Exercise Sheet 4 Neuroinfo

jerfranz 967647
mikang 971312
jzerfowski 967786

~~1a)~~ 1a) Bernoulli-Distribution:

$$p(x_i|\mu) = \mu^{x_i}(1-\mu)^{1-x_i}$$

Assumption i.i.d $p(\vec{x}|\mu) = p(x_1, \dots, x_N) = \prod_{i=1}^{N} p(x_i|\mu)$

$$p(\vec{x}|\mu) = \prod_{i=1}^{N} \mu^{x_i}(1-\mu)^{1-x_i}$$

"$\log(p\dots) = \log(\prod\dots) = \sum \log(\dots)$"

$$\Rightarrow \log(p(\vec{x}|\mu)) = \sum_{i=1}^{N} \log\left[\mu^{x_i}(1-\mu)^{1-x_i}\right] = \sum_{i=1}^{N}\left[\log(\mu^{x_i}) + \log((1-\mu)^{1-x_i})\right]$$

"$\log(a^b) = b\cdot\log(a)$" $\Rightarrow$ $= \sum_{i=1}^{N}\left[x_i\log(\mu) + (1-x_i)\log(1-\mu)\right] = f(\mu)$

$$\frac{d}{d\mu}f(\mu) = \frac{\partial}{\partial\mu}f(\mu) \underset{\substack{\uparrow \\ \text{to maximize}}}{\overset{!}{=}} 0 \Leftrightarrow \sum_{i=1}^{N}\left[\frac{x_i}{\mu} + \frac{1-x_i}{1-\mu}(-1)\right] \overset{!}{=} 0 \qquad |\mu\cdot(1-\mu)$$

$$\Rightarrow \sum_{i=1}^{N} \underbrace{x_i(1-\mu) - (1-x_i)\mu}_{= x_i - \cancel{x_i\mu} - \mu + \cancel{x_i\mu}} \overset{!}{=} 0 \Rightarrow \sum_{i=1}^{N}(x_i - \mu) \overset{!}{=} 0$$

$$\Rightarrow \sum_{i=1}^{N} x_i - \sum_{i=1}^{N}\mu = \sum_{i=1}^{N}x_i - N\mu \overset{!}{=} 0 \Rightarrow \sum_{i=1}^{N}x_i = N\mu \Leftrightarrow \mu_{ML} = \frac{1}{N}\sum_{i=1}^{N}x_i$$

The maximum likelihood estimation for μ is $\mu_{ML} = \frac{1}{N}\sum_{i=1}^{N}x_i$

1b) Binomial Distribution assumed

$$p(m|\mu,N) = \binom{N}{m} \mu^m (1-\mu)^{N-m} \qquad | \log(a \cdot b \cdot c) = \log a + \log b + \log c$$

$$\log(p(m|\mu,N)) = \log\left(\binom{N}{m}\right) + \log(\mu^m) + \log\left((1-\mu)^{N-m}\right)$$

$$= \log\left(\binom{N}{m}\right) + m \log(\mu) + (N-m)\cdot\log(1-\mu) \qquad \left| \frac{\partial}{\partial\mu}\right.$$

$$\frac{\partial}{\partial\mu}\log(p) = 0 + \frac{m}{\mu} + \frac{N-m}{1-\mu}\cdot(-1) \stackrel{!}{=} 0 \qquad | \cdot \mu\cdot(1-\mu)$$

$$0 \stackrel{!}{=} m\cdot(1-\mu) + (N-m)\cdot\mu\cdot(-1) = m - \mu\cdot N$$

$$\Rightarrow \underline{\underline{\mu_{ML} = \frac{m}{N}}}$$

The maximum likelihood estimation for μ for a binomial distribution is $\mu_{ML} = \frac{m}{N}$

Neuroinformatics E4

1c) ML-estimate for λ after observing k spikes in an Interval

$p(k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}$ Maximize this for λ: $\underset{\lambda}{\operatorname{argmax}} \frac{\lambda^k e^{-\lambda}}{k!}$

$\Rightarrow \log\left(\frac{\lambda^k e^{-\lambda}}{k!}\right) = -\log(k!) + \log(\lambda^k) + \log(e^{-\lambda}) = -\log(k!) + k \cdot \log(\lambda) - \lambda \log(e)$

derive for λ: $\frac{d}{d\lambda} -\log(k!) + k \cdot \log(\lambda) - \lambda \cdot 1 = \frac{k}{\lambda} - 1$

set $= 0$: $\frac{k}{\lambda} - 1 = 0 \Rightarrow \frac{k}{\lambda} = 1 \Rightarrow \lambda = k$

The maximum likelihood estimate for λ is $\underline{\underline{\lambda_{ML} = k}}$