Neuroinformatics Exercise
Fall 2017

Exercise E1

Name: _____
Pascal Nieters

Instructions: (10 points) Solve the following problems. Write clearly and use same symbols as used in the lecture. Add comments, explanations or questions to your solution if necessary.
Solutions to exercises **1** and **2** are to be submitted as physical copies in groups of three to four. Please label each page you hand in **clearly** and **carefully** with the name of each group member and their student ID. Deadline for this exercise sheet is: **10.11.2017**

(5 pts) **1.** A bag contains 15 balls distinguishable only by their colours; 10 are blue and 5 are red. You reach into the bag with both hands and pull out 2 balls (one with each hand, at once) and record their colours. Let's assume you always pick a ball with the left hand first and probabilities to pick each individual ball are uniform (*i.e.* $\frac{1}{15}$ *for the first draw.*)

5 pts

(a) What is the random variable?

(b) What is the sample space?

(c) Express the event that the ball in my left hand is red as a subset of the sample space.

(d) What is the probability for the event that the ball in my left hand is red?

(e) What is the probability for the event that exactly one of the two balls i drew is blue?

(5 pts) **2.** You want to know how probable it is for you to meet a student from a different field of study and expand your horizons in a bar you and your Cognitive Science friend like to visit. However, you feel it would be awkward to walk around the Schloss and ask strangers where they spend their nights. So instead you decide to infer this using the sum and product rule of probability. A very representative inquiry among your Coxi-friends tells you that about 70% of all Coxis seem to frequent the bar. Since the bar is rather small, you know that in total (at most) 6% of all students frequent the bar. Let's assume that there are roughly 650 enrolled Cognitive Science students and a total of about 13000 enrolled students at Osnabrück University.

5 pts

(a) What's the joint probability that a random student is a Coxi **and** frequents your favourite bar?

(b) What's the joint probability that a random student is not a Coxi **and** frequents your favourite bar?

(c) What's the conditional probability that, if they are not a Coxi, they frequent you favourite bar?

(d) What's the probability at least one of the first ten people you see at the bar is not a Cognitive Science student?

3. Your friend plays football for the local football team and he is a bit superstitious. He believes that he'll score a goal if there's no rain on the day of the match. You want to prove him wrong and decide to note, for 100 games, whether it was raining (R) or not raining ($\neg R$) and whether he scored a goal (G) or not ($\neg G$).

(a) Fill in the missing values in a table.

	R	$\neg R$	
G	0.21		
$\neg G$			$P(\neg G) = 0.4$
		$P(\neg R) = 0.65$	

(b) Prove that the variables R and G are independent.

4. Suppose that Bob can decide to go to work by one of three modes of transportation, car, bus, or commuter train. Because of high traffic, if he decides to go by car, there is a 50% chance he will be late. If he goes by bus, which has special reserved lanes but is sometimes overcrowded, the probability of being late is only 20%. The commuter train is almost never late, with a probability of only 1%, but is more expensive than the bus.

(a) Suppose that Bob is late one day, and his boss wishes to estimate the probability that he drove to work that day by car. Since he does not know which mode of transportation Bob usually uses, he gives a prior probability of 1/3 to each of the three possibilities. What is the bosss estimate of the probability that Bob drove a car to work?

(b) Suppose that a coworker of Bobs knows that he almost always takes the commuter train to work, never takes the bus, but sometimes, i.e. 10% time, takes the car. What is the coworkers probability that Bob drove to work that day, given that he was late?