FACULTY OF ENGINEERING SCIENCE
DEPARTMENT OF COMPUTER SCIENCE
MASTER OF ARTIFICIAL INTELLIGENCE
CELESTIJNENLAAN 200 A BOX 2402 3001 LEUVEN

KU LEUVEN

Assignment For Financial Engineering(G0Q22a)

Junshu Jiang - r0870105

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Contents

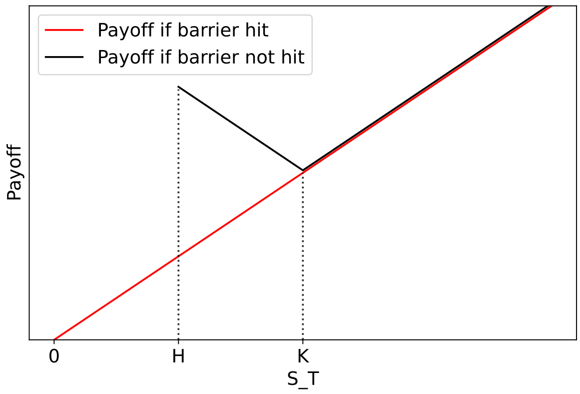


Figure 1.1: Payoff of Twin-Win

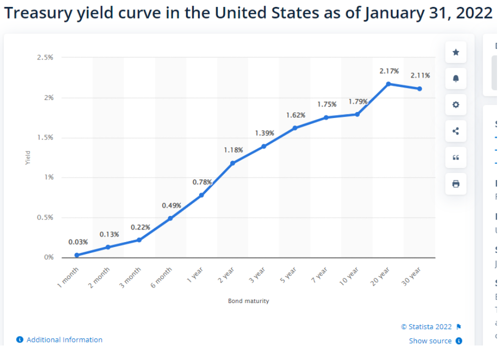
1 Product Description

In this report, a particular structuralized product called Twin-Win certificate is built. This Twin-Win certificate, whose underlying asset is Regeneron Pharmaceuticals, Inc.(NASDAQ: REGN), can help customs avoid part of downside risk and enjoy the appreciation of underlying assets simultaneously. Twin-Win Certificates are a particular variant of the bonus certificate that generate a positive return on their investment in both downside and upside markets, as long as the underlying asset price does not hit through a predetermined barrier.

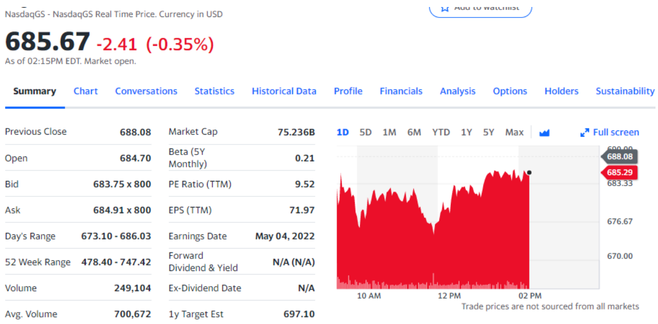
It suits customs that have no strong feeling about the directions of the future market movement and hope to protect the invested capital. The clients will suffer loss only when the prices fall below the Barrier price. By consulting the implied volatility given by Heston(See Section 3 for more) which is 0.359, we set the **barrier price one sigma away** from the current stock price. Under this setting, **the probability that our clients will not suffer loss will be** $1 - \Phi(-\sigma) = 84.1\%$**.**

The payoff structure of our Twin-Win products is demonstrated in Figure 1.1. The price for one Twin-Win certificate is 762.8. The strike price(K) is 688, the lower barrier(H) is 512 and the lockout period is 6 months from now. The red line is the payoff when the market does hits the barrier price during the lockout period. If the barrier price is not hit, the black line represents the payoff.

The rest of this report organize as follows: chapter 2 describes the sources of the data, other parameters, and the settings that we used in this report. Chapter 3 describe the Heston model we adapt to modeling the process of underlying assets. Also the calibration to estimate the parameters which best fit the market are also covered in Chapter 3. Chapter 4 demonstrates the details of the Monte Carlo simulation that we used to price the Twin-Win product. Theoretically, the Twin-Win can be perfectly replicated by a portfolio consisting of one underlying stock and two Down-Out-Barrier-Put(DOBP) options. According to arbitrage-free theory, the price of those two should be the same. In chapter 5, we study the sensitivity of our product to the price change of underlying assets and conduct the delta hedge. Since delta risk is a significant component of the overall risks, this analysis can guide us to do the delta hedge and reconcile the risk to a large extent. Chapter 5 also includes the detailed recipe to hedge the delta risk when 1,000,000 dollar is invested in our account. Section 6 concludes this report by highlight the merits of this report.



(a) Risk-free rate



(b) Stock Price

Items	Value	Sources/ Notes
Start date	05-May-2022	
End data	05-Nov-2022	
Lockup period	0.5 yr	
Current Stock Price	687.67	collected From Nasdaq
Strike Price	688	Current stock price
Risk-free Rate	0.5%	collected From Statista
Barrier Price	512	Barrier price with 1 std. lower
Margin charged	5%	
Option Price	See attached file	collected From Nasdaq(See attached file)

Table 2.1: Aggregation of the settings

2 Data Source and Setting-ups

This section clarifies each source of the data we used in this report. Also, some critical settings are explained in this section.

The start date we adapt is 5th May 2022 when the stock price is 687.67. The options price we used is from NASDAQ. Since we also need the risk-free rates, we take the interest rate of US Treasure as the counterpart to the risk-free rate. We also investigated the dividend history of REGN and found no dividend was paid in the history of this company. Therefore, we assume the dividend payments will not happen in the lockup period.

The lockup period is set to be six months. We believe this is a reasonable and attractive period. Currently, the volatility of the market is high. Hence, investors are inclined to be more risk-averted this year. The strike price is set to be the nearest price to the current price, and the barrier price is set one standard deviation away from the current price. **Setting K and H in this way can make we are in the lowest point of V-shape(See Figure 1.1) and guarantee profits in both downside and upside. Tables 2.1 summarize all the setting ups.**

The margin we charge is 5% and we justify our relative high margin with two reasons. First, this structure product is complicated, and we ask a high margin for our efforts. **Second, we only do the delta hedge and leave other Greeks exposure uncovered so we need more margin to compensate these risks.**

The source code for this assignment is attached in separate files. The 'FE.ipynb' demonstrate detailed steps about how to reproduce this result. Option datas are also included to ensure the result can be reproduced in a one-click manner.

3 Heston Model and calibration

Heston model is an advanced version of the Balck-Scholes-Merton model, which also models the stochasticity in the volatility. The SPDE of the Heston model is given in Formula 2 below:

$$\frac{dS_t}{S_t} = (r - q)dt + \sqrt{V_t}dW_t,\ S_0 >= 0 \tag{1}$$

$$dV_t = \kappa(\eta - V_t)dt + \theta\sqrt{V_t}\tilde{W}_t,\ V_0 = \sigma^2 >= 0 \tag{2}$$

The process of volatility is modeled by a CIR process(As known as the Feller square root process). This process can model the property of **mean-reverting**, which consists of the phenomenon of volatility clustering in markets. Another popular mean-reverting process is the Ornstein-Uhlenbeck process(OU). However, CIR is superior since it constrains the volatility larger than zero. The W_t and $\tilde{W}_t$ are correlated Brownian motions with coefficient ρ.

To make the Heston applicable in practice, there are some implied constraints in the parameters for Heston.

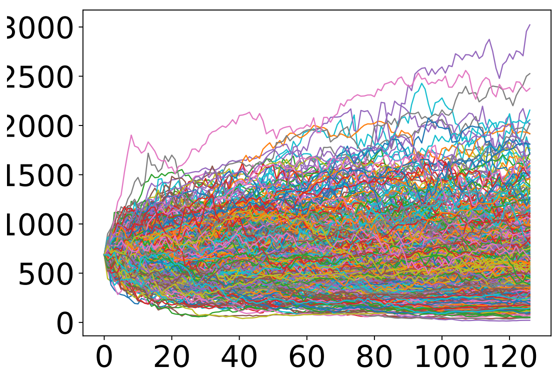
- $\kappa > 0$, so the speed of mean-reverting is larger than 0.
- $\eta > 0$, level of means reverting & volatility at the stable point
- $\theta > 0$, the level of the volatility of the volatility
- $-1 < \rho < 1$, coefficiency between two Brownian motions. Usually, in the stock market, this parameter is smaller than 0, and in the exchange market, the parameter is larger than zero. **The sign of this parameter reflects the direction of the fear in the market. When the market moves in an unwanted direction, the participant begins to panic, and the market's volatility increases**.
- $V_0 > 0$ is the initial volatility.
- $2\kappa\eta >= \theta^2$, This is Feller conditions which prevent the volatility from hitting zeros in **sheer continuous situation**. However, because of the discrete error that we introduced in the numerical experiment, the volatility still hit the zero line with some chances. **In this study, we handle this problem by simply use the absolute value of the volatility(This can be viewed as volatility bouncing back when hit the zero)**.

Carr-Madan formula gives the formula for European options and here we use Faster Fourier Transformation(FFT) to calculate the integral part efficiently. The formula can be found in the slides of this course.

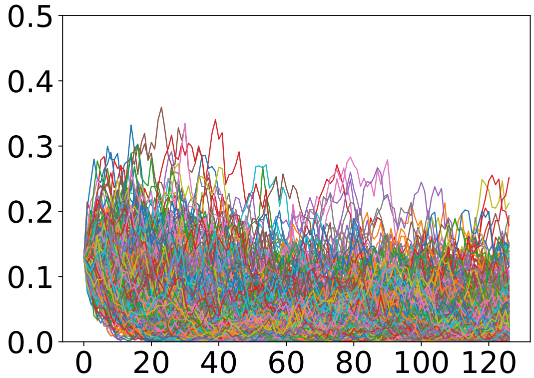
To get the best parameter to represent the market feature, we do the calibration with the market data. First, we define the error function, which measures the difference between the theoretically price given by the model and the agreed price in the market. Then, we use initial guesses to initialize the Heston model, and lastly, we optimize the error function by adjusting those 5 Parameters. **The error function we used here is Mean Squared Error(MSE), and the optimization method we used is Simplex**. Empirically, the first row of table 3.1 lists the parameters that give good results, and we use these as seed parameters for initialization. Another thing that needs to bear in mind is not all the market data is helpful, and they need to be used selectively. **We filter the data far away from the market since its liquidity is not enough to reveal its intrinsic price.** After 157 iterations, the error has been reduced from 20.2 to 1.6. **Also, those parameters have been checked to ensure they comply with constrains introduced above**.

	Kappa	Eta	Theta	Rho	V_0	Error(MSE)	Iterations
Initial Value	0.5	0.05	0.2	-0.75	0.5	20.2	0
Converged Value	Kappa	Eta	Theta	Rho	0.13	1.6	157

Table 3.1: Parameters for Heston



(a) 500 path samples for REGN stock price



(b) 500 path samples for REGN stock volatility

4 Pricing Twin-Win by Monte Simulation

After calibration with the market data, we obtain the Heston model with the best parameters. Given that the payoff of Twin-Win is path-dependent and that the analytical solution for Twin-Win is not available under Heston, the only method we can resort to is Monte Carlo simulation. With the help of the Heston model, we first simulate many paths, and then we calculate the average payoff.

We use the Euler scheme to simulate the path. Another scheme is Milstein which takes into account the second order approximation to reduce the error. We choose Euler because of its simplicity and efficiency. Their formulas are given in 3 below:

$$\begin{aligned} S_{t_{i+1}} &= S_{t_i}(1 + (r-q)\Delta t + \sqrt{V_{t_i}\Delta t}\varepsilon_1) \\ V_{t_{i+1}} &= V_{t_i} + \kappa(\eta - V_{t_i})\Delta t + \lambda\sqrt{V_{t_i}\Delta t}\varepsilon_2, \quad \textit{Euler Scheme} \\ V_{t_{i+1}} &= V_{t_i} + (\kappa(\eta - V_{t_i}) - \lambda^2/4)\Delta t + \lambda\sqrt{V_{t_i}\Delta t}\varepsilon_2 + \lambda^2\Delta t(\varepsilon_2)^2/4, \quad \textit{Milstein Scheme} \end{aligned} \tag{3}$$

We set the number of paths to be 200,000 and the number of timesteps to 100. **The number of paths should be big enough to make the average price converge to the expected price in probability**. Figure 4.1a and 4.1b demonstrate the sampling path of stock price and the volatility. **It shows the volatility will fluctuate around 0.359 from the long range and the stock moves resembling to diffusion process.**

As discussed in Section 1, the Twin-Win is equivalent to a zero-strike call option plus 2 DOBP options, and the equation $TW = S_T + 2DOBP$ should hold. Therefore, we first do the pricing for the DOBP and then calculate the price for the Twin-Winas a whole. **The price for DOBP is 18.8 and the price for the given Twin-Win certificate for one share is 762.8 which includes a margin of 5% and one stock and two DOBP.**

Items	Value
Stock	834,544(1213 shares)
Bank account	115,456
Margin charged	50,000
Sum	**1,000,000**

Table 5.1: Components for an investment with notional value 1,000,000

5 Sensitivity Analysis and Delta Hedge

To hedge the risk as much as possible, exposure to different kind of risks need to analyze. Typically, the five most significant risks are delta, gamma, vega, rho and theta which represents the risk of price movement of underlying in first order, the risk of underlying in second order, the risk of the change of volatility, the risk or the change of interest rate, and the risk of time elapsing. They are called Greeks in short.

There is no analytical solution for the Twin-Winand hence there is no analytical solution for different kind of risks. Luckily, we can still use Monte Carlo to estimate the current Greeks by changing the S_0. Then with the formula $delta = \lim_{\Delta \to 0} \frac{O(S_t)-O(S_t-\Delta)}{\Delta}$, we can get the approximation of the delta.

Among those Greeks, delta is the most significant one. Given that the Twin-Winis the collection of one stock and two DOBP and the **delta of the portfolio is the summation of the delta for each of components**. Therefore, we only measure the delta for the DOBP options. To do this, we vary the $S0$ from 450 to 900 and price the corresponding Twin-Win. **Since this process will conduct Monte Carlo for dozens of times, we do it in a parallel manner to accelerate it.**

The plot of the DOBP and the S_0 is in Figure 5.1 and the slope -0.0385 of the tangent line is the Delta for the DOBP at the start times. **So the overall delta for our Twin-Win is** $1+2\times\Delta_{DOBP}$ **is 0.925**. One can spot the delta is changing when the underlying asset's price varies. When the stock price falls a little, the value of DOBP increases since it is more in the money. But it will decrease when the price drops too much since the likelihood that the stock will hit the barrier will increase. Those two effects cancel each out and jointly make the highest point of the price somewhere between the H and the K. **Given the factor the delta(AKA the slope of the curve in Figure 5.1) varying in the stock price and the factor that the curve itself evolves in time, delta hedging in a dynamic manner is needed. The delta needs to recalculate, and each asset's weight in our account needs to rebalance dynamically.**

Therefore, by selling certificates with notional values equaling to 1,000,000 USD, we short 1311 number of certificates. To maintain the neutrality of delta, we need to buy $\frac{N}{Pirce_{TwinWin}} \times delta \approx 1213$ shares. The detailed recipe for such construction is listed in table 5.1 below.

6 Conclusion

In this study, we price Twin-Win certificate by decomposing it into the margin, stock, and DOBP parts. The same idea is also applied to the delta sensitivity analysis since the delta for a portfolio is the summation of the components' delta. Strike price and barrier price are set in a way that the client can benefit in both bearish and bullish markets(if the barrier is not breached). Python code has been built, and other technical details have been applied to make it more practical. For example, the delta sensitivity is accelerated by parallel computing.

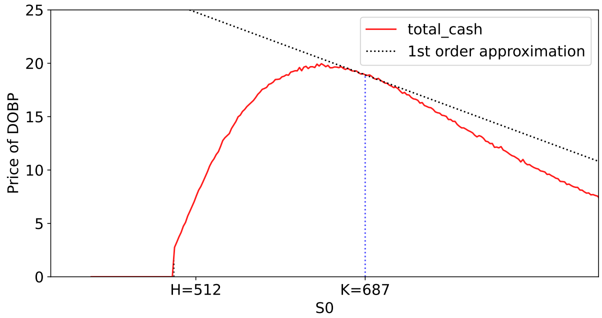


Figure 5.1: Payoff of Twin-Win