

4장 확률분포

4.1

$$(1) k = 1 - 0.2 - 0.25 - 0.3 = 0.25, E(X) = 1 \times 0.2 + 2 \times 0.25 + 4 \times 0.25 + 8 \times 0.3 = 4.1$$

$$(2) k = \frac{1}{1 + 0.5 + 0.5^2} = \frac{4}{7}, E(X) = 1 \times \frac{0.5}{1.75} + 2 \times \frac{0.5^2}{1.75} = \frac{4}{7}$$

$$(3) \int_0^1 kx dx = 1 \Rightarrow k = 2, E(X) = \int_0^1 2x^2 dx = \frac{2}{3}$$

$$(4) \int_0^\infty ke^{-x} dx = 1 \Rightarrow k = 1, E(X) = \int_0^\infty xe^{-x} dx = 1$$

4.2

$$(1) P(X = 2 | X \neq 1) = \frac{0.5}{1 - 0.25} = \frac{2}{3}$$

$$(2) E(X) = 1 \times 0.25 + 2 \times 0.5 + 3 \times 0.15 + 4 \times 0.1 = 2.1$$

4.3

$$(1) \Omega = \{H, TH, TTH, TTTH, TTTT\}$$

근원사건	H	TH	TTH	TTTH	TTTT
확률	0.4	0.24	0.144	0.0864	0.1296

x	0	1
$f(x)$	0.1296	0.8704

$$(3) E(X) = 0 \times 0.1296 + 1 \times 0.8704 = 0.8704$$

$$E(X^2) = 0 \times 0.1296 + 1 \times 0.8704 = 0.8704$$

$$Var(X) = E(X^2) - E(X)^2 = 0.1128$$

$$(4) \text{ i) } P(A \text{의 던진 횟수} = 1) \times P(B \text{의 던진 횟수} \neq 1) = 0.4 \times 0.6 = 0.24$$

$$\text{ ii) } P(A \text{의 던진 횟수} = 2) \times P(B \text{의 던진 횟수} = 3 \text{ or } 4) = 0.24 \times 0.36 = 0.0864$$

$$\text{ iii) } P(A \text{의 던진 횟수} = 3) \times P(B \text{의 던진 횟수} = 4) = 0.144 \times 0.216 = 0.0311$$

$$\therefore P(A \text{ 승리}) = 0.3575$$

4.4

x	0	1	2	3
$f(x)$	0.064	0.288	0.432	0.216

$$(2) P(X = 2) = 0.432$$

$$(3) E(X) = 0 \times 0.064 + 1 \times 0.288 + 2 \times 0.432 + 3 \times 0.216 = 1.8$$

$$E(X^2) = 0 \times 0.064 + 1^2 \times 0.288 + 2^2 \times 0.432 + 3^2 \times 0.216 = 3.96$$

$$Var(X) = E(X^2) - E(X)^2 = 0.72$$

4.5

$$(1) \Omega = \{R, FR, FFR, FFFR, FFFF\}$$

$$P(R) = 0.4, P(FR) = 0.24, P(FFR) = 0.144, P(FFFR) = 0.0864, P(FFFF) = 0.1296$$

$y \backslash x$	1	2	3	4	$f_Y(y)$
0	0	0	0	0.1296	0.216
1	0.4	0.24	0.144	0.0864	0.784
$f_X(x)$	0.4	0.24	0.144	0.216	1

$$(2) \text{ ① 독립이 아님. } P(X = 4, Y = 0) = 0.0864 \neq P(X = 4) \times P(Y = 0) = 0.0467$$

$$\text{ ③ } E(X + Y) = E(X) + E(Y) = 2.716 + 0.784 = 2.96, E(XY) = \sum_x \sum_y xyf(x, y) = 1.6576$$

$$\text{ ④ } E(Y) = 0.784, E(Y^2) = 0.784, Var(Y) = E(Y^2) - E(Y)^2 = 0.1693$$

4.6 : 5전3선승제로 문제수정

(1)

y	0	1	2	3
$f_Y(y)$	$\frac{1}{2^3}$	$\frac{3}{2^4}$	$\frac{6}{2^5}$	$\frac{1}{2^3} + \frac{3}{2^4} + \frac{6}{2^5} = \frac{1}{2}$

(2) $E(Y) = 0 \times 1/2^3 + \dots + 3 \times 0.5 = 2.0625 < 2.5$,
 $E(Y^2) = 0^2 \times 1/2^3 + \dots + 3^2 \times 0.5 = 5.4375$
 $Var(Y) = E(Y^2) - E(Y)^2 = 1.1836$

(3)

$x \backslash y$	3	4	5	$f_Y(y)$
0	$1/2^3$	0	0	$1/8$
1	0	$3/2^4$	0	$3/16$
2	0	0	$6/2^5$	$3/16$
3	$1/2^3$	$3/2^4$	$6/2^5$	$1/2$
$f_X(x)$	$1/4$	$3/8$	$3/8$	1

(4) 세 번 해야하는 경우는 {DDD, LLL}
 $P(D_1 D_2 D_3) = P(D_1)P(D_2|D_1)P(D_3|D_1 D_2) = 0.5 \times 0.6 \times 0.6 = 0.13$
 $\Rightarrow P(DDD) + P(LLL) = 0.13 \times 2 = 0.26$

4.7

(1)

$x \backslash y$	1	2	3	$f_Y(y)$
1	0.3	0.6	0	0.9
3	0	0	0.1	0.1
$f_X(x)$	0.3	0.6	0.1	1

(2) 독립이 아님. $\because P(X=1, Y=1) = 0.3 \neq 0.27 = P(X=1)P(Y=1)$
(3) $Cov(X, Y) = 0.24$, $Corr(X, Y) = 2/3$

4.8

(1) $E(X) = 1$, $Var(X) = 0.7$
(2) $P(X \geq 2 | X \geq 1) = \frac{5}{14}$
(3) $1 - P(X=0)^5 = 0.9976$

4.9

(1) $\Omega = \{AA, AB, AC, AD, BA, BB, BC, BD, CA, CB, CC, CD, DA, DB, DC, DD\}$
 $\Omega = \{8, 7, 6, 5, 4, 3, 2\}$

성적	AA	AB	AC	AD	BA	BB	BC	BD
확률	0.09	0.12	0.075	0.015	0.12	0.16	0.1	0.02
성적	CA	CB	CC	CD	DA	DB	DC	DD
확률	0.075	0.1	0.0625	0.0125	0.015	0.02	0.0125	0.0025

(2)

평균학점	1	1.5	2	2.5	3	3.5	4
확률	0.0025	0.025	0.1025	0.23	0.31	0.24	0.09

(3) $P(X \leq 1.5) = 0.0275$
(4) $E(X) = 2.95$, $Var(X) = 0.37375$

4.10

(1)

x	60	100	110	150	200	510	550	600
$f(x)$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{2}{15}$

(2)

 $P(X \leq 100) = 1/5$

(3)

 $E(X) = 270, \text{ } Var(X) = 44200$

(4)

x	20	60	100	110	150	200	510	550	600	1000
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{4}{36}$	$\frac{4}{36}$	$\frac{8}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{4}{36}$	$\frac{1}{36}$