EN.580.694: Statistical Connectomics
Final Project Report

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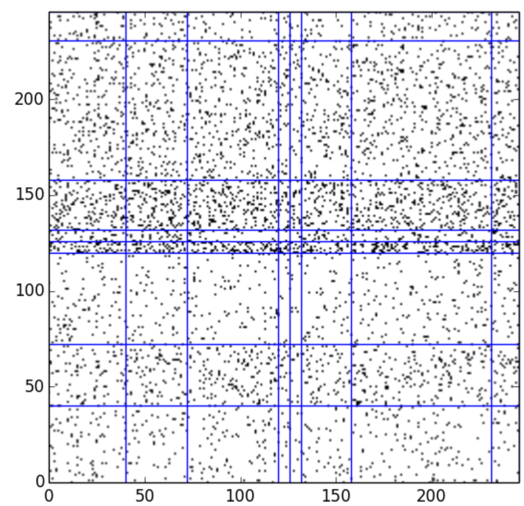
Using a Random Dot Product Model to Approximate the C. Elegans Connectome



Opportunity Graph Models allow us to generate similar graphs from a smaller amount of parameters. If we can model connectomes well, then a good model allows us to generate similar connectomes to perform statistical analysis on similar graphs. The paper by Pavlovic [1] used the Erdos Reyni mixture model as an approximation of the C. Elegans connectome. Using other graph models to perform the same analysis will aid future researchers in selecting tools for connectomics work.

Challenge Representing graphs in terms of graph random variables is difficult because there will always be a loss of information. Our generated data will be biased based on the parameters we choose, so it's important that our graph model is actually useful. Also, choosing appropriate graph metrics for various analysis is important and still an active area of research.

Action The Random Dot Product Graph Model (RDPG) represents each node by a random vector, and assigns the probability of edges between nodes to be the dot product between the two vectors [2]. Each vector is in $\mathbb{R}^d$ with coordinates independently and identically distributed according to a uniform distribution: $\frac{1}{\sqrt{d}}\mathbb{U}^{\alpha}$.

Here, we use a variant of RDPG, the Random Dot Product Mixture Model, where we construct blocks in the graph. Each block j contains vectors whose coordinates are independently and indentically distributed according to the uniform distribution: $\frac{1}{\sqrt{d}}\mathbb{U}^{\alpha_j}$. For n blocks, we have n parameters which specify the model and determine how connected blocks will be.

We use the node-block clustering from Figure 2 of [1], and generate Random Dot Product Graphs with these blocks.

We applied parameter estimation to the alpha parameters for each block, using a Covariance Matrix Adaption Evolutionary Strategy optimizer to converge to a solution.

The graph metric that we are using in optimization is the clustering coefficient, which is defined [1] as the amount of triplets of nodes that are all connected to each other, divided by the amount of nodes with degree of more than 2. We are trying to minimize the absolute difference between the estimated graph's clustering coefficient and the true graph's clustering coefficient.

In an attempt to push the algorithm to converge faster, we also added a loss function. If the mean degree of nodes in a block is outside of one standard deviation of the truth's mean node degree for that block, then a penalty is given to the score by the amount of extra edges (or how many fewer edges there are) in that block.

Each iteration of the optimization algorithm creates a Random Dot Product Graph and calculates the graph metric. An optimal generated graph is then implicitly defined as having a near-identical clustering coefficient to the true graph, and having the mean degree of nodes in every block being within one standard deviation of the mean degree of nodes in blocks from the true graph.

Resolution Shown above is the adjacency matrix that's the result of the parameter estimation. It's suboptimal for a few reasons. First, the way that I organized the RDP mixture model assigns a random vector to each vertex. However, when we block nodes, even if the blocks have their own α parameter, the dot product between nodes in different blocks will probably be non-zero, and will generate an edge with a somewhat higher than non-zero probability.

Since the edge probability is pairwise between two vertices, each vertex needs to have a non-zero vector so that it can have a non-zero degree inside its own block, but we also want to have very few edges outside of its own local block.

Future Work It would be cool to implement the same Random Dot Product Model with blocks that have orthogonal or near-orthogonal vectors. This would enable there to be fewer edges between nodes across blocks. But this would require the ability to generate orthogonal random vectors, or random near-orthogonal vectors with a bounded probability by another parameter. There may be some way to exploit this geometrically.

I also wrote code to do spectral embedding but did not get to play around with that for clustering purposes. It would be interesting to see spectral embedding for the different blocks in this model, as I've only seen theory related to spectral embedding for a single RDP graph.

References

[1] Dragana M. Pavlovic, Petra E. Vrtes, Edward T. Bullmore, William R. Schafer, and Thomas E. Nichols. Stochastic blockmodeling of the modules and core of the ¡italic¿caenorhabditis elegans¡/italic¿ connectome. *PLoS ONE*, 9(7):e97584, 07 2014.

[2] StephenJ. Young and EdwardR. Scheinerman. Random dot product graph models for social networks. In Anthony Bonato and FanR.K. Chung, editors, *Algorithms and Models for the Web-Graph*, volume 4863 of *Lecture Notes in Computer Science*, pages 138–149. Springer Berlin Heidelberg, 2007.