

3장 확률

3.1

- (1) ① C를 대장암이라고 할 때, $P(A^c \cup C) = P(A^c) + P(C) - P(A^c \cap C) = 0.6 + 0.2 - 0.1 = 0.7$
 ② $P(B^c \cap A) = 0.05 + 0.1 + 0.1 + 0.1 = 0.35$
 (2) $P(A \cap B) = 0.05 \neq P(A)P(B) = 0.4 \times 0.25 = 0.1$ 이므로 독립아님
 (3) ① $0.6 \times 0.4 + 0.4 \times 0.5 = 0.44 \Rightarrow 44\%$
 ② $0.6 \times 0.4 / (0.6 \times 0.4 + 0.4 \times 0.5) = 0.24 / 0.44 = 6/11$
 ③ 완치된 여자의 비율은 전체 환자의 20%(=0.4×0.5)이므로 $40/0.2 = 200$ 명

3.2 암-A : $p(+|A) = 0.95$, $p(+|A^c) = 0.05$, $p(A) = 0.005$

- (1) $P(A^c) = 1 - 0.005 = 0.995$
 (2) $P(+) = P(A)P(+|A) + P(A^c)P(+|A^c) = 0.005 \times 0.95 + 0.995 \times 0.05 = 0.0545$
 (3) $P(A^c \cap +) = P(A^c)P(+|A^c) = 0.995 \times 0.05 = 0.04975$
 (4) $P(+^c|A) = 1 - 0.95 = 0.05$
 (5) $P(A|+) = \frac{P(A)P(+|A)}{P(A)P(+|A) + P(A^c)P(+|A^c)} = \frac{0.005 \times 0.95}{0.005 \times 0.95 + 0.995 \times 0.05} = \frac{0.00475}{0.0545} = 0.0872$

3.3 A: 50명, B: 40명, C:10명,

A(찬성) : 35명, B(찬성): 8명, C(찬성): 4명

- (1) $\frac{35+8+4}{100} = \frac{47}{100} = 0.47$
 (2) $P(C|\text{찬성}) = \frac{P(C)P(\text{찬성}|C)}{P(C)P(\text{찬성}|C) + P(A)P(\text{찬성}|A) + P(B)P(\text{찬성}|B)} = \frac{0.1 \times 0.4}{0.1 \times 0.4 + 0.4 \times 0.2 + 0.5 \times 0.7} = 0.0851$
 (3) ① $\Omega_1 = \{(\text{찬}, \text{반}), (\text{반}, \text{찬}), (\text{찬}, \text{찬}), (\text{반}, \text{반})\}$
 ② (찬성, 반대) $\rightarrow \frac{47}{100} \times \frac{53}{99}$, (반대, 찬성) $\rightarrow \frac{53}{100} \times \frac{53}{99}$, (찬성, 찬성) $\rightarrow \frac{47}{100} \times \frac{46}{99}$, (반대, 반대) $\rightarrow \frac{47}{100} \times \frac{52}{99}$
 (4) $\frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}} = \frac{4 \times 3 \times 2}{10 \times 9 \times 8} = \frac{1}{30}$

3.4

- (1) $P(\text{①}) \times P(\text{②}) = 0.9 \times 0.8 = 0.72$
 (2) $P(\text{①} \cup \text{②}) = P(\text{①}) + P(\text{②}) - P(\text{①} \cap \text{②}) = 0.9 + 0.8 - 0.72 = 0.98$
 (3) $\frac{0.9 \times 0.8}{0.9 \times 0.8 + 0.9 \times 0.2 + 0.8 \times 0.1} = \frac{0.72}{0.98} = 0.735$

3.5

- (1) $\Omega = \{(\text{ON}, \text{OFF}, \text{OFF}), (\text{OFF}, \text{ON}, \text{OFF}), (\text{OFF}, \text{OFF}, \text{ON}), (\text{ON}, \text{OFF}, \text{ON}),$
 $(\text{OFF}, \text{ON}, \text{ON}), (\text{ON}, \text{ON}, \text{OFF}), (\text{ON}, \text{ON}, \text{ON}), (\text{OFF}, \text{OFF}, \text{OFF})\}$
 (2) $P(A)=P(B)=P(C)=0.6$
 ① i) $0.6 \times 0.4 \times 0.4 = 0.096$
 ii) $0.4 \times 0.6 \times 0.4 = 0.096$
 iii) $0.4 \times 0.4 \times 0.6 = 0.096$
 iv) $0.6 \times 0.4 \times 0.6 = 0.144$
 v) $0.4 \times 0.6 \times 0.6 = 0.144$
 vi) $0.6 \times 0.6 \times 0.4 = 0.144$
 vii) $0.6 \times 0.6 \times 0.6 = 0.216$
 viii) $0.4 \times 0.4 \times 0.4 = 0.064$

$$\textcircled{2} \quad 0.144+0.144+0.144=0.432$$

$$\textcircled{3} \quad \frac{0.144+0.144}{0.432} = 0.667$$

$$\textcircled{4} \quad \frac{0.096+0.144+0.144}{3 \times 0.096+0.432+0.064} = \frac{0.384}{0.784} = 0.49$$

3.6 $P(A)=0.4, P(B)=0.3, P(C)=0.3,$

D: 찾을 사건이라고 하면 $P(D|A)=0.5, P(D|B)=0.6, P(D|C)=0.4$

(1) $P(A \cap D) = P(A)P(D|A) = 0.4 \times 0.5 = 0.2, P(B \cap D) = 0.18, P(C \cap D) = 0.12$ 이므로 A지역

(2) $P(D)=P(A)P(D|A)+P(B)P(D|B)+P(C)P(D|C) = 0.4 \times 0.5 + 0.3 \times 0.6 + 0.3 \times 0.4 = 0.5$

(3) E: A 지역을 수색한 결과 비행기를 찾지 못했을 사건

$$P(E) = P(A \cap E) + P(A^c \cap E) = P(A)P(E|A) + P(A^c)P(E|A^c) = 0.4 \times 0.5 + 0.6 \times 1 = 0.8$$

$$\textcircled{1} \quad P(A|E) = P(A)P(E|A)/P(E) = 0.2/0.8 = 0.25$$

$$\textcircled{2} \quad P(C|E) = P(C)P(E|C)/P(E) = 0.3/0.8 = 3/8$$

3.7 D: 다름 사건이라고 하면 $P(D)=0.2$

(1) 0.5648

0	0	0	0	→	$0.8 \times 0.8 \times 0.8 \times 0.8 = 0.4096$
0	0	1	0	→	$0.8 \times 0.8 \times 0.2 \times 0.2 = 0.0256$
0	1	0	0	→	$0.8 \times 0.2 \times 0.2 \times 0.8 = 0.0256$
1	0	0	0	→	$0.2 \times 0.2 \times 0.8 \times 0.8 = 0.0256$
1	1	0	0	→	$0.2 \times 0.8 \times 0.2 \times 0.8 = 0.0256$
1	0	1	0	→	$0.2 \times 0.2 \times 0.2 \times 0.2 = 0.0016$
0	1	1	0	→	$0.8 \times 0.2 \times 0.8 \times 0.2 = 0.0256$
1	1	1	0	→	$0.2 \times 0.8 \times 0.8 \times 0.2 = 0.0256$

(2) $P(1\text{발신})=0.4, P(0\text{발신})=0.6$

$$P(0\text{발신}|1\text{수신}) = \frac{P(0\text{발신})P(1\text{수신}|0\text{발신})}{P(0\text{발신})P(1\text{수신}|0\text{발신}) + P(1\text{발신})P(1\text{수신}|1\text{발신})} = \frac{0.6 \times (1 - 0.5648)}{0.6 \times (1 - 0.5648) + 0.4 \times 0.5648} = 0.536$$

(3) - 제대로 전달 : 잡음 0개 or 2개 or 4개

- 제대로 전달 안됨: 잡음 1개 or 3개

- 제대로 전달될 확률 : $P(X=0)+P(X=2)+P(X=4)$

$$= 0.8^4 + \binom{4}{2} 0.8^2 0.2^2 + 0.2^4 = 0.5648$$

- 제대로 전달 안될 확률 : $P(X=1)+P(X=3)$

$$= \binom{4}{1} 0.8^3 + \binom{4}{3} 0.8 0.2^3 = 0.4352$$

- 2개 이상일 확률 : $1 - P(0\text{개}) - P(1\text{개})$

$$1 - \binom{5}{0} 0.5648^0 0.4352^5 - \binom{5}{1} 0.5648^1 0.4352^4 = 0.8831$$

3.8 5전3선승제로 문제수정

(1) $\Omega = \{(DDD), (DDL D), (DLDD), (LDDD), (DDL L), (DL D L), (DLL D), (LDD L), (LDL D), (LLDD), (LLL), (LLDL), (LDLL), (DLLL), (LLDDL), (LDLDL), (LDDL), (DLLDL), (DLDDL), (DDL L L)\}$

(2) ① $1/8 + 3 \times 1/16 + 6 \times 1/32 = 0.5$

② $\Omega' = \{(DDD), (DDL D), (DLDD), (DLLL), (DDL L), (DL D L), (DLL D), (LDDL), (LDL D), (LLDD), (LDL D), (LLDD), (LLDD)\}$
 $(1/16 + 3 \times 1/32)/0.5 = 0.3125$

③ $\Omega' = \{(DDD), (DDL D), (DLDD), (LDDD), (DDL L), (DL D L), (DLL D), (LDDL), (LDL D), (LLDD), (LLDD)\}$
 $(1/16 + 3 \times 1/32)/0.5 = 0.3125$

3.9

(1) $\Omega = \{(RBB), (BRB), (BBR), (RRB), (RBR), (BRR), (RRR)\}$

$$(RBB) \rightarrow \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{1}{10}, (BRB) \rightarrow \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{10}, (BBR) \rightarrow \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3} = \frac{1}{10}, (RRB) \rightarrow \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5}$$

$$(RBR) \rightarrow \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5}, (BRR) \rightarrow \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{5}, (RRR) \rightarrow \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{1}{10}$$

$$(2) \quad 1/10 + 1/5 + 1/5 + 1/10 = 0.6$$

$$(3) \quad \text{정확히 알 수 있는 경우: } BBR \Leftrightarrow P(BBR) = 1/10$$

$$(4) \quad (1/10 + 1/5 + 1/5 + 1/10) / (1 - 1/10) = 6/9 = 2/3$$

3.10 D: 불량이라고 하면 $P(L1)=0.2$, $P(L2)=0.5$, $P(L3)=0.3$, $P(D|L1)=0.15$, $P(D|L2)=0.08$, $P(D|L3)=0.1$

$$(1) \quad P(L2 \cap D^c) = P(L2)P(D^c|L2) = 0.5(1 - 0.08) = 0.46$$

$$(2) \quad P(D) = P(L1)P(D|L1) + P(L2)P(D|L2) + P(L3)P(D|L3) = 0.2 \times 0.15 + 0.5 \times 0.08 + 0.3 \times 0.1 = 0.1$$

$$(3) \quad P(L1|D) = \frac{P(L1)P(D|L1)}{P(D)} = \frac{0.2 \times 0.15}{0.1} = 0.3$$

3.11

$$\begin{aligned} P(\text{전기 전달}) &= P((C \cap D) \cup (E \cap F)) \\ &= P(C \cap D) + P(E \cap F) - P((C \cap D) \cap (E \cap F)) \\ &= P(C)P(D) + P(E)P(F) - P(C)P(D)P(E)P(F) \end{aligned}$$

$$3.12 - P(\text{Yes}) = P(\text{Yes} \cap \text{앞면}) + P(\text{Yes} \cap \text{뒷면}) = P(\text{앞면})(\text{Yes}|\text{앞면}) + P(\text{뒷면})(\text{Yes}|\text{뒷면})$$

$$\Leftrightarrow 40/100 \approx 0.5 \times 0.5 + 0.5P(\text{부정행위})$$

$$\therefore P(\text{부정행위}) \approx (0.4 - 0.25)/0.5 = 3/10$$