

Instructions: (10 points) Solve the following problems. Write clearly and use same symbols as used in the lecture. Add comments, explanations or questions to your solution if necessary.

Solutions to exercise 1 is to be submitted as physical copies in groups of three or four. Please label each page you hand in **clearly** and **carefully** with the name of each group member and their student ID.

Please staple all of your sheets.

Deadline for this exercise sheet is: **30.11.2017**

(10^{pts})

1. Let's look at a single spiketrain with spikes described by a binary process (i.e. a 0 means no spike, 1 means spike). We observe N bins in an interval between 0 and T , for each bin we record whether or not a spike occurred. The figure below is an example of how your data might look like after recording $N = 20$ bins. For each of the problems below, give the general solution for any spike train and N bins. Do not use results from the lecture.

10 pts



Figure 1: An example spike train after observing $N = 20$ bins.

- (a) Use the Bernoulli model and assume i.i.d. bins to calculate the maximum likelihood (ML) estimate of the probability of a spike occurring in a bin.
 - (b) Rethink your approach and remember that we derived the binomial model in the lecture as well. Derive the ML estimator for μ (or p , the probability of a spike occurring) for the same problem using the binomial distribution.
 - (c) The Poisson distribution is often used to model spike trains. Here, however, we do not consider any bin size, but instead model how many spikes k occur in a fixed time interval between 0 and T . Derive a ML estimate for λ , after having observed k spikes in an interval.
2. **ADVANCED** Show that the sum of two independent Poisson random variables $X \sim \text{Poisson}(X = x|\lambda)$ and $Y \sim \text{Poisson}(Y = y|\mu)$ is again a Poisson distributed random variable $Z \sim \text{Poisson}(Z = z|\lambda + \mu)$
3. You have decided that a binary experiment you observe is best modeled by a Bernoulli distribution. Previously you have measured mean 0.3 from which you can also calculate the variance. The question you ask yourself is this: In the next 100 trials, what is the probability that I get 40 or less successes? Try and approach your problem using the Central Limit Theorem presented in the lecture.