Instructions: (10 points) Solve the following problems. Write clearly and use same symbols as used in the lecture. Add comments, explanations or questions to your solution if necessary.

Solutions to exercises **1** and **2** are to be submitted as physical copies in groups of three to four. Please label each page you hand in **clearly** and **carefully** with the name of each group member and their student ID. **Please staple all of your sheets.**
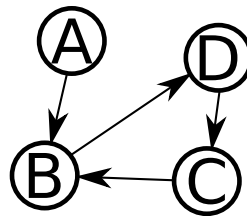
Deadline for this exercise sheet is: **17.11.2017**

(5^{pts}) **1.** For the following Graphs, check if they are cyclic or not. If they are cyclic, write down the cycle. If they are not, write down the joint probability distribution
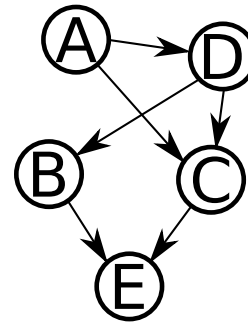
<div style="border:1px solid">5 pts</div>
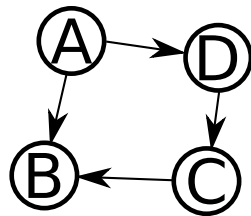
a)   b)   c)

(d) Draw the graph that corresponds the following joint probability distribution:
$$p(a,b,c,d) = p(a|d)p(b|c,a)p(c)p(d|c)$$

(e) Draw the graph that corresponds the following joint probability distribution:
$$p(a,b,c,d,e) = p(a|b,c)p(b|d)p(c|d,e)p(d)p(e|b)$$

(5^{pts}) **2.** Given the graph below, answer the following questions using the D-separation property. Your answers should contain the following information:

<div style="border:1px solid">5 pts</div>

(i) Write down the path from the first to the second node of interest.

(ii) Indicate how the arrows on that path meet.

(iii) Is the path blocked? Are the nodes d-separated and therefore independent?
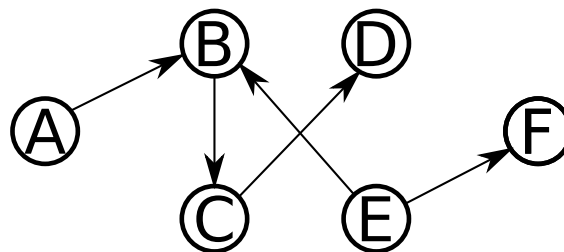
(a) Node **B** is observed, are nodes **A** and **E** independent?

(b) Node **B** is observed, are nodes **A** and **D** independent?

(c) Node **E** is observed, are nodes **A** and **F** independent?

3. Let's start with the following probability distribution:

$$p_x(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Verify that $p_x(x)$ is a probability distribution.

(b) Let $X \sim p_x(x)$. Calculate the expected value $\mathbb{E}[X]$

(c) Let the random variable $Y \sim p_y(y)$ and $y = y(x) = x^2$. Determine $p_y(y)$.

(d) Verify that $p_y(y)$ is a probability distribution.

(e) Calculate the expected value $\mathbb{E}[Y]$.

4. For the following graph, first check whether a and d are independent using the d-separation property. Then, write the joint distribution and prove independence analytically.