Summative Assessment 002

ECO-6004B:
The Economics of Alternative Investments

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Portfolio Insurance: Exploring the Safe-Haven Nature of Commodities During Stock Market Turmoil

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Contents

Section 1 – Introduction:

Periods of market turmoil, such as the Global Financial Crisis, and the COVID-19 shock, intensify investor concerns around capital preservation and systemic risk. Traditional safe assets, notably government bonds, are increasingly scrutinised due to inflationary pressures, low yields, and geopolitical instability. This has shifted attention toward alternative investments, particularly commodities, for their potential role as portfolio insurance (Erb & Harvey, 2006).

Portfolio insurance broadly refers to strategies that reduce downside exposure while preserving growth potential. Beyond dynamic hedging rules (e.g., CPPI), insurance can emerge via strategic allocation into assets that behave defensively during crises. "Safe-haven" assets, by definition, retain value or decorrelate from equities in periods of heightened volatility. Yet, the empirical literature remains inconclusive. While (Baur & Lucey, 2010) document safe-haven properties in gold, others highlight the sensitivity of such findings to crisis definitions, sample periods, and model design (Klein, et al., 2018).

This study advances the literature by assessing whether alternative investments offer effective insurance during equity market turmoil. Combining crisis-sensitive econometric techniques, performance and risk-adjusted diagnostics, we evaluate asset behaviour across normal and distressed regimes. The analysis tests whether alternative assets deliver more stable returns, exhibit lower betas, and enhance downside protection, particularly when incorporating diversification or optimal portfolio structures. In doing so, we continue the debate on whether commodities act as reliable hedging instruments, or require broader allocation frameworks to fulfil an insurance role (Ratner & Klein, 2008) (Mitchell, et al., 2010).

Hypotheses to be Tested:

H1: Alternative investments deliver stable or positive returns in crisis compared to equities.
H2: Beta and correlation between alternatives and S&P500 drop significantly during turmoil.
H3: Portfolios with broader diversification (TRCC, BCOM) offer greater portfolio insurance than narrow metal-only allocations.

Overview of Investment Universe:

Asset Class	Asset Name / Index	Ticker	Source	Sample Period
Equities	S&P500 Index	SP500	Yahoo Finance	01/1985-04/2025
Commodity Index	TRCC ex Energy Index	TRCC	Investing.com	02/1994-04/2025
Commodity Index	Bloomberg Commodity Index	BCOM	Investing.com	02/1991-04/2024
Precious Metal	Gold Futures	GC	Investing.com	01/1985-04/2025
Precious Metal	Silver Futures	SI	Investing.com	01/1985-04/2025
Precious Metal	Platinum Futures	PL	Investing.com	01/1985-04/2025
Fixed Income Proxy	US 1-Month Treasury-Bill	US1M	FRED	08/2001-04/2025

Where US1M data were unavailable (pre-2001), raw (log) returns = excess returns, ensuring sample consistency for earlier crisis periods and analysis.

Section 2 – Empirical Analysis:

This section presents empirical results based on the Section 2 framework. We begin by outlining key statistical properties of the assets and portfolios, before examining how their behaviour evolves under financial stress. Using rolling regressions, drawdowns, and quantile-models, we assess not only reward-to-risk trade-offs, but also whether any assets exhibit persistent safe-haven characteristics during market shocks.

Descriptive Statistics:	Metals Portfolio	TR/CC CRB ex Energy TR Index	Bloomberg Commodity Index	S&P500 (Market Index)
Mean (Log Returns)	0.0043	0.0038	0.0004	0.0066
Standard Deviation (Volatility)	0.0497	0.0360	0.0439	0.0438
Skewness	-0.377	-0.584	-0.721	-0.761
Kurtosis	4.83	6.35	5.56	4.31
Variance	0.002469	0.001296	0.001926	0.001920
Minimum	-0.2463	-0.1923	-0.2400	-0.1856
Maximum	0.1321	0.1156	0.1221	0.1194
N	374	374	374	374

Log returns used to capture continuous compounding effects and distributional properties | Metal Futures via Appendix A1.

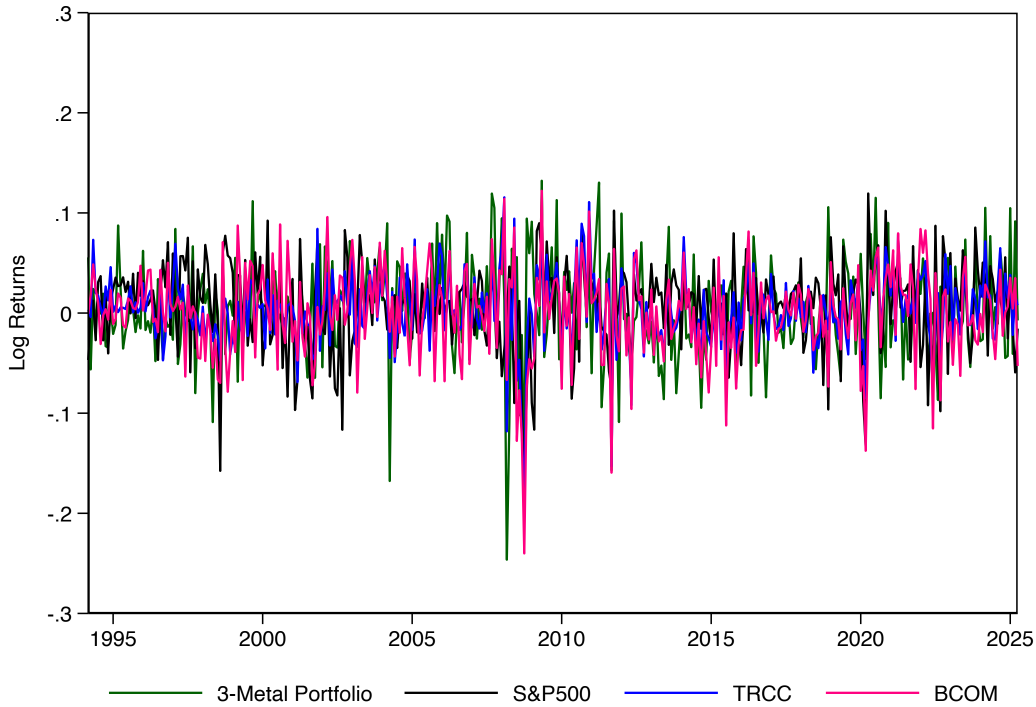


Figure 1: Time-Series Plot of (Log) Returns Across Assets

Figure 1 displays the distributional properties of log returns across the key asset classes and the naïvely diversified Metals Portfolio. The S&P500 posts the highest mean return (0.66%) but displays pronounced left-skew (-0.761) and leptokurtosis (4.31), signalling vulnerability to sharp drawdowns. The Metals Portfolio delivers solid average returns (0.43%) with higher volatility (4.97%) and heavy tails (kurtosis = 4.83), suggesting episodic safe-haven potential offset by embedded crash risk.

TRCC shows more stable returns with lower volatility (3.60%) and a less extreme minimum, but its high kurtosis (6.35) points to latent tail-risk. BCOM combines weak returns (0.04%) with elevated volatility (4.39%) and pronounced left-skew (-0.721), reflecting poor downside efficiency. All return series exhibit negative skewness and excess kurtosis, violating normality and justifying the use of tail-sensitive methods (e.g., CVaR, quantile regressions). These characteristics emphasise the need to assess not only average returns but conditional downside behaviour and distributional asymmetry when evaluating portfolio insurance potential.

CAPM-Style and Crises Regression Models:

We begin with the following CAPM-Style regression:

$$ExRet_{i,t} = \alpha_i + \beta_i\left(ExRet_{SP500,t}\right) + \varepsilon_{i,t} \qquad \textbf{(1)}$$

Equation (1): Baseline CAPM-style regression of asset excess returns on S&P500 excess returns.

Table 1: Portfolio Excess Return Progression on S&P500

	(1)	(2)	(3)
	3-Metal Portfolio	TRCC Index	BCOM Index
S&P500	0.246***	0.437***	0.515***
	(0.066)	(0.045)	(0.054)
_cons (α)	-0.007**	-0.007***	-0.011***
	(0.003)	(0.002)	(0.003)
R-Squared	0.046	0.250	0.242
N	285	285	285

Standard Errors in Parentheses | * p<0.10, ** p<0.05, *** p<0.01

The CAPM regressions on the S&P500 illustrates that the Metals Portfolio exhibits a modest but significant beta (0.246, $p<0.01$), suggesting limited equity sensitivity and diversification potential. Mid- and high-diversification indices (TRCC, BCOM) show stronger systematic exposure (0.437 and 0.515 respectively). All portfolios generate significantly negative alphas, with BCOM underperforming the most (-1.1% per month). R^2 values (≈25% for both indices) indicate a more tight-link to equity markets compared to metals, limiting insurance value during downturns. These baseline exposures offer a benchmark against which crisis-period behaviour is evaluated, enabling us to identify structural vs state-dependent sensitivity shifts.

We then extend this framework by using Crisis dummies (VIX indicator (2), and systemic shock events (3)) to test safe-haven characteristics under market stress:

$$ExRet_{i,t} = \alpha_i + \beta_1\left(ExRet_{SP500,t}\right) + \beta_2\left(Crisis_t\right) + \beta_3\left(ExRet_{SP500,t} \times Crisis_t\right) + \varepsilon_{i,t} \qquad \textbf{(2)}$$

Equation (2): Portfolio returns modelled with VIX threshold and market DD to capture beta dynamics under stress.

Panel A

VIX Crisis Interaction	(1) 3-Metal Portfolio	(2) TRCC Index	(3) BCOM Index
S&P500	0.408***	0.526***	0.572***
	(0.095)	(0.065)	(0.079)
Crisis	0.025**	0.004	0.003
	(0.010)	(0.007)	(0.008)
Crisis * S&P500	-0.154	-0.144	-0.086
	(0.143)	(0.099)	(0.119)
_cons (α)	-0.011***	-0.008***	-0.011***
	(0.003)	(0.002)	(0.003)
R-Squared	0.086	0.260	0.246
N	285	285	285

Panel B

Global Financial Crisis Interaction	(1) 3-Metal Portfolio	(2) TRCC Index	(3) BCOM Index
S&P500	0.251***	0.391***	0.457***
	(0.071)	(0.048)	(0.057)
GFC Dummy	0.033*	0.006	-0.005
	(0.018)	(0.012)	(0.015)
GFC * S&P500	0.052	0.352***	0.413**
	(0.201)	(0.135)	(0.163)
_cons (α)	-0.008**	-0.007***	-0.010***
	(0.003)	(0.002)	(0.003)
R-Squared	0.053	0.251	0.244
N	285	285	285

COVID-19 Crisis Interaction	(1) 3-Metal Portfolio	(2) TRCC Index	(3) BCOM Index
S&P500	0.229***	0.443***	0.513***
	(0.069)	(0.047)	(0.056)
COVID-19 Dummy	0.034	-0.007	-0.016
	(0.024)	(0.016)	(0.020)
COVID-19 * S&P500	0.250	-0.091	0.035
	(0.259)	(0.177)	(0.212)
_cons (α)	-0.008**	-0.007***	-0.010***
	(0.003)	(0.002)	(0.003)
R-Squared	0.,062	0.25	0.246
N	285	285	285

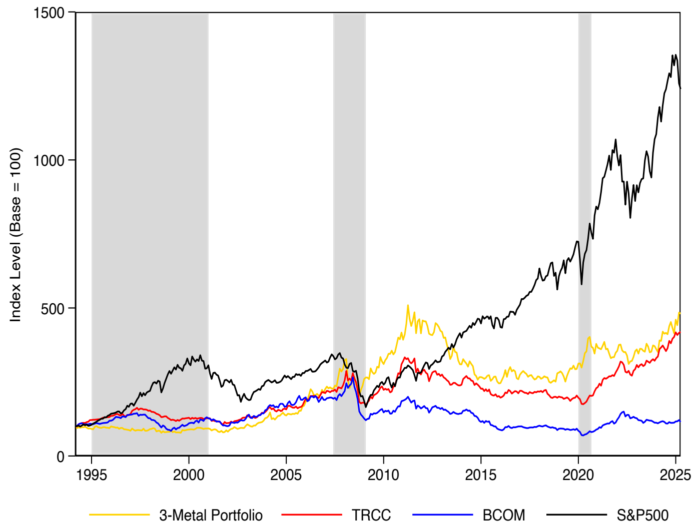
Tech-Bubble Crisis Interaction	(1) 3-Metal Portfolio	(2) TRCC Index	(3) BCOM Index
S&P500	0.177***	0.368***	0.420***
	(0.065)	(0.043)	(0.054)
Tech-Bubble Dummy	-0.007	-0.004	-0.003
	(0.007)	(0.005)	(0.006)
Tech-Bubble * S&P500	-0.038	-0.249**	-0.344***
	(0.150)	(0.101)	(0.124)
_cons (α)	0.005	0.003	-0.002
	(0.003)	(0.002)	(0.002)
R-Squared	0.024	0.167	0.143
N	374	374	374

Table 2: Standard Errors in Parentheses | * p<0.10, ** p<0.05, *** p<0.01

Coefficients Reflect Baseline (β) and Stress-Adjusted Sensitivity (β + β × Crisis). Panel B Crisis Dummies =1 if Within the Dates of event, =0 if Otherwise

$$ExRet_{i,t} = \alpha_i + \beta_1\left(ExRet_{SP500,t}\right) + \sum_k \beta_{2k}\left(D_{k,t}\right) + \sum_k \beta_{3k}\left(ExRet_{SP500,t} \times D_{k,t}\right) + \varepsilon_t \quad \textbf{(3)}$$

Equation (3): Portfolio returns modelled with Crisis-Event interaction dummies to capture beta dynamics under stress.



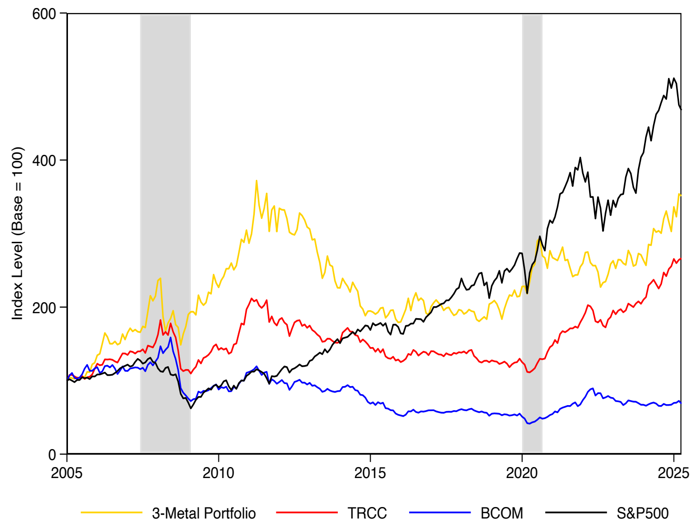


Figure 2&3: Normalised Plots of Entire Time Period and 2005-Present Respectively. Grey Zones Indicate Tech-Bubble, GFC, and COVID-19

The Metals Portfolio demonstrates crisis resilience, particularly during the Tech-Bubble, reflecting structural detachment from equity-driven shocks. Post-2005, its divergence from TRCC and BCOM widens, as shown in Figures 2&3, highlighting heterogeneous commodity dynamics and enhanced insulation as diversification narrows towards select metals.

VIX values >30 signal acute market distress and elevated uncertainty (Fidelity, 2019), as shown in Figure 4. Interaction regressions reveal the Metals Portfolio exhibits a meaningful crisis-beta contraction ($\beta = -0.154$), suggesting conditional decoupling from equities during volatility spikes. State-dependent beta-decline highlights safe-haven behaviour under stress. Significant positive crisis-period alpha is also delivered ($\alpha = 0.025, p < 0.05$), reflecting idiosyncratic resilience uncorrelated with equities, underscoring diversification value of select alternatives during volatility-driven selloffs. TRCC and BCOM show weaker, less significant beta-contractions, implying partial, less reliable hedging.

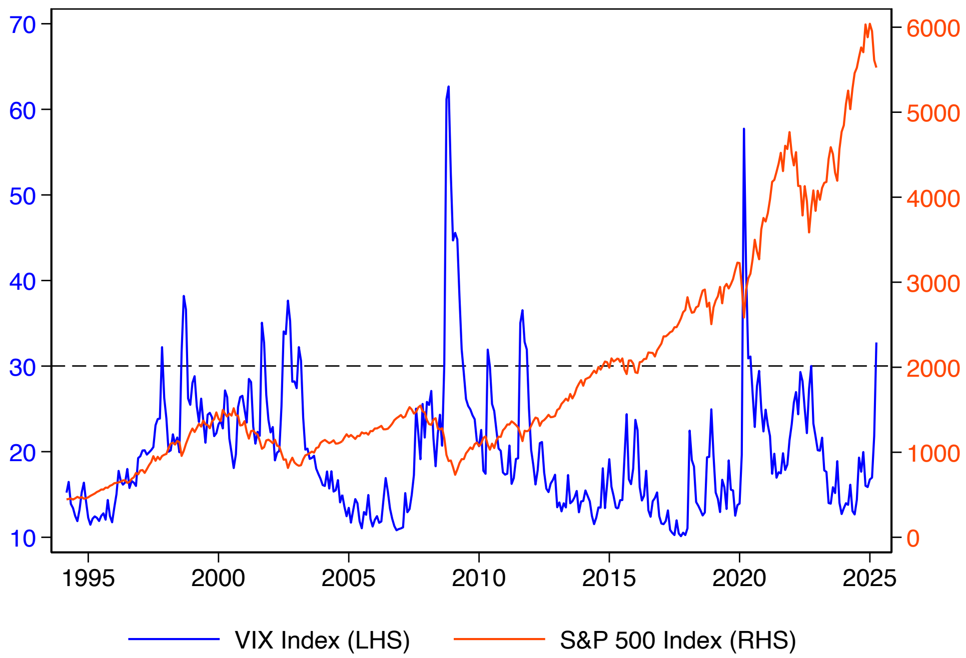


Figure 4: VIX and S&P500 Value Overlay | Indication of Crisis-Period Measurement. VIX & Other Assets via Appendix A2

Panel B regressions capture crisis-specific shifts in equity sensitivity. During the GFC, TRCC ($\beta = 0.352, p < 0.01$) and BCOM ($\beta = 0.413, p < 0.05$) experience sharp beta expansions, signalling correlation breakdowns and heightened market integration. The Metals Portfolio, contrastingly, maintains stable equity exposure. COVID-19 interaction terms are uniformly insignificant, suggesting unchanged co-movement, consistent with transitory, liquidity-driven shocks. In the Tech-Bubble, TRCC ($\beta = -0.249, p < 0.01$) and BCOM ($\beta = -0.344$, $p < 0.01$) display significant negative crisis-betas, reflecting decoupling and short-term hedging potential (Georgiev, 2001). Again, metals remain largely inert, reinforcing structural insulation across heterogeneous stress-episodes.

Rolling Systematic Risk and Co-Movement Dynamics:

To assess the stability and regime-dependence of equity market exposure, we estimate rolling betas and correlations at 12-, 60-, and 120-month windows for each asset class. This multi-horizon approach, consistent with (Ang & Bekaert, 2002), allows for a nuanced view of how risk dynamics evolve from short-term turbulence to long-term structural shifts. Figures 5a and 5b plot these trajectories, overlaying crisis zones to reveal time-varying diversification breakdowns.

Short-term windows (12-months) exhibit considerable noise, particularly for the 3-Metals Portfolio and BCOM, reflecting episodic decoupling amid volatility clustering. However, the metals portfolio frequently dips to near-zero or negative beta during shocks, a potential hedge signal absent in TRCC or BCOM. Medium-term (60-months) smoothing exposes clearer behavioural trends: the metals maintain moderate, stable beta and correlation, while TRCC begins its post-GFC integration with equities. BCOM demonstrates early signs of equity convergence, consistent with rising cross-asset correlation in financialization literature (Tang & Xiong, 2012).

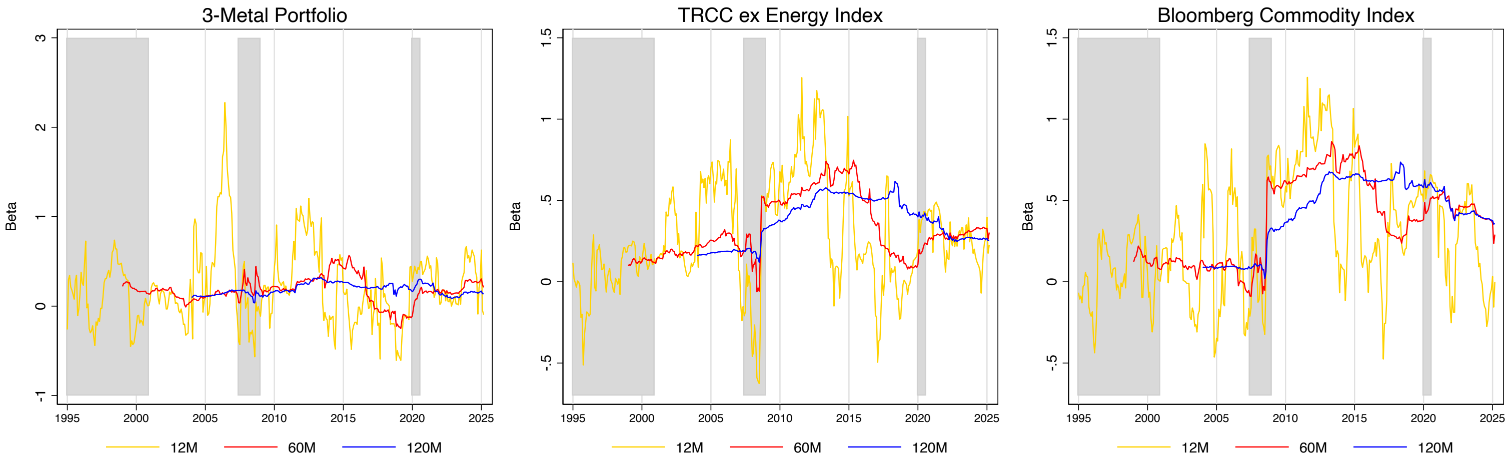


Figure 5a: Rolling Beta Estimated Plots Across Short-, Medium- and Long-Term Time Horizons for Each Asset

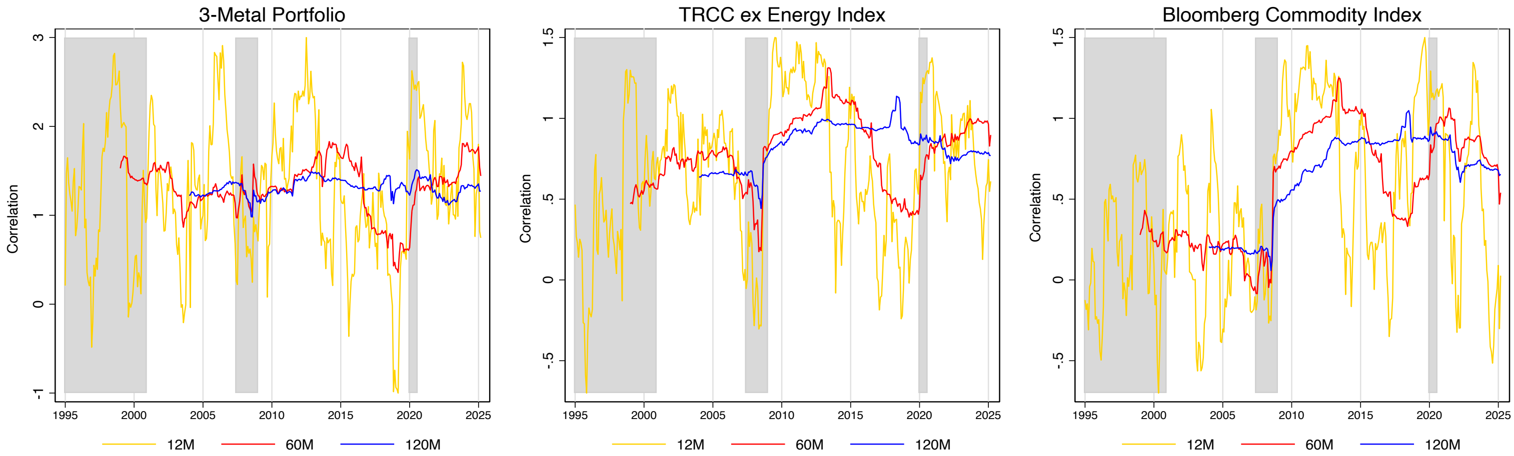


Figure 5b: Rolling Correlation Estimated Plots Across Short-, Medium- and Long-Term Time Horizons for Each Asset

Over long-term horizons (120-months), structural differences sharpen. The metals portfolio sustains low beta (≈0.25) and weak equity correlations, even across crises – strong evidence of strategic diversification. TRCC and BCOM, contrastingly, show rising and persistent equity alignment. TRCC's beta and correlation rise steadily post-2008, peaking around 0.6, while BCOM approaches parity with equities, with beta nearing 1.0 and correlation surpassing 0.6, particularly post-2010 amid QE-led asset inflation. These shifts erode their viability as standalone insurance vehicles.

Together, crisis-period results support H2, but challenge H3. The evidence suggests that short-term noise may obscure underling co-movement patterns, but long-term views expose persistent equity linkages in diversified commodity indices. Only the precious metals portfolio maintains structural independence across timeframes. This reinforces the argument that broader commodity diversification does not guarantee better protection. If anything, it amplifies systemic risk exposure in financialised markets.

Quantile Regression and Performance Metrics:

To model asymmetric dependence in downside regimes, we estimate quantile regressions of asset returns on S&P500 excess returns at 1%, 2.5% and 5% quantiles (Baur & Lucey, 2010):

$$Q_\tau\left(ExRet_{i,t} \,\middle|\, ExRet_{SP500,t}\right) = \alpha(\tau) + \beta(\tau) \times ExRet_{SP500,t} \quad \textbf{(4)}$$

Equation (4): Quantile regression on asset returns across the distribution. Notation via Technical Glossary.

Quantile Regression: 3-Metal Portfolio vs S&P500				Quantile Regression: TRCC Index vs S&P500				Quantile Regression: BCOM Index vs S&P500			
Model	(1)	(2)	(3)		(1)	(2)	(3)		(1)	(2)	(3)
Quantiles:	1%	2.50%	5%		1%	2.50%	5%		1%	2.50%	5%
S&P500	0.326	0.291	0.332***	S&P500	0.643***	0.611***	0.649***	S&P500	0.736***	0.811***	0.753***
	(0.563)	(0.242)	(0.083)		(0.168)	(0.141)	(0.123)		(0.163)	(0.145)	(0.152)
_cons (α)	-0.133***	-0.096***	-0.088***	_cons (α)	-0.084***	-0.069***	-0.061***	_cons (α)	-0.105***	-0.090***	-0.078***
	(0.036)	(0.012)	(0.005)		(0.012)	(0.006)	(0.003)		(0.012)	(0.009)	(0.005)
Pseudo R-Squared	0.075	0.057	0.060		0.367	0.274	0.209		0.356	0.261	0.229
N	285	285	285		285	285	285		285	285	285

Table 3: Standard Errors in Parentheses | * $p<0.10$, ** $p<0.05$, *** $p<0.01$
Quantiles Aligned to That Explored by *(Baur & Lucey, 2010)*. Testing Worse-Case Shock Scenarios

This method provide high-resolution views of asset-sensitivity to equities vary in extreme lower-tails of return distributions, where portfolio insurance is critical. Results show that the 3-Metal Portfolio exhibits modest equity beta at the 5% quantile ($\beta = 0.332, \mathrm{SE} = 0.083$), but loses significance deeper into the tail, suggesting decoupling during extreme stress. This aligns with safe-haven behaviour observed by (Naeem, et al., 2022). Conversely, TRCC and BCOM display consistently high and significant downside betas ($\beta = 0.61 - 0.81$) with BCOM most exposed ($\beta = 0.811$ at 2.5%). High pseudo R^2 values (TRCC: 0.367; BCOM: 0.356) indicate tight linkages to equity returns even in extreme states.

These findings challenge the presumption again that broader diversification enhances portfolio insurance. Instead, diversified indices appear more exposed to systemic co-movements, while the narrower, naïvely-diversified allocation provides stronger left-tail resilience, offering conditional hedging when most-required. While quantile-based models highlight conditional hedging under stress, they do not speak to realised performance outcomes, an essential dimension of true insurance value.

Performance Metrics:

Performance Metrics:	Metals Portfolio	TR/CC CRB ex Energy TR Index	Bloomberg Commodity Index	S&P500 (Market Index)
Sharpe Ratio	-0.180	-0.275	-0.322	-0.223
Conditional Sharpe Ratio	-0.128	-0.214	-0.218	-0.159
Sortino Ratio	-0.260	-0.399	-0.433	-0.297
Jensen's Alpha	-0.0071	-0.0068	-0.0105	≈ 0
Beta (Systematic Risk)	0.246	0.437	0.515	1
Maximum Drawdown (%)	-51.85	-47.50	-73.87	-52.56
Parametric VaR ($)	7597.28	5444.40	6993.73	6391.26
Historical VaR ($)	7009.94	4565.82	6985.60	7484.91
Excess Returns on VaR	-0.128	-0.21	-0.228	-0.166
CVaR	10290.28	7519.63	9747.44	9756.73

Table 4: Performance Metrics Across Investments. Metal Futures via Appendix A1
[Note: VaR and CvaR Calculations Reflect 95% CLs, and a Max Expected Loss of a $100,000 Investment]

Despite marginally lower losses, the 3-Metal Portfolio underperforms broader benchmarks on most risk-adjusted metrics. Its Sharpe (-0.180), Conditional Sharpe (-0.128), and Sortino Ratio (-0.260) all trail TRCC and BCOM, implying weaker return-to-risk efficiency. Yet its low beta (0.246 vs 0.437 for TRCC; 0.515 for BCOM) supports its role as a lower-risk, defensive asset.

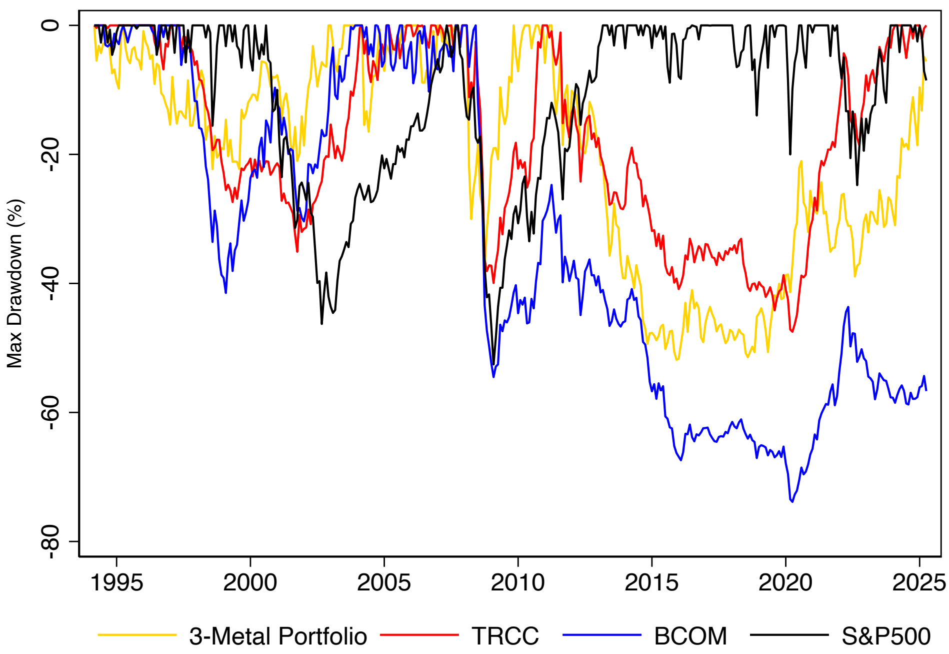


Figure 6: Maximum Drawdown Across Assets

Drawdowns in Figure 6 offer nuance: metals fell -51.25% peak-to-trough, substantially less than BCOM's -73.87%. VaR and CvaR metrics signal material tail exposure, but BCOM's realised losses suggest metal's risk may be overstated by parametric assumptions. Overall, the evidence highlights the limitations of metals as standalone insurance and questions the robustness of commodity-only diversification. Broader protection likely requires asset classes less correlated with systemic equity and market-cycles.

A long-only MVO portfolio, spanning the full commodity universe, is included below. Its persistent underperformance highlights the difficulty of achieving robust insurance, even under theoretically optimal diversification.

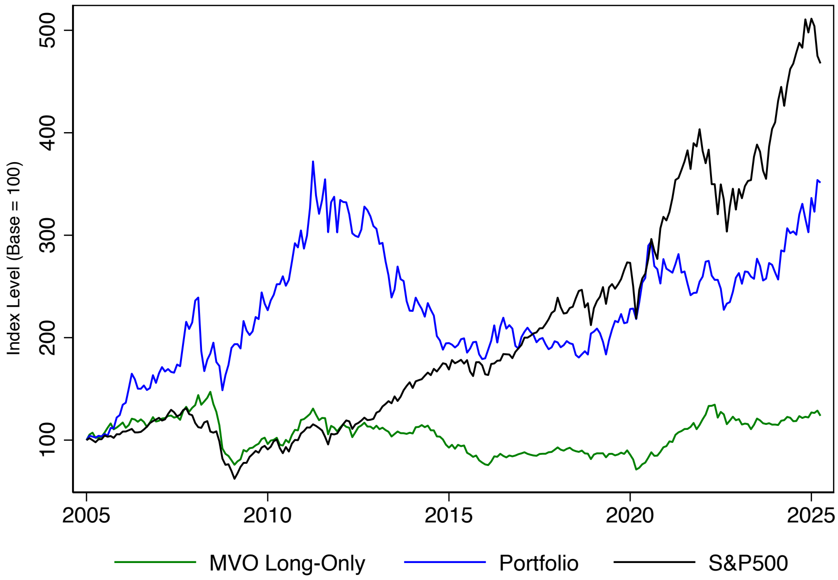


Figure 7: Normalised plots of naïve portfolio and mean-variance optimised portfolio against market. Weights via Appendix A3

Section 3 – Literature Review:

The concept of portfolio insurance is foundational in financial theory, traditionally linked to strategies that limit downside risk while preserving upside potential. Early models, such as dynamic hedging (Perold & Sharpe, 1988) and option-based replication frameworks (Grossman & Zhou, 1993), defined insurance in formal, system-driven terms. More recent empirical research has broadened this lens to real-world asset allocation, particularly via safe-haven alternatives. Safe-havens are defined by their ability to retain value or exhibit negative correlation with equities during market stress. (Baur & Lucey, 2010) provide a widely adopted classification, distinguishing hedges, diversifiers, and safe-havens by conditional correlation patterns across cycles. Their findings, alongside (Baur & McDermott, 2010), highlight gold's episodic safe-haven behaviour, but also its inconsistency across crises. This unreliability has prompted a broader academic re-evaluation, of whether commodity assets can deliver robust protection under systemic shocks.

Subsequent literature refines the criteria for safe-haven assets, emphasising not only correlation breakdowns, but also beta compression and tail decoupling as key insurance indicators. These behaviours are inherently regime-dependent, often varying by crisis severity and market structure (Georgiev, 2001). Our empirical framework builds on this foundation by applying interaction regressions, multi-horizon rolling diagnostics, and quantile techniques to test whether such state-contingent behaviour translates into tangible insulation within a broader commodity context.

Although gold and other precious metals are frequently positioned as safe-havens, their protective qualities are often overstated or mischaracterised. (Capie, et al., 2005) and (Baur & Lucey, 2010) find evidence of gold's decorrelation from equities during extreme conditions, supporting its role as a conditional hedge. However, these studies rely heavily on correlation-based measures, which conflate statistical independence with true capital preservation. Our findings corroborate episodic decoupling, especially during the GFC and Tech-Bubble, but also reveal substantial drawdowns, persistently negative Sharpe ratios, and high downside exposure within the metals portfolio. These metrics, more directly tied to realised investor outcomes, call into question the practical efficacy of metals as reliable portfolio insurance instruments.

Recent work by (Klein, et al., 2018) highlights the conditional nature of gold's hedging capacity, which varies across crisis typologies. This aligns with our findings: while the 3-Metal Portfolio exhibits episodic co-movement breakdowns, it fails to offer consistent protection, particularly during liquidity-driven episodes like COVID-19. Such evidence challenges the notion that decorrelation alone constitutes effective insurance. Broader indices like TRCC and BCOM have been promoted as superior hedges due to sectoral diversification (Tang & Xiong, 2012). Yet our rolling-window analysis shows that these indices sustain elevated and persistent betas across horizons, particularly post-GFC, undermining their role as systematic buffers. The evolving financialization of commodities, extended by (Kang, et al., 2023), may explain this diminished insulation, as market integration offsets diversification gains. Thus, broader commodity exposure does not necessarily enhance insurance relative to metals.

Critically, safe-haven claims grounded in correlation must be tested against realised downside risk. (Sortino & van der Meer, 1991) argue that downside-sensitive metrics better capture protection quality under non-normal return distributions. Our analysis reinforces this: all three portfolios (Metals, TRCC, and BCOM) deliver negative Sharpe and Sortino ratios, with BCOM suffering the deepest drawdown (-73.87%). These outcomes suggest that short-term decoupling or correlation shifts may not translate into durable capital preservation, weakening the case for commodities as standalone insurance. (Choueifaty & Coignard, 2008) warn that conventional allocation models overweight volatile assets in pursuit of alpha, inadvertently increasing crash exposure.

This is reflected in our MVO portfolios, which, despite theoretical diversification, both underperform S&P500 and naïve portfolio in cumulative returns. the structural limitations of commodity-only diversification remains apparent, even under optimised allocation.

(Mujtaba, et al., 2023) reinforces this critique, finding that precious metals serve as, at best, weak hedges, particularly during liquidity-driven crises. Their conclusions echo our quantile regression results: while beta compression emerges in the extreme left tail for metals, it remains persistently high for broader indices like TRCC or BCOM. These findings suggest that although select commodities (e.g., gold, silver, platinum) may exhibit episodic safe-haven characteristics, neither diversified indices nor focused metal portfolios provide robust standalone insurance. Our rolling window results further confirm this fragility: while the 3-Metal Portfolio maintains low-beta and correlation long-term, diversified indices increasingly mirror equity dynamics across time-horizons.

Section 4 - Conclusion:

This study critically assessed whether alternative investments, particularly precious metals and diversified commodity indices, offer effective portfolio insurance during equity market turmoil. While the 3-Metal Portfolio displayed intermittent safe-haven traits, particularly via beta and correlation breakdowns during select crises, this did not translate into sustained downside protection. Broader indices such as TRCC and BCOM remained persistently correlated with equities, undermining their role as shock-absorbers.

Risk-adjusted performance metrics, drawdowns, and quantile regressions further revealed that commodities, individually or optimally combined, fail to deliver standalone insurance. These findings challenge the assumption that correlation decoupling implies true hedging capacity, echoing arguments in (Blitz & van Vliet, 2008), who highlight fragility of traditional risk metrics under systemic stress.

Our results align with recent literature suggesting that commodities provide crisis-contingent hedges, vulnerable to liquidity shocks and systemic co-movements. Even MVO could not overcome structural limitations. Future work should examine Private Equity, Hedge Fund strategies, and REIT investment as potential stabilisers - assets whose return profiles may offer more consistent risk mitigation across cycles and regimes. For institutional investors seeking true capital preservation, reliance on commodities alone may induce a false sense of security during market extremes. Ultimately, true insurance appears to require multi-asset diversification beyond commodities, a conclusion reinforced by the systematic erosion of diversification benefits revealed through our multi-horizon risk diagnostics.

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Regression Equations – For Methodology:

Baseline CAPM Regression (For Model 1 and Jensen's Alpha):

$$ExRet_{i,t} = \alpha_i + \beta_i\left(ExRet_{SP500,t}\right) + \varepsilon_{i,t}$$

Where:
- $ExRet_{i,t}$ is the excess return on portfolio/asset i
- $ExRet_{SP500,t}$ is the excess return on the market (S&P 500)
- α_i = Jensen's alpha
- β_i = Market beta
- ε_t = Error term

Crisis Interaction Regression:

$$ExRet_{i,t} = \alpha_i + \beta_1\left(ExRet_{SP500,t}\right) + \beta_2(Crisis_t) + \beta_3\left(ExRet_{SP500,t} \times Crisis_t\right) + \varepsilon_t$$

Where:
- $Crisis_t = 1$ during defined turmoil (e.g., VIX > 30 or event dummies), 0 otherwise.
- $Crisis_t$ = 1 for GFC, COVID-19, and Dot-Com Bubble dates, but $r_f = 0$ for latter as US1M T-Bill was introduced 08/2001 (so excess returns here = raw (log) returns).
- β_3 shows differential beta during crisis, which is evidence of safe haven behaviour if significantly lower or negative.

Event Specific Crisis Regression (Tech-Bubble, GFC, COVID-19):

$$ExRet_{i,t} = \alpha_i + \beta_1\left(ExRet_{SP500,t}\right) + \textstyle\sum_k \beta_{2k}\left(D_{k,t}\right) + \textstyle\sum_k \beta_{3k}\left(ExRet_{SP500,t} \times D_{k,t}\right) + \varepsilon_t \quad \textbf{(3)}$$

Where:
- $D_{k,t}$ are individual crisis event dummies.
- Captures heterogeneity of asset behaviour across different market shocks.

Event Specific Dummies:

- Tech Bubble: January 1995 – January 2001
- GFC: June 2007 – February 2009
- COVID-19: January 2020 – September 2020

Quantile Regressions (Tail Sensitivity):

$$Q_\tau\left(ExRet_{i,t} \,\middle|\, ExRet_{SP500,t}\right) = \alpha(\tau) + \beta(\tau) \times ExRet_{SP500,t}$$

Where:
- $Q_\tau(.)$ is the conditional quantile function (e.g., $\tau = 0.01, 0.025, 0.05$).
- $\beta(\tau)$ is the quantile-dependent sensitivity to market returns.

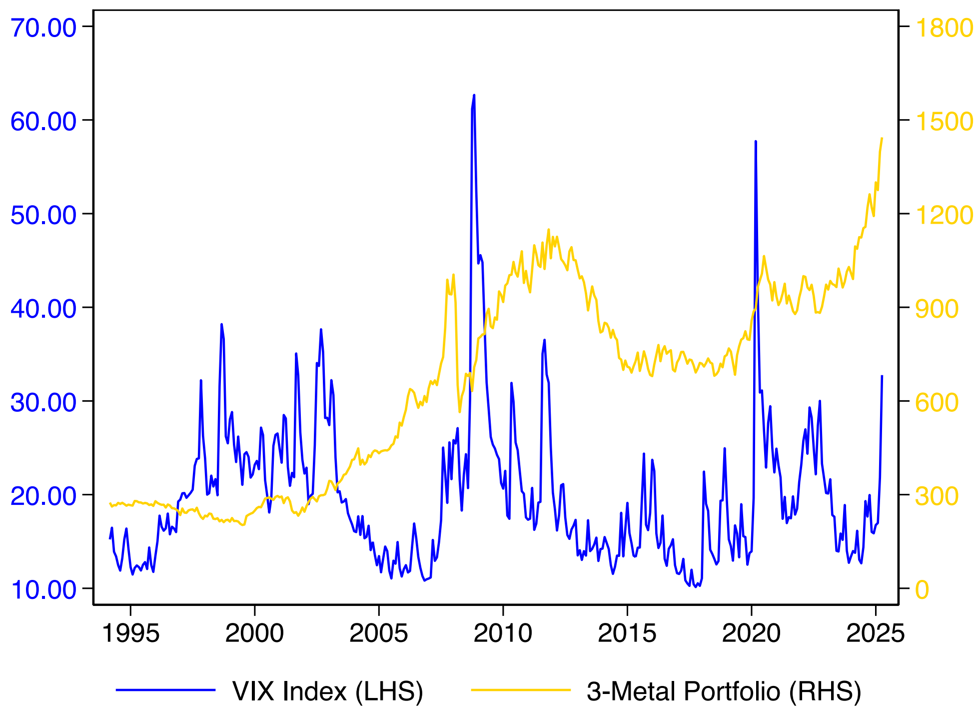
Appendix:

A1 – Descriptive Statistics and Performance Metrics of Precious Metals:

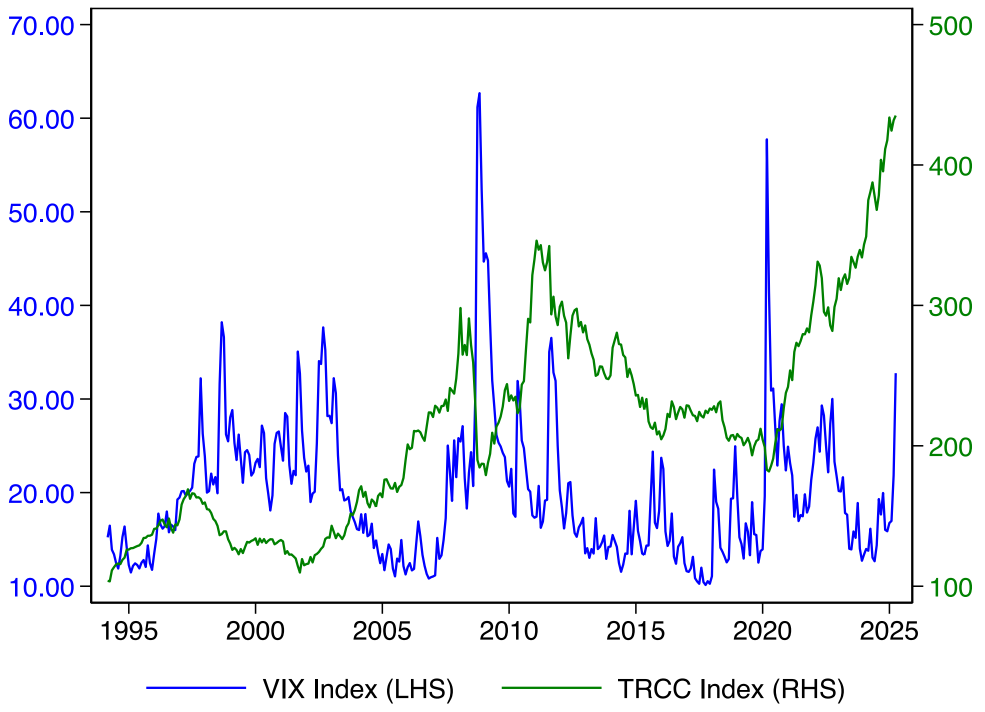
Descriptive Statistics:	Gold Futures	Silver Futures	Platinum Futures
Mean (Log Returns)	0.0049	0.0034	0.0026
Standard Deviation (Volatility)	0.0440	0.0793	0.0683
Skewness	0.000	0.003	-1.029
Kurtosis	4.17	4.29	11.74
Variance	0.001940	0.006290	0.004660
Minimum	-0.2041	-0.3272	-0.5448
Maximum	0.1519	0.2659	0.2980
N	374	374	374
Performance Metrics:			
Sharpe Ratio	-0.146	-0.097	-0.181
Conditional Sharpe Ratio	-0.011	-0.069	-0.123
Sortino Ratio	-0.229	-0.152	-0.222
Jensen's Alpha	-0.0055	-0.0038	-0.0121
Beta (Systematic Risk)	0.151	0.464	0.122
Maximum Drawdown (%)	-48.25	-71.60	-66.40
Parametric VaR ($)	6700.01	12957.38	10243.59
Historical VaR ($)	5887.83	11509.78	9203.07
Excess Returns on VaR	-0.106	-0.067	-0.131
CVaR ($)	8364.83	16318.32	13996.78

Descriptive Statistics utilise (log) returns for cumulative and compounding effects.
[Note: VaR and CvaR calculations reflect 95% CLs, and a Max Expected Loss of a $100,000 investment].

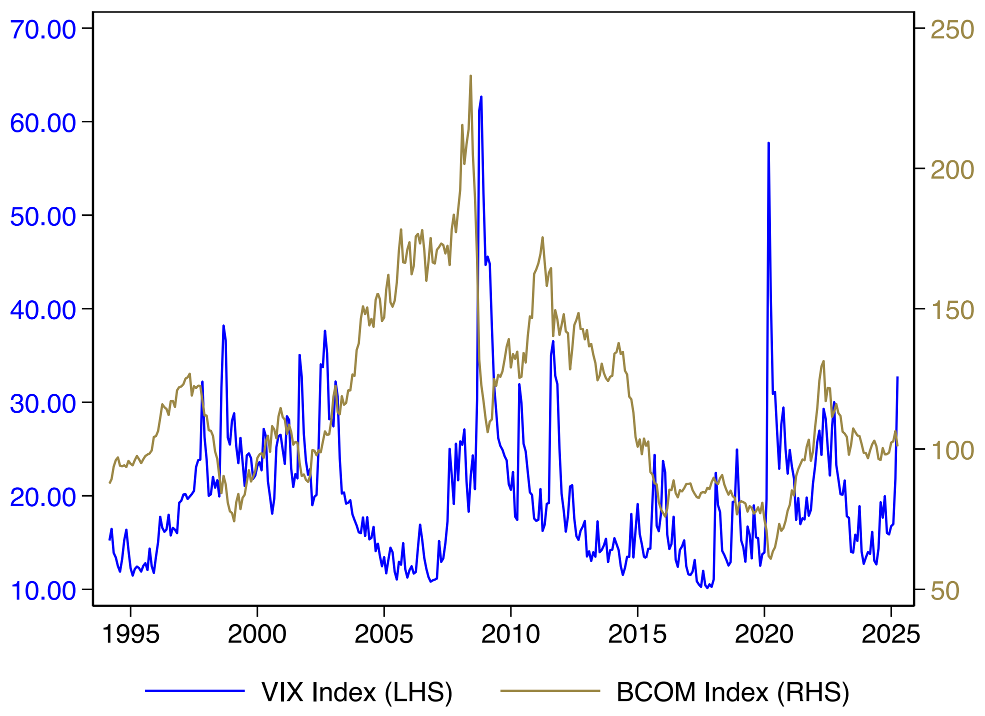
A2 – VIX Plot Overlay with 3-Metal Portfolio, TRCC and BCOM Indices:



Cross-Comparison of VIX Value and (Log) Price of 3-Metal Portfolio



Cross-Comparison of VIX Value and (Log) Price of TRCC Index



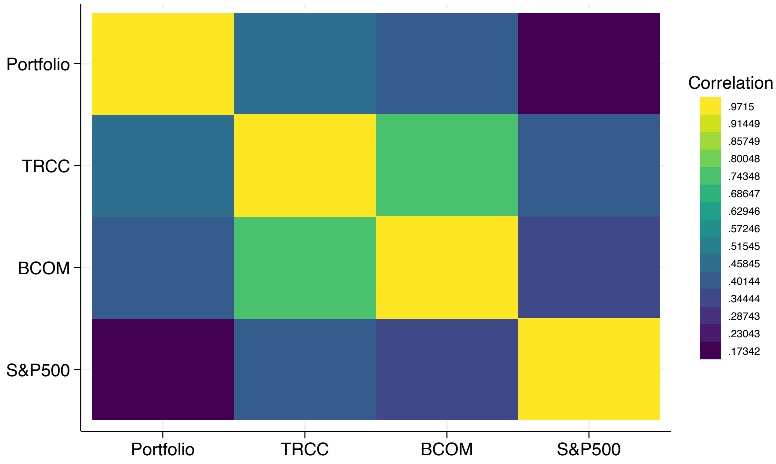
Cross-Comparison of VIX Value and (Log) Price of BCOM Index

A3 – Asset Weightings Across Manually Constructed Portfolios:

Naïve 3-Metals Portfolio	MVO Portfolio (Short Selling)	MVO Portfolio (No Short Selling)
Gold (33.33%)	3-Metal Portfolio (8.83%)	3-Metal Portfolio (8.68%)
Silver (33.33%)	TRCC (-1.69%)	TRCC (0.00%)
Platinum (33.33%)	BCOM (69.20%)	BCOM (68.05%)
-	S&P500 (23.66%)	S&P500 (23.27%)

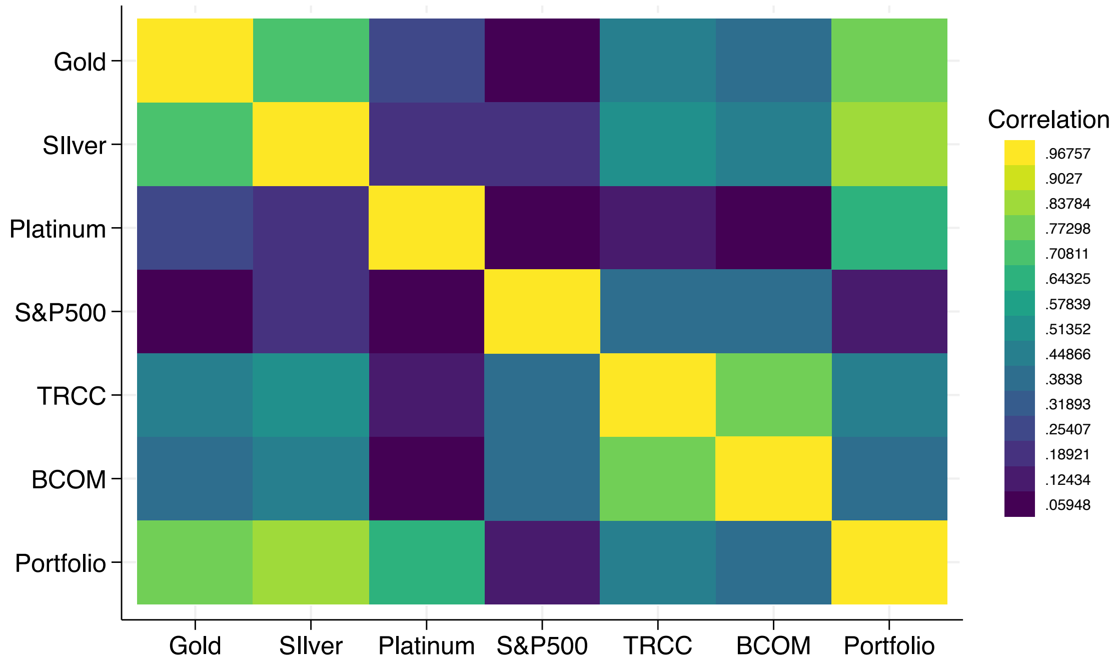
Capital allocation weightings in parentheses. Short selling MVO requires margin account.

A4a – Correlation Matrix and Heatmap Across Assets of Focus:



	Portfolio	TRCC	BCOM	S&P500
Portfolio	1.0000			
TRCC	0.4589	1.0000		
BCOM	0.3974	0.7568	1.0000	
S&P500	0.1449	0.3839	0.3543	1.0000

A4b – Correlation Matrix and Heatmap Across Investable Universe:



	Gold	Silver	Platinum	S&P500	TRCC	BCOM	Portfolio
Gold	1.0000						
Silver	0.7210	1.0000					
Platinum	0.2279	0.1806	1.0000				
S&P500	0.0430	0.2146	0.0270	1.0000			
TRCC	0.4388	0.4857	0.1249	0.3839	1.0000		
BCOM	0.3867	0.4389	0.0810	0.3543	0.7568	1.0000	
Portfolio	0.8048	0.8558	0.6217	0.1449	0.4589	0.3974	1.0000

A4c – Correlation Matrices of Returns During Theoretical and Systemic Shocks:

GFC Correlation Matrix

	Portfolio	TRCC	BCOM	S&P500
Portfolio	**1.0000**			
TRCC	0.5615	**1.0000**		
BCOM	0.4226	0.9183	**1.0000**	
S&P500	0.0816	0.3776	0.4912	**1.0000**

COVID-19 Correlation Martrix

	Portfolio	TRCC	BCOM	S&P500
Portfolio	**1.0000**			
TRCC	0.4908	**1.0000**		
BCOM	0.5159	0.9836	**1.0000**	
S&P500	0.6847	0.7304	0.7594	**1.0000**

Tech-Bubble Correlation Matrix

	Portfolio	TRCC	BCOM	S&P500
Porfolio	**1.0000**			
TRCC	0.2158	**1.0000**		
BCOM	0.2041	0.5404	**1.0000**	
S&P500	0.1599	0.2211	0.0865	**1.0000**

VIX Threshold Correlation Matrix

	Portfolio	TRCC	BCOM	S&P500
Portfolio	**1.0000**			
TRCC	0.6193	**1.0000**		
BCOM	0.5360	0.8471	**1.0000**	
S&P500	0.2792	0.4626	0.4276	**1.0000**

Technical Glossary:

Descriptive Statistics:

Continuously Compounded (Log) Returns:

$$\mathrm{r_t} = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

- r_t: Log return at time t.
- P_t: Price at time t.

Excess Returns:

$$r_t^{excess} = r_t - r_{f,t}$$

- $r_{f,t}$: Risk-free rate (yield from US 1-Month Treasury Bill)

Equally Weighted Portfolio Returns (for Gold, Silver, Platinum):

$$r_{p,t} = \frac{1}{N}\sum_{t=1}^{N} r_{i,t}$$

- N: Number of assets (here is the three precious metals).
- $r_{i,t}$: Return of asset i at time t.

Cumulative Return to Price Index:

$$I_t = I_0 \times \exp\sum_{t=1}^{t} r_t$$

- I_t: Price index at time t.
- I_0: Base value (e.g., 100 at $t = 0$).

Crisis Dummy Variable:

$$\mathrm{Crisis_t} = \begin{cases} 1 & \text{if } \mathrm{VIX_t} > 30 \text{ or } \Delta\mathrm{SP500_t} \leq -5\% \\ 0 & \text{otherwise.} \end{cases}$$

Standard Deviation (Volatility):

$$\sigma_i = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

- $\mathrm{r_t} - \bar{\mathrm{r}}$: Difference between returns at time t and mean returns.
- Measures the volatility of asset i across periods or over time.

Skewness:

$$\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\sigma_i}\right)^3$$

Kurtosis:

$$\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_t - \bar{r}}{\sigma_i}\right)^4$$

<u>Performance Metrics:</u>

Sharpe Ratio:

$$ShR_i = \frac{E\left[R_{i,t} - r_{f,t}\right]}{\sigma_i}$$

- $E[R_{i,t} - r_{f,t}]$: Expected excess returns at time t.
- σ_i: Volatility of asset i.

Conditional Sharpe Ratio:

$$CShR_i \frac{R_{i,t} - r_{f,t}}{CVaR}$$

- $R_{i,t} - r_{f,t}$: Excess returns of asset i at time t.
- $CVaR$: Conditional value at Risk / Expected Shortfall.

Sortino Ratio:

$$SoR_i = \frac{E\left[R_{i,t} - r_{f,t}\right]}{\sigma_{d,i}}$$

- $E[R_{i,t} - r_{f,t}]$: Expected excess returns at time t.
- $\sigma_{d,i}$: Downside volatility of asset i.

Jensen's Alpha (CAPM Framework):

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i\left(R_{SP500,t} - r_{f,t}\right) + \varepsilon_{i,t}$$

- α_i: Jensen's Alpha (intercept).
- β_i: Market Beta (systematic risk).
- $R_{SP500,t} - r_{f,t}$: Market excess returns.

Market Beta (Systematic Risk, CAPM Framework):

$$\beta_i = \frac{Cov(R_i, R_{SP500})}{Var(R_{SP500})}$$

- $Cov(R_i, R_{SP500})$: Covariance in returns between asset i and the S&P500.
- $Var(R_{SP500})$: Variance in returns of the S&P500

Maximum Drawdown:

$$DD_t = \frac{I_t - \max_{k \le t} I_k}{\max_{k \le t} I_k}, \qquad Max\ DD = \min_t (DD_t)$$

- DD_t: Drawdown at time t
- I_t: Value of investment or index at time t.
- $max_{k \le t}$: The peak value of investment or index at time t.

Rolling Beta:

$$\beta_{i,\tau} = \frac{Cov(R_{i,\tau}, R_{SP500,\tau})}{Var(R_{SP500,\tau})}, \qquad \tau \in [t - \omega, t]$$

- ω: Window size of analysis (12, 60, and 120 months).
- τ: Denotes the particular time window.

Rolling Correlation:

$$\rho_{i,m,\tau} = \frac{Cov(R_{i,\tau}, R_{SP500,\tau})}{\sigma_{i,\tau} \sigma_{SP500,\tau}}$$

- $\sigma_{i,\tau}\sigma_{SP500,\tau}$: STDEV of asset i or S&P500's returns in window τ.

Parametric VaR:

$$VaR_\alpha = -(\mu_i - z_\alpha \sigma) \times V$$

- Z_α: Critical value for confidence level (95% ≈ 1.645).
- V: Value of portfolio investment (simulated at $100,000).
- μ: Mean return of asset i.

Historical VaR:

$$VaR_\alpha = -Quantile(r, \alpha) \times V$$

- $Quantile(r, \alpha)$: Empirical α-quantile of historical returns (5th percentile).

Excess Return on VaR:

$$ERoVaR = \frac{R_{i,t} - r_{f,t}}{VaR}$$

Conditional VaR (CvaR):

$$CVaR_\alpha = -\mathbb{E}[\mathrm{r} \mid r \le VaR_\alpha] \times V$$

- $\mathbb{E}[.\mid.]$: Conditional expectation (average loss beyond VaR).
- $\mathrm{CVaR}_{95\%} = \text{Average of the Worst 5\% of Returns}$

Mean-Variance Optimisation Portfolio - Methodology:

Unconstrained Portfolio (short-selling allowed):

To construct the Mean-Variance Optimised (MVO) portfolios, we solve the standard Markowitz optimisation problem using sample estimates of expected returns and the variance-covariance matrix. For the shorting-allowed case (margin account considered), weights are derived via:

$$w^* = \frac{\Sigma^{-1}(\mu_i - r_{f,t})}{1^T \Sigma^{-1}(\mu_i - r_{f,t})}$$

- μ: Vector of expected excess returns of asset i.
- Σ: Covariance matrix.
- 1: Vector of ones (ensures weights sum to 1).

This formulation maximises the Sharpe ratio and permits negative weights. In the long-only specification, a non-negativity constraint is imposed, and weights are re-normalised to ensure full-investment:

Constrained Portfolio (no short-selling):

$$w^* = \arg\min_{\mathrm{w}}(w^T \Sigma w) \quad s.t. \quad w^T \mu = \mu_p, \quad w \geq 0$$

- μ_p: target portfolio returns.
- $w \geq 0$: Short-selling constraint.

where μ_p is the target portfolio return, and the constraint $w \geq 0$ ensures no short positions are held. Weights are re-normalised to ensure full capital allocation.