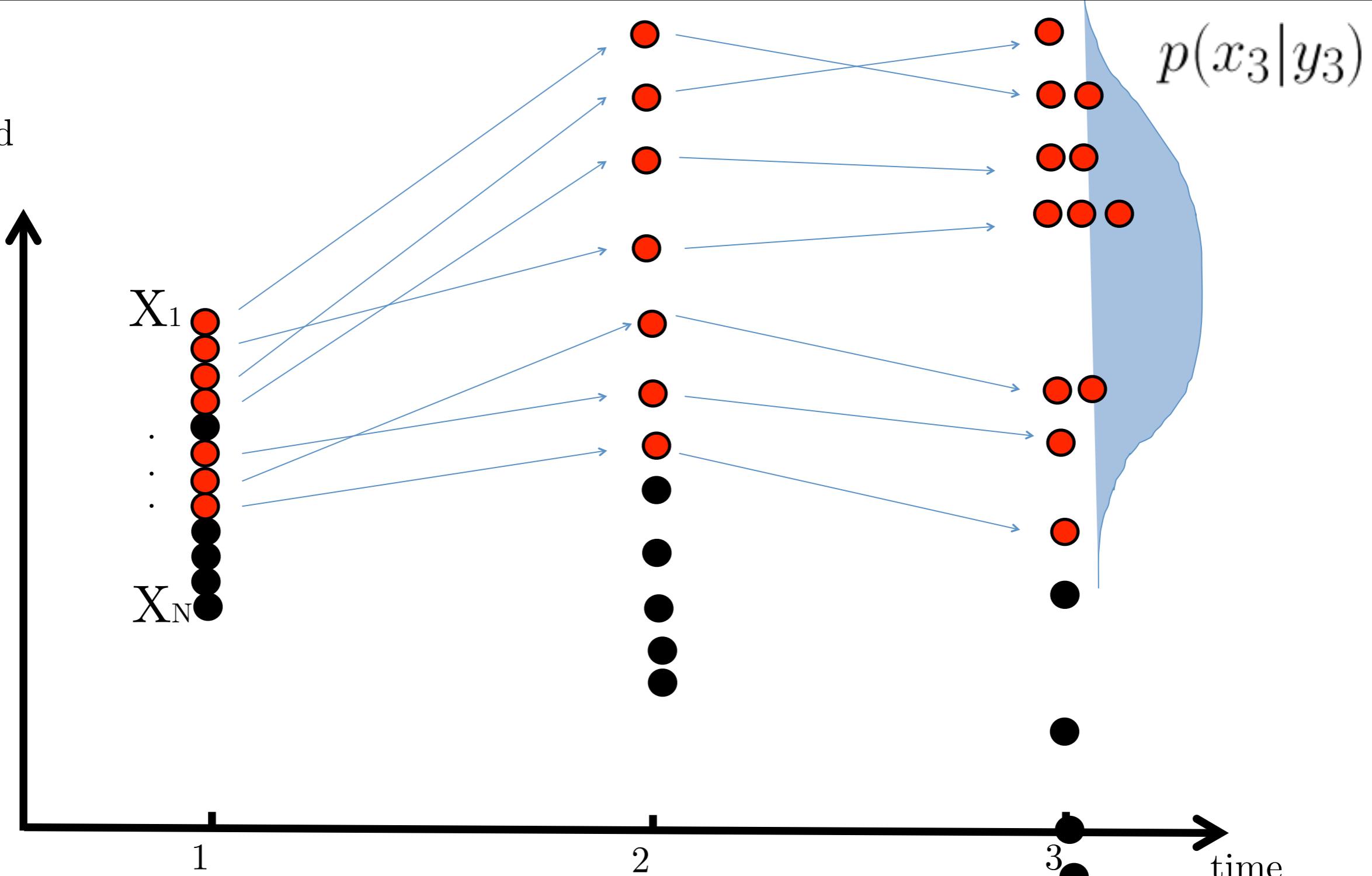
$$W_2^i = W_1^i \times p(y_2|X_2^i)$$
$$([X_1^i, X_2^i], W_2^i) \sim p(x_{1:2}|y_{1:2})$$

$$p(x_3|y_3)$$

Measured
quantity



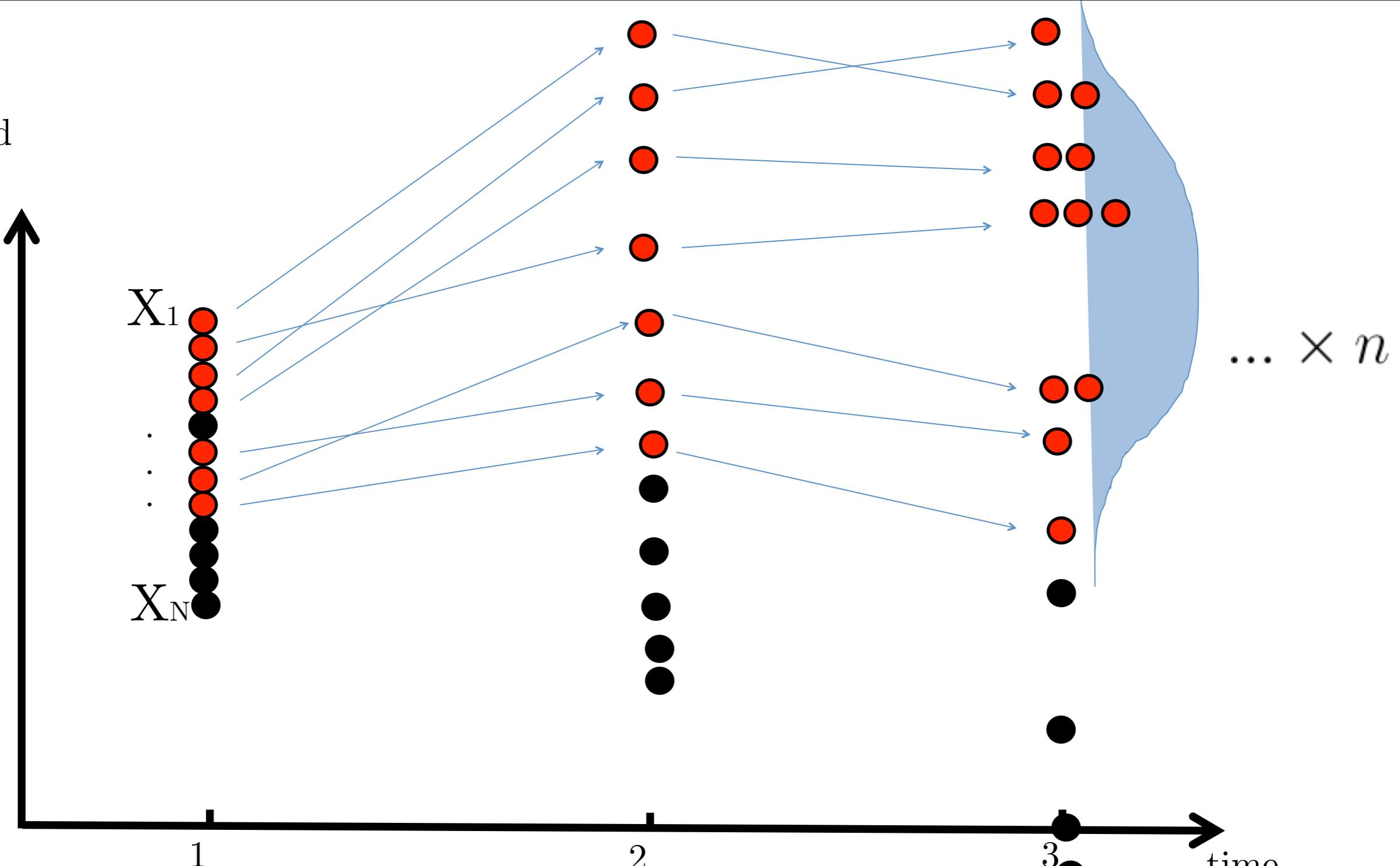
Measured
quantity



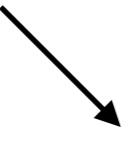
Resample $[X_1^i, X_2^i, X_3^i]$ according to W_3^i

$$p(x_3|y_3)$$

Measured
quantity



Unbiased
but noisy
estimator


$$p(y|\theta) = \prod_i \left(\sum_j p(y_i|X_{t_i}^{(j)}, \theta) p(X_{t_i}^{(j)}|\theta) \right)$$

Particle filtering, a.k.a. Sequential Monte Carlo

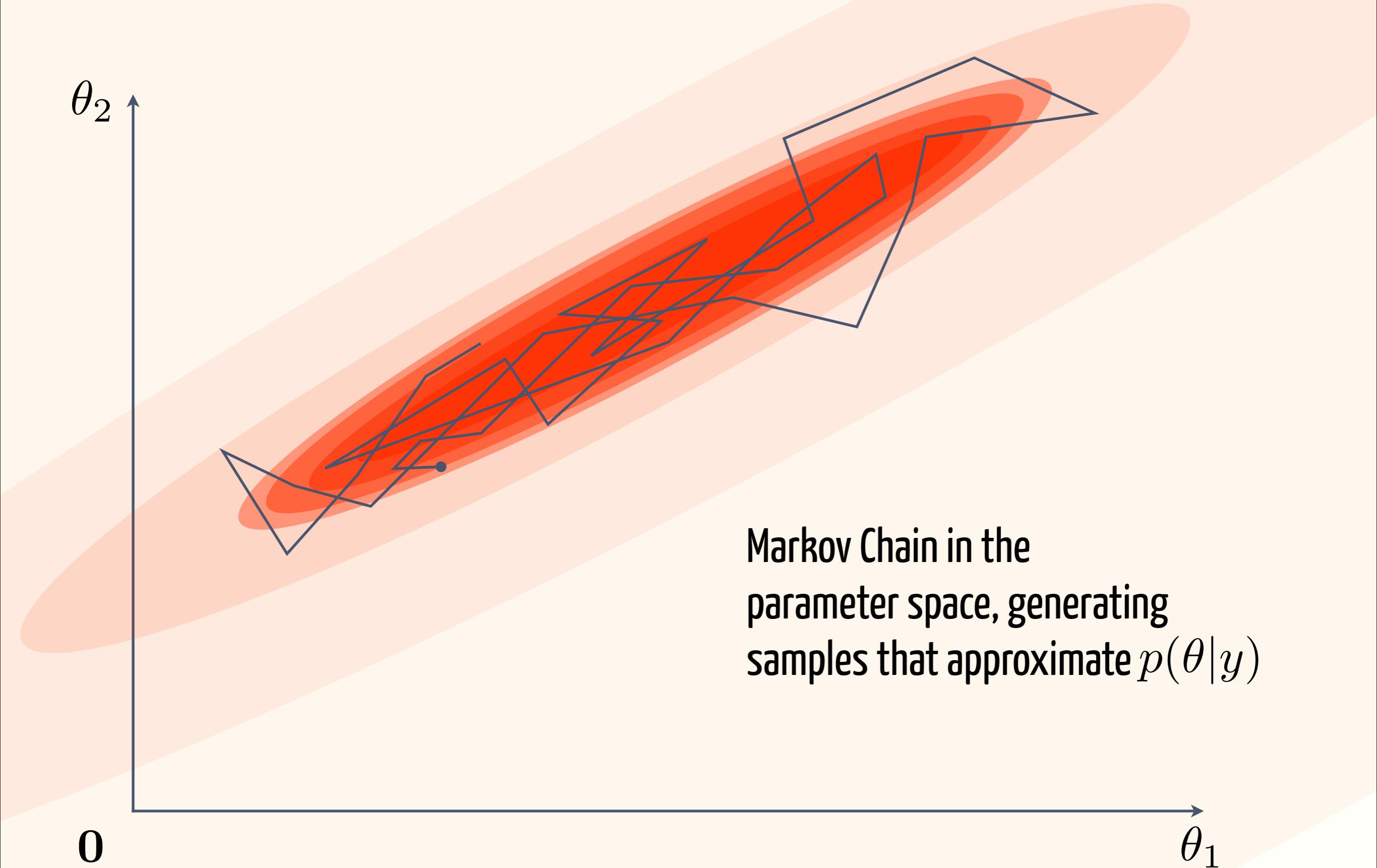
Number of
particles

Print out
likelihood

```
cat ./theta.json | ./smc psr -J 1000 -v -hat
```

Try it

Particle filter + MCMC: PMCMC



```
cat ../theta.json | ./smc psr -J 20000 -v -I 2
```

Number of particles

Print out likelihood

Seed

```
graph TD; A[Number of particles] --> J["-J 20000"]; B[Print out likelihood] --> V["-v"]; C[Seed] --> I["-I 2"];
```

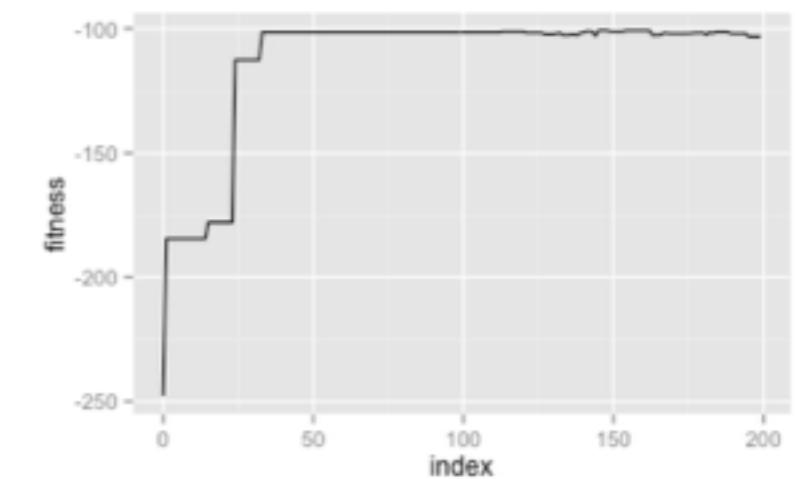
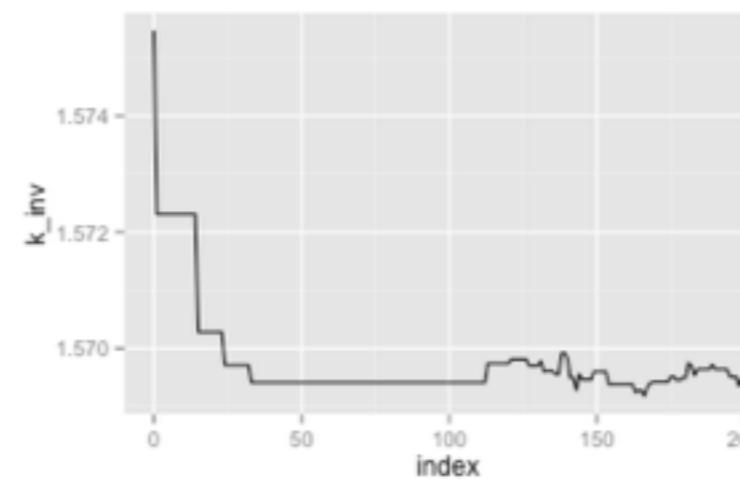
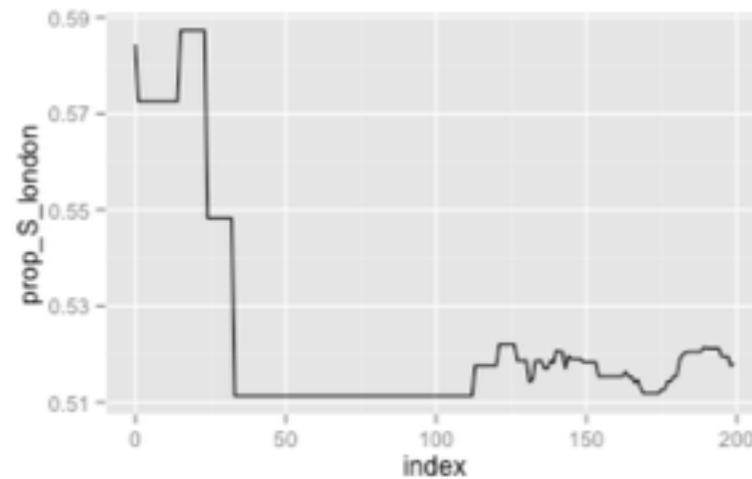
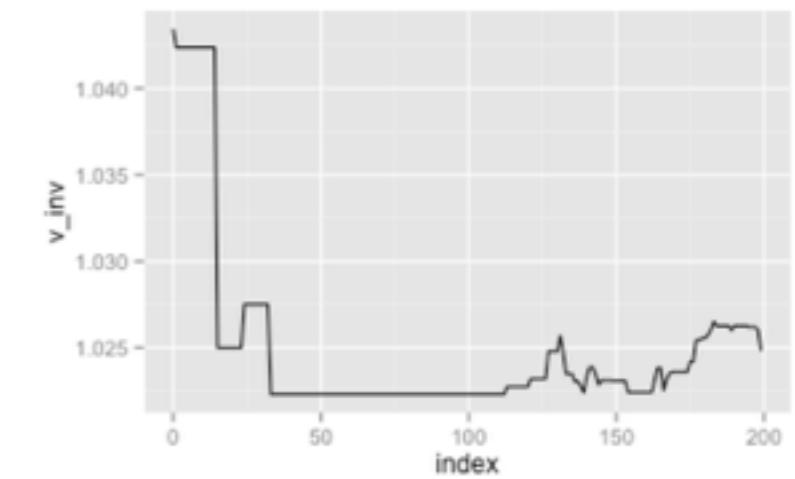
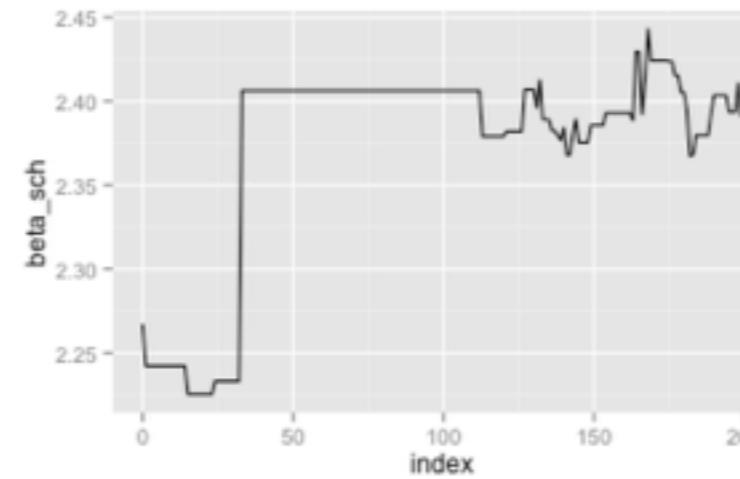
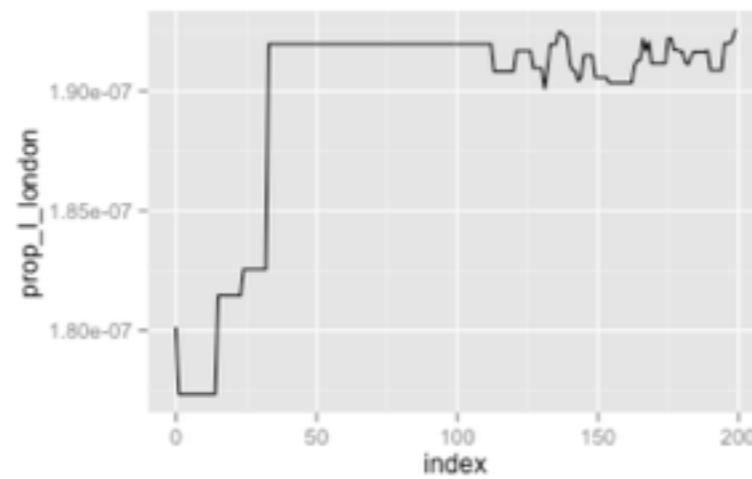
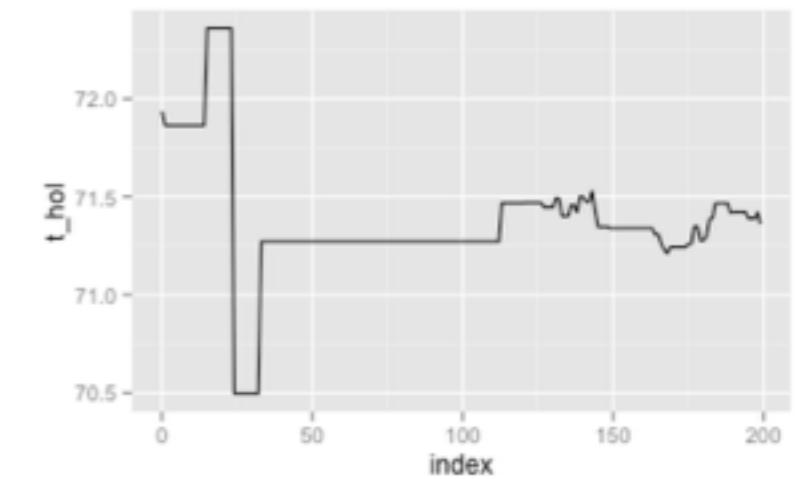
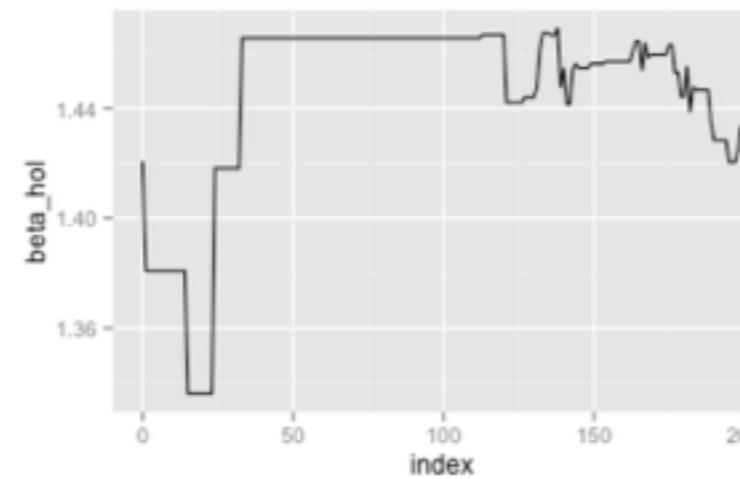
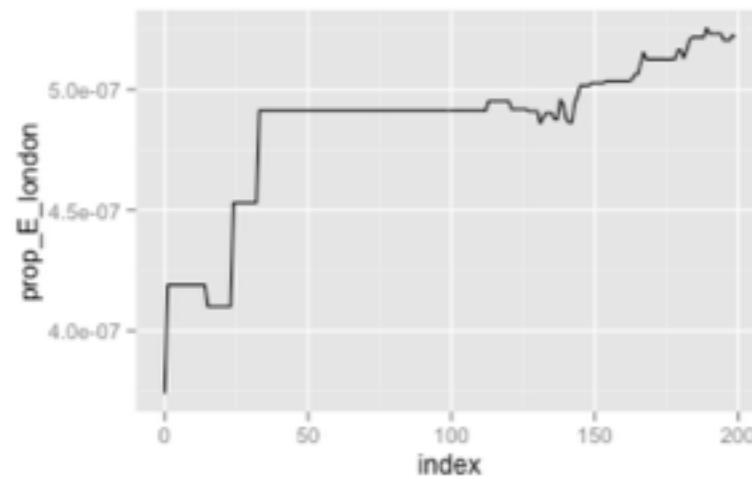
!! Particle Filters
are sensitive to model mis-specification

```
cat ./theta.json | ./smc psr -J 1000 -v -I 2
```

Back to H1N1 and sir-hol

```
cat ./theta.json | ./pmcmc psr -J 1000 -v -M 100
```

Back to H1N1 and sir-hol



Burn-in is costly. Could we make a first check of the posterior density while capturing stochasticity?

Extended Kalman filter

hypothesis

$$p(x_i|y_{1:i}, \theta) = \mathcal{N}(\mu_i^x, \Sigma_i^x)$$
$$p(y_i|x_i, \theta) = \mathcal{N}(h(x_i), \Sigma_i^{obs})$$

hence,

$$\frac{d\mu^x}{dt} = f^{det}(\mu^x)$$
$$\frac{d\Sigma^x}{dt} = \text{Jac}(f^{det})\Sigma^x + \Sigma^x \text{Jac}(f^{det})^t + Q$$
$$\longrightarrow p^{EKF}(y_{1:n}|\theta)$$

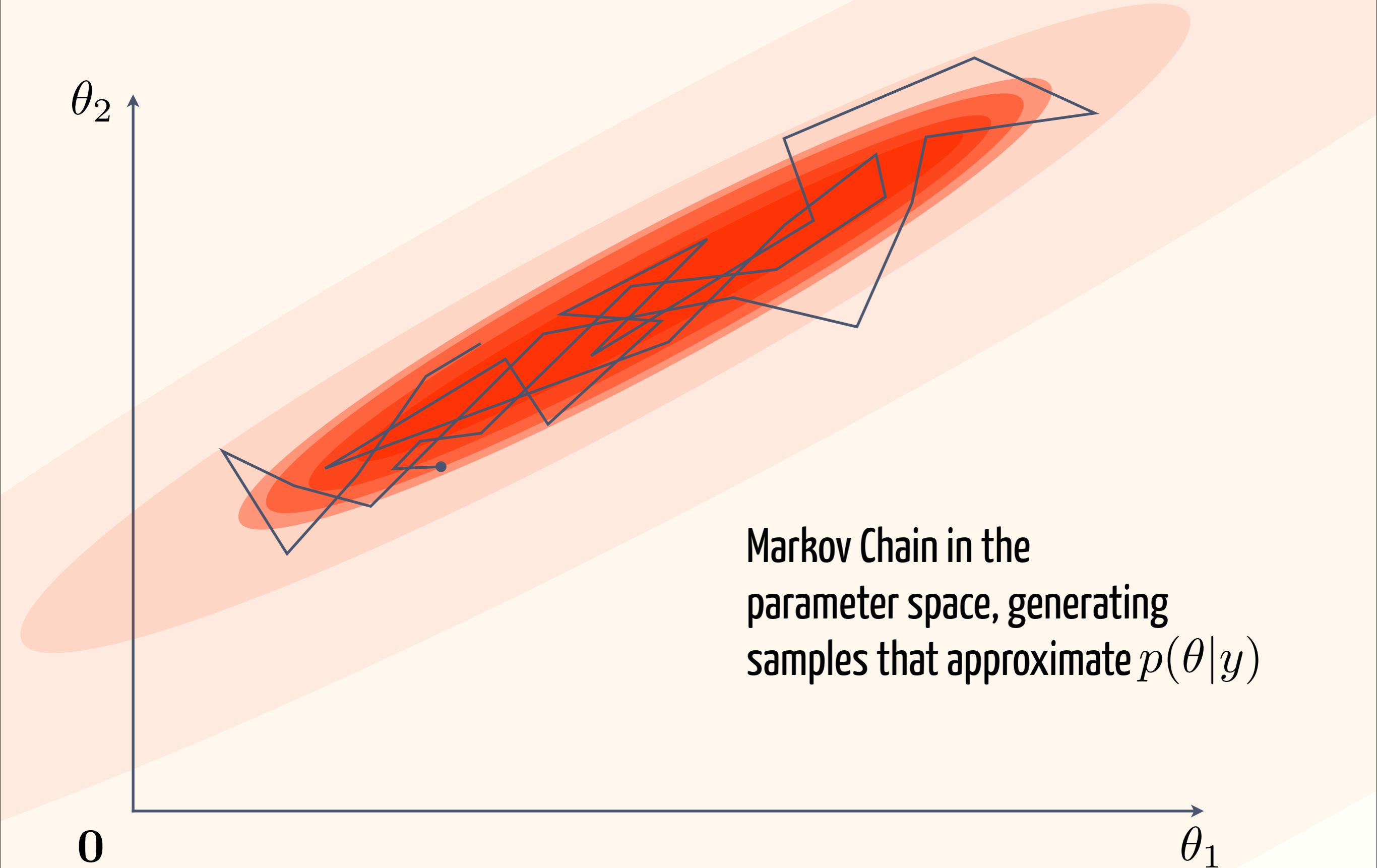
In practice:

Up to 1000 times quicker than particle filter

Uncontrolled approximation

Some numerical unstabilities to be taken care of

Extended Kalmen filter + MCMC : KMC

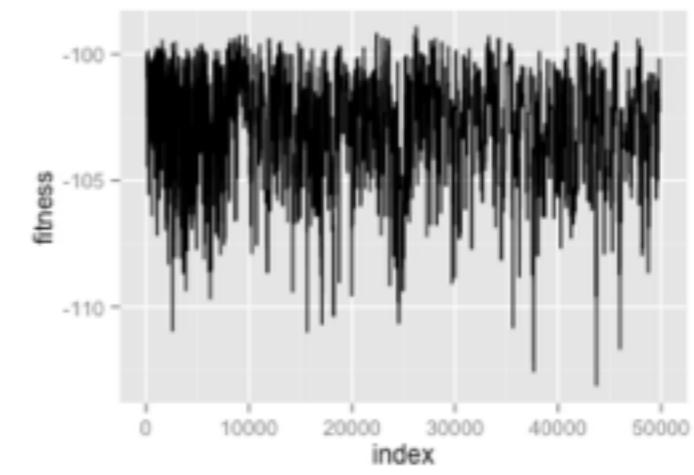
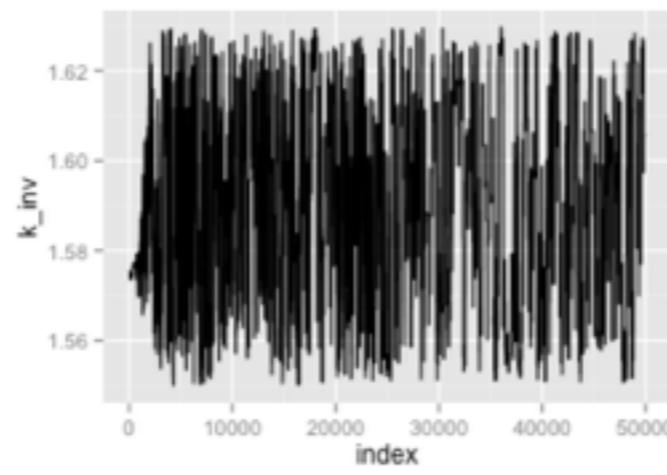
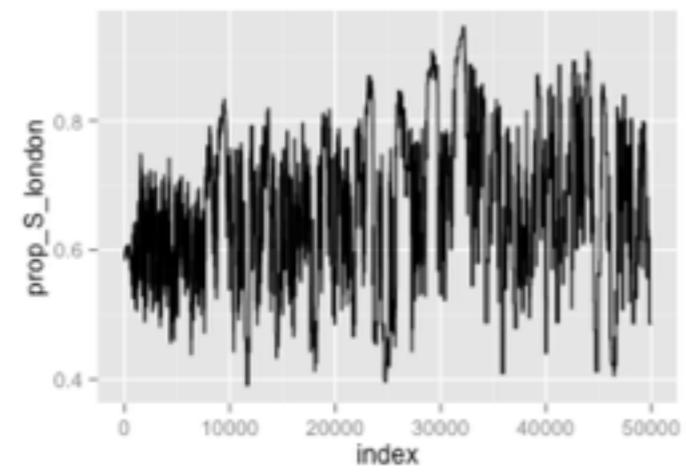
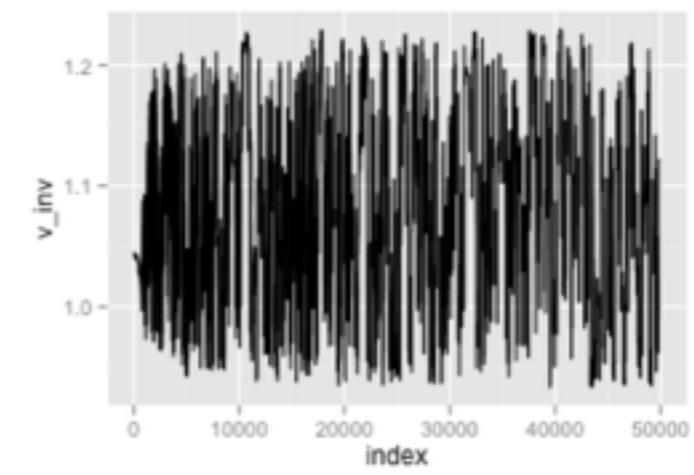
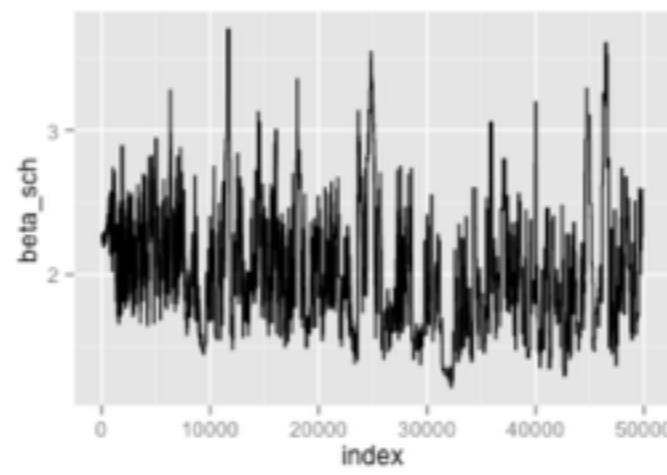
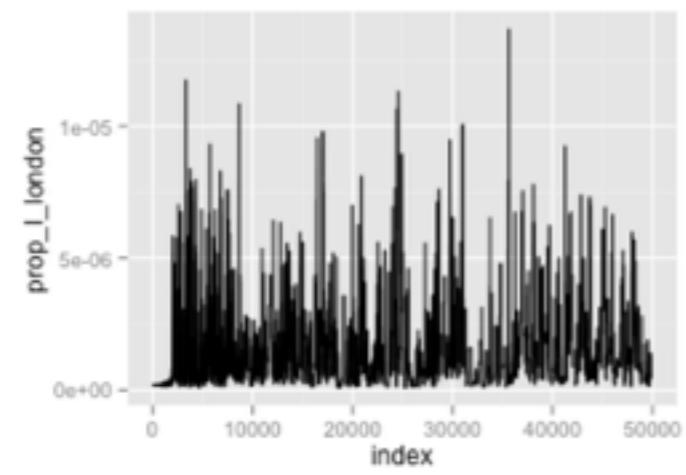
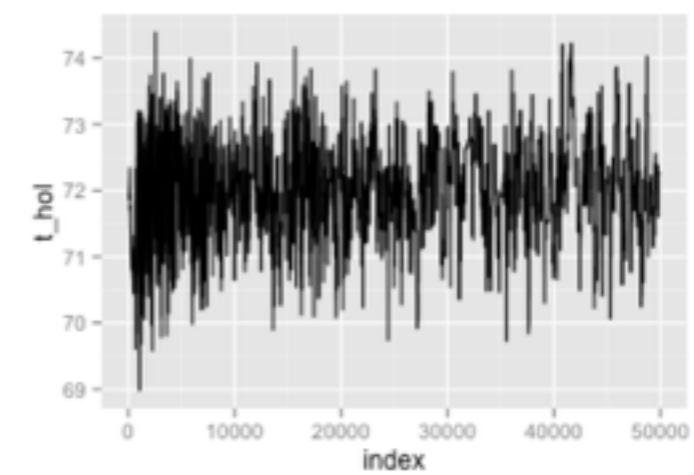
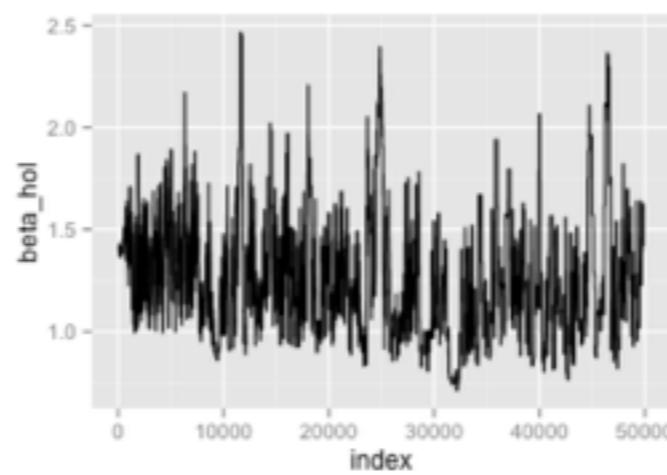
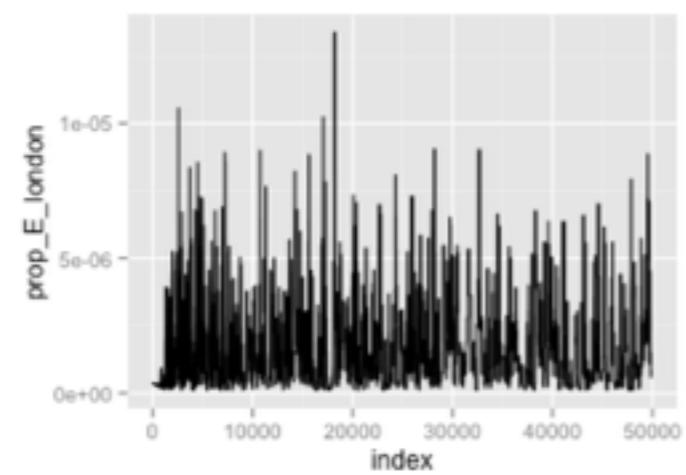


```
cat ./theta.json | ./kmcmc -v -M 50000 -trace
```



No need for implementation
specification
(sde is impled)

Back to H1N1 and sir-hol



10000 iterations of KMCMC

Going Further!

Parallel computing: -N

Environmental stochasticity: stochastic rates and
time-varying parameters

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update README

3 days ago



file structure of JS code

3 days ago



working install

3 days ago



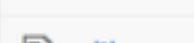
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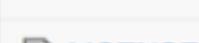
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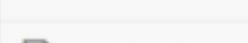
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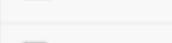
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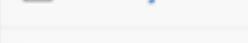
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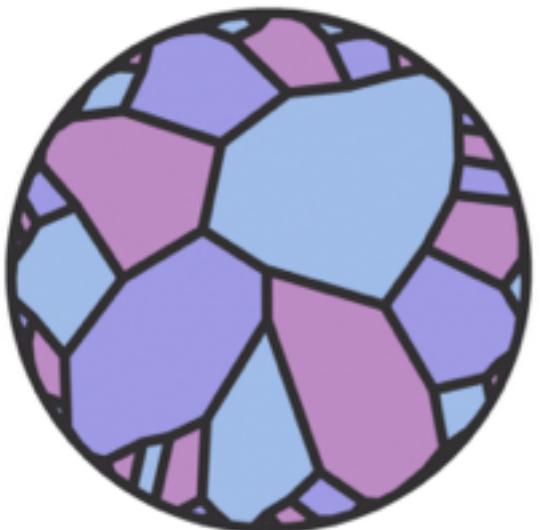
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Trevor Bedford

trvrb

Fred Hutchinson Cancer Research Center

Seattle, WA

<http://bedford.io>

Joined on Nov 06, 2011

Anton

ntncmch

London, UK

anton.camacho@lshtm.ac.uk

Joined on Feb 06, 2012



Serge Stinckwich

SergeStinckwich

UCBN & IRD

France

Serge.Stinckwich@gmail.com

<http://doesnotunderstand.org/>

Joined on Jul 09, 2010

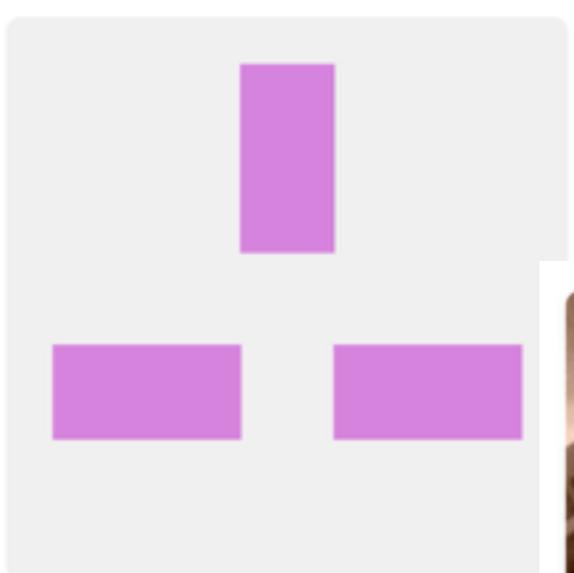
Oliver

olli0601

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Cazelles Bernard

cazelles

UPMC

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cazelles@biologie.ens.fr

Joined on Nov 25, 2013

and all the **cool kids** are there !

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