

Methods of Artificial Intelligence

midterm recap

...

General remarks

Yesterday, only 15 people had filled out the test exam. The test exam provides a good overview of the questions that will be asked in the midterm exam, so please take time to test your knowledge!

CSP consistency

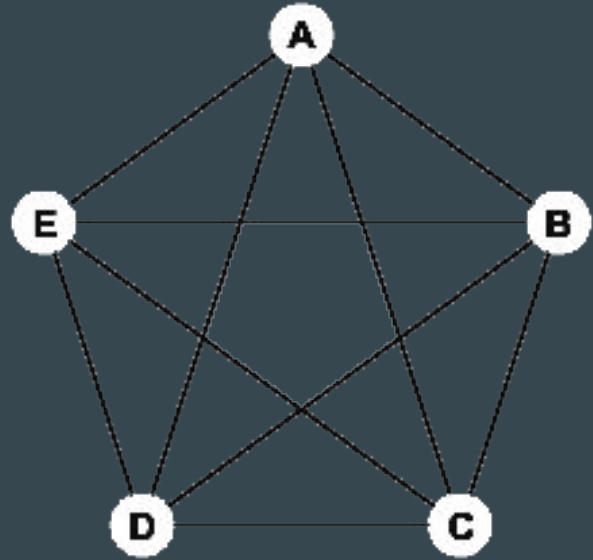
Consistency types

- Node consistency
- Arc consistency
- Path consistency
- k-consistency

Node consistency

suppose the domain $\{1,2,3,4,5\}$

with the constraint node $E \neq 3$



Node consistency

Consider the domain $D = \{1,2,3,4,5\}$

with the constraint of an arbitrary node $N \neq 3$

Since $3 \in D$, the graph is Node inconsistent!

Deleting 3 from the domain would make the graph node-consistent again:

$D = \{1,2,4,5\}$

Arc consistency

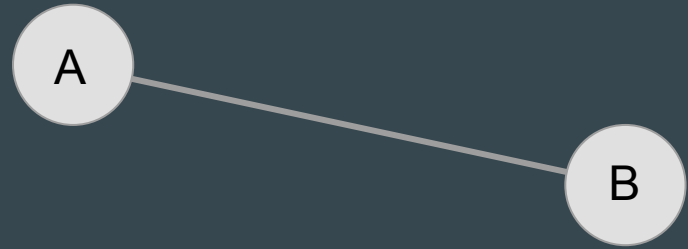
Suppose the set $\{1,2\}$ and a graph G

if:

$$A = 1$$

$$B = 1$$

The graph is **arc-inconsistent!**



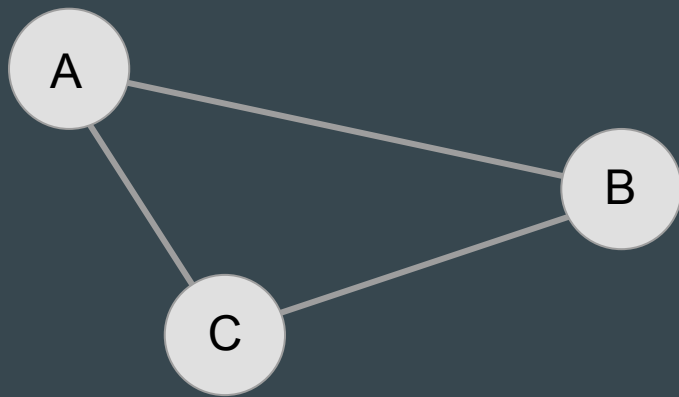
Path consistency

Suppose the Domain $D = \{1,2,3\}$:

We establish **arc-consistency** between A and B by assigning $A = 1$ and $B = 2$

Nodes A and B are path consistent with node C only if we assign $C = 3$, making it arc consistent with A and B!

(path consistency is arc consistency between an already arc consistent pairing of nodes with another node)



k-consistency

Node consistency = 1-consistent

Arc consistency = 2-consistent







path-consistency = 3-consistent

strong k consistent: consider a $j \in \{1, 2, \dots, k\}$. A graph is strong k-consistent if it is consistent for every $j \leq k$

A graph is k-consistent if it can be extended from a (k-1)-consistent state to a k-consistent state.

A k-consistent graph DOES NOT need to be consistent in general! i.e. with k+1 nodes

VIPS questions concerning consistency

- Every arc consistent CSP is node consistent. 
- Every node consistent CSP is consistent. 
- Every node consistent CSP is arc consistent. 
- Every strong k -consistent CSP is strong $(k-1)$ -consistent. 
- Every strong $(k-1)$ -consistent CSP is strong k -consistent. 
- Every consistent CSP is node consistent. 

Local search

Hill climbing vs Simulated annealing

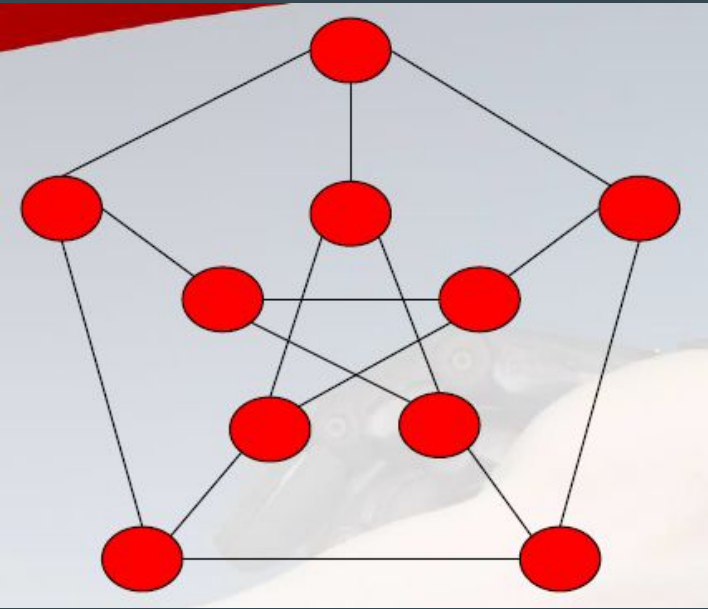
Stochastic Hill Climbing can make choices that decrease the objective function.

->wrong

```
current ← select random initial state
do
  neighbor ← random neighbor of current with higher value
  if neighbor = null
    return current
  current ← neighbor
until termination condition is met
```

Solving CSPs with Local Search

Suppose that the CSP has n variables and m constraints.



$$n = 10$$

$$m =$$

$$\text{domain} = \{1, 2, 3\}$$

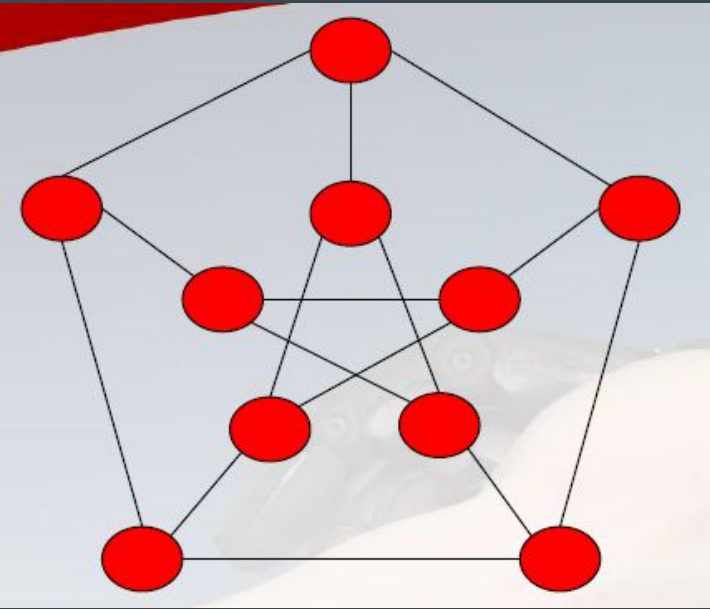
a state is a complete assignment. In this example: $(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$

$\# \text{neighbors} = \# \text{variables} * \# \text{values in domain}$
other than the current one

$$= 10 * 2 = 20$$

Solving CSPs with Local Search

Suppose that the CSP has n variables and m constraints.



Objective function:

value of state is defined as the number of constraints that are satisfied in the state

$$n = 10$$

$$m = 15$$

$$f(s) = 0$$

Solving CSPs with Local Search

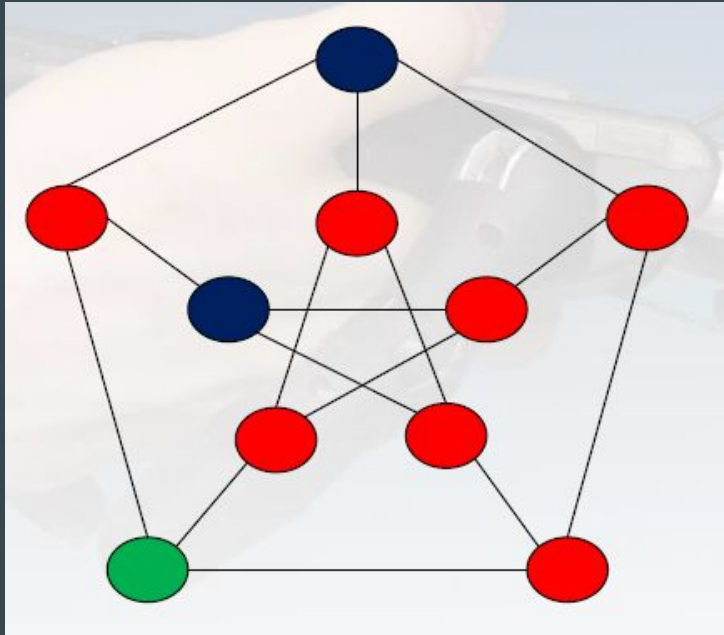
Suppose that the CSP has n variables and m constraints.

$$n = 10, m = 15$$

1. When our local search algorithm finds a solution with value n , it found a consistent assignment and the CSP is consistent.
2. When our local search algorithm finds a solution with value m , it found a consistent assignment and the CSP is consistent.
3. When our local search algorithm finds a solution with value less than n , there is no consistent assignment and the CSP is inconsistent.
4. When our local search algorithm finds a solution with value less than m , there is no consistent assignment and the CSP is inconsistent.

Solving CSPs with Local Search

Suppose that the CSP has n variables and m constraints.



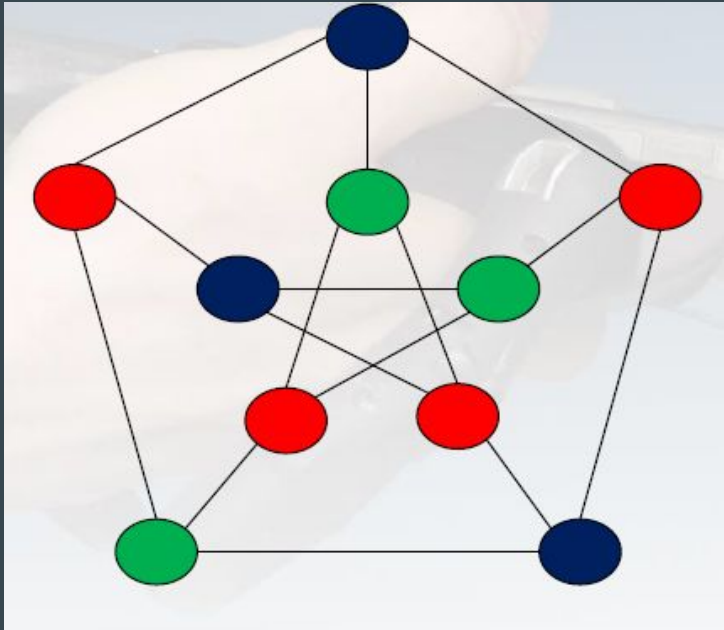
$$n = 10$$

$$m = 15$$

$$f(s) = 9$$

Solving CSPs with Local Search

Suppose that the CSP has n variables and m constraints.



$$n = 10$$

$$m = 15$$

$$f(s) = 15$$

Genetic Algorithms

Uniform-Order-Crossover

Child 1 is created by using genes of parent 1 at 1-positions. Fill gaps with the remaining elements according to the order given by parent 2.

Child 2 is created by switching the roles of parent 1 and 2.

parent 1: 456123

parent 2: 312546

template: 010101

child 1: _____

child 2: _____

Uniform-Order-Crossover

Child 1 is created by using genes of parent 1 at 1-positions. Fill gaps with the remaining elements according to the order given by parent 2.

Child 2 is created by switching the roles of parent 1 and 2.

parent 1: 456123 leftover: 4,6,2

parent 2: 312546 leftover: 3,2,4

template: 010101

child 1: _5_1_3

child 2: _1_5_6

Uniform-Order-Crossover

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parent 1: 456123 leftover: 4,6,2

parent 2: 312546 leftover: 3,2,4

template: 010101

child 1: 254163

child 2: 412536

Theorem Proving

First-Order Predicate Logic

Terms, Predicates, Formulas, Function Symbol, Relation Symbol, Constants, Variables

socrates Mammal(A) \rightarrow Animal(A) “ \leq ” 3 Human(X)

Mammal(johannes) “=” “+” 4+Z Human 2+3=5

First-Order Predicate Logic

Terms, Predicates, Formulas, Function Symbol, Relation Symbol, Constants, Variables

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Terms:

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First-Order Predicate Logic

Terms, Predicates, Formulas, Function Symbol, Relation Symbol, Constants, Variables

Terms: `socrates` `3` `Human` `4+Z`

`Mammal(A) -> Animal(A)`

“`≤`”

`Human(X)`

`Mammal(johannes)`

“`=`”

“`+`”

`2+3=5`

First-Order Predicate Logic

Terms, Predicates, Formulas, Function Symbol, Relation Symbol, Constants, Variables

Terms: `socrates` `3` `Human` `4+Z`

Predicate symbols:

`Mammal(A) -> Animal(A)`

“ \leq ”

`Human(X)`

`Mammal(johannes)`

“ $=$ ”

“ $+$ ”

`2+3=5`

First-Order Predicate Logic

Terms, Predicates, Formulas, Function Symbol, Relation Symbol, Constants, Variables

Terms: socrates 3 Human 4+Z

Predicate symbols: Animal(...), Human(...), Mammal(...), “ \leq ”, “ $=$ ”

Mammal(A) \rightarrow Animal(A)

Human(X)

Mammal(johannes)

2+3=5

First-Order Predicate Logic

Terms, Predicates, Formulas, Function Symbol, Relation Symbol, Constants, Variables

Terms: $socrates$ 3 $Human$ $4+Z$

Predicate symbols: $Human(\dots)$, $Mammal(\dots)$, " \leq ", " $=$ "

Formulas:

$Mammal(A) \rightarrow Animal(A)$

$Human(X)$

$Mammal(johannes)$

$2+3=5$

First-Order Predicate Logic

Terms, Predicates, Formulas, Function Symbol, Relation Symbol, Constants, Variables

Terms: socrates 3 Human 4+Z

Predicate symbols: Human(...), Mammal(...), " \leq ", " $=$ "

Formulas: Mammal(A) \rightarrow Animal(A), Human(X), Mammal(johannes), 2+3=5

Normal Clause Form

$$\exists x(P(x) \rightarrow \forall yQ(y))$$

Steps for conversion:

- (1) Remove Implications.
- (2) Reduce scope of negations.
- (3) Skolemization (remove existential quantifiers).
- (4) Standardize (rename) variables.
- (5) Prenex-form.
- (6) Conjunctive normal form (CNF).
- (7) Eliminate conjunctions and rename variables if necessary. Each conjunct should have a different set of variables.
- (8) Eliminate universal quantifiers.

Normal Clause Form

$$\exists x(P(x) \rightarrow \forall yQ(y))$$

$$(1) \exists x(\neg P(x) \vee \forall yQ(y))$$

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Normal Clause Form

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$$(1) \exists x(\neg P(x) \vee \forall yQ(y))$$

$$(3) \neg P(c_x) \vee \forall yQ(y)$$

Steps for conversion:

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Normal Clause Form

$$\exists x(P(x) \rightarrow \forall yQ(y))$$

$$(1) \exists x(\neg P(x) \vee \forall yQ(y))$$

$$(3) \neg P(c_x) \vee \forall yQ(y)$$

$$(8) \neg P(c_x) \vee Q(y)$$

Steps for conversion:

- (1) Remove Implications.
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Markov decision processes

Basic idea

For a given problem, we want to find the most optimal (or good enough) solution-strategy

if there exists a strategy: it is to be evaluated

if there exists no strategy: it is to be found

Policy iteration

consider an MDP with (S, A, p, r) :

S = set of states

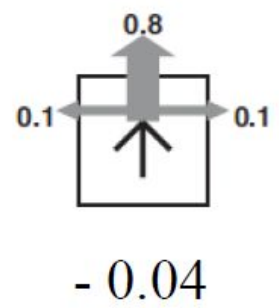
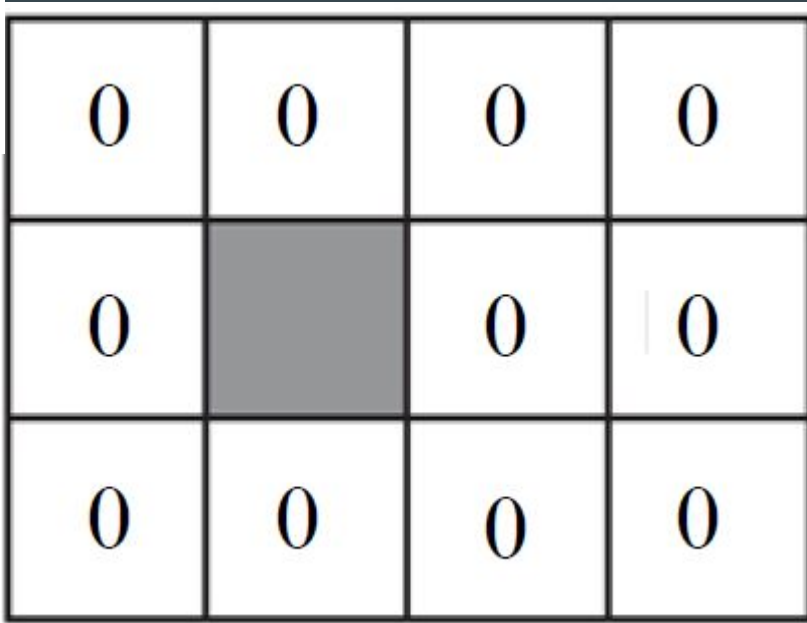
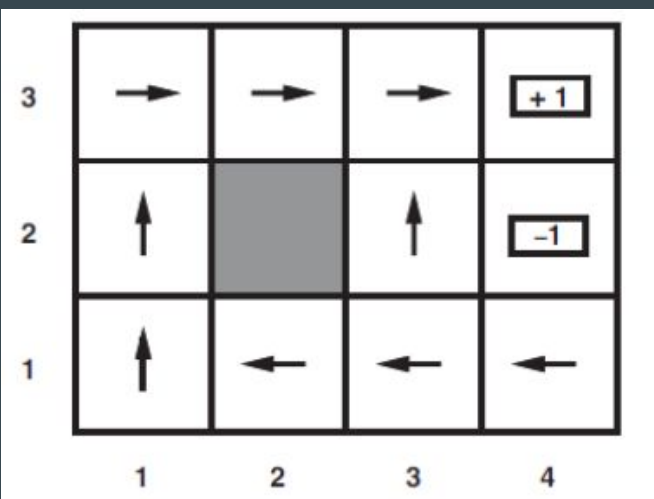
A = set of actions

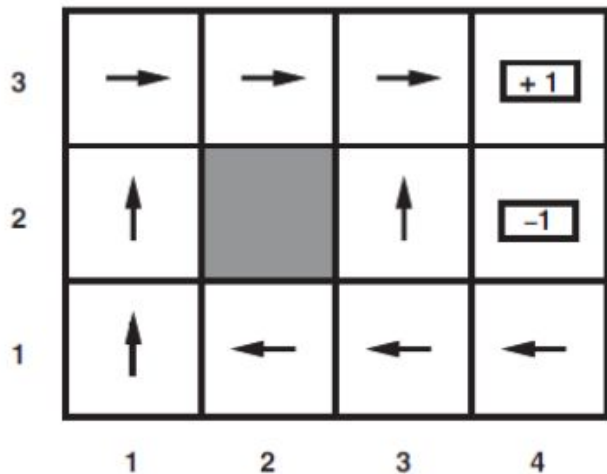
p = probability that action a in state s will lead to states s'

r = expected rewards from going from s to s'

Discount factor γ

deterministic policy π

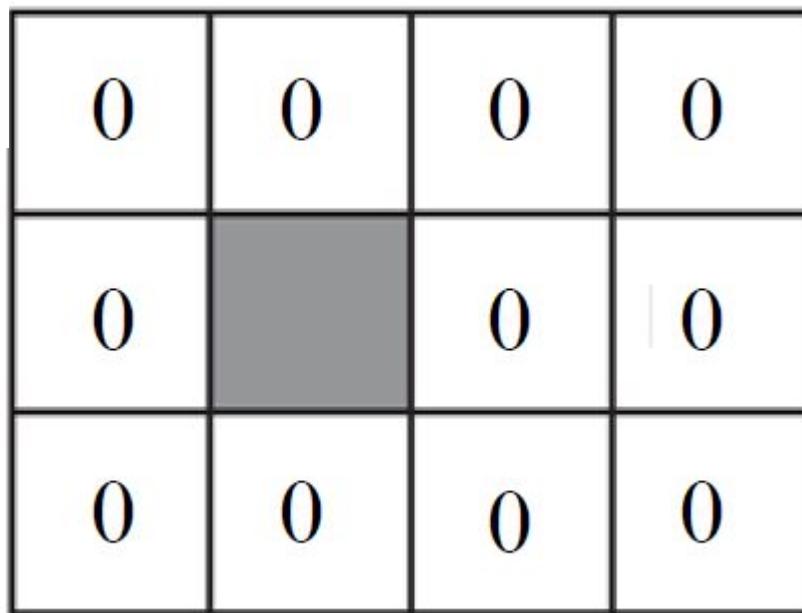




Initial values

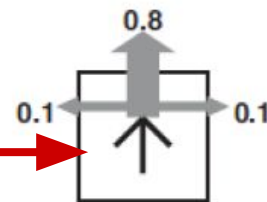
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1



Initial policy

Probability of
state-action
function



- 0.04

expected
reward
per move

Initialize value estimate v and policy π arbitrarily

do

$v \leftarrow \text{evaluate } \pi$

(perform policy evaluation)

for each s in S

$\pi(s) \leftarrow \text{greedy action with respect to } v$

until policy is stable (does not change anymore)

return π

3	-0.04	-0.04	-0.04	1
2	-0.04		-0.04	-1
1		-0.04	-0.04	-0.04

$$\begin{aligned}
 & -0.04 \\
 & + 0.8 * -0.04 \\
 & + 0.1 * -0.04 \\
 & + 0.1 * -0.04
 \end{aligned}$$

$$= -0.08$$

**Let's continue on the blackboard
(if there is time)**

Convergence

Convergence is guaranteed under some assumptions such as:

$\gamma \neq 1!$

why?

Convergence

Convergence is guaranteed under some assumptions such as:

$\gamma \neq 1$!

why?

Because if $\gamma = 1$, the algorithm will search for a reward after infinite steps, this may lead to a jumping in between equally optimal policies

A possible solution to this is to simply let the algorithm terminate after a number of iterations rather than when it falls under a certain ϵ value.

Good luck in the exam!

