

# Demographic inference using the SFS with **moments** and **demes**

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## Abstract

The site frequency spectrum (SFS) describes the distribution of allele frequencies in one or more populations and sees wide use in evolutionary inference. Here we show how to use **moments**, a Python library for working with the SFS and other measures of genetic diversity, and **demes**, a standard format for parameterizing population models, to infer demographic history. Following an introduction to SFS inference, we illustrate the general usage of **moments** and its **demes** interface, and then show two specific examples, using both simulated and human genetic data. We conclude with caveats and considerations regarding the **moments** library and SFS inference more broadly.

## Introduction

The genetic composition of a sample of individuals is shaped by their genome biology and evolutionary history. Variation resulting from this history can be fully represented by the ancestral relationships among samples at each locus in the genome and how those gene-genealogical relationships change along a chromosome due to recombination (that is, by information stored in the Ancestral Recombination Graph (Nielsen et al., 2025)). However, the ARG can be large and unwieldy, and methods for reconstructing history directly from the ARG, while showing promise (e.g., Brandt et al., 2022; Fan et al., 2023; Brandt et al., 2024), are in their infancy and so far limited in application and scalability. Instead, evolutionary inference using informative summaries of genetic variation remains a tractable and powerful alternative for learning parameters of population history, natural selection and genome biology.

One such summary that has seen wide use is the site frequency spectrum (SFS), which stores the counts (observed or expected) of alleles carried by a given number of genomes in a set of samples. Like any summary of the data, the SFS discards some information stored in the ARG – in this case, loci are treated independently so that haplotypic information is lost. Even so, the relative densities of allele frequencies and the overall scale of the SFS are both sensitive to demographic and non-demographic processes.

This chapter focuses on multi-population demographic inference from the SFS using **moments** (Jouganous et al., 2017). Past population processes, including changes in population size, splits, gene flow and population structure, impact the SFS in predictable ways. A population size expansion (or contraction) leads to a relative excess (or deficit) of low-frequency variants, for example, and commonly used statistics measuring population divergence, such as  $F_{ST}$ , are themselves summaries of the SFS. Thus, the development of methods to learn demographic history from the SFS came soon after the publication of whole-genome sequencing from multiple individuals (Marth et al., 2004; Williamson et al., 2005). More recent methodological advances allow for increased sample sizes and numbers of populations, providing a rich ecosystem of software for SFS-based demographic inference (Gutenkunst et al., 2009; Excoffier and Foll, 2011; Gravel et al., 2011; Jouganous et al., 2017; Ragsdale et al., 2018; Kamm et al., 2020; Dilber and Terhorst, 2024). We return to this in the final section: Considerations and Caveats.

Other evolutionary mechanisms are known to affect allele frequency dynamics and the SFS. This presents a challenge for demographic inference, as the observed SFS may be distorted by non-demographic processes. However, this also presents an opportunity to learn about different evolutionary processes, if demographic history can be controlled for. The SFS has been used to infer the distribution of fitness effects of new mutations, primarily for nonsynonymous variation in coding regions (e.g., Eyre-Walker et al., 2006; Boyko et al., 2008; Kim et al., 2017); to scan for historical selective sweeps (e.g., Kim and Stephan, 2002; Nielsen et al., 2005); and to understand how selection on quantitative traits impacts the genetic architecture underlying those traits (eg., Patel et al., 2024; Ragsdale, 2024). The SFS has also been used to infer relative mutation rates and how they have changed over time (DeWitt et al., 2021). Each of these analyses typically requires partitioning the data in some way – by functional annotation, mutation context, local recombination rate, etc – so that comparisons can be made across SFS from different classes of mutations.

## Interfacing moments with demes

Specifying demographic models requires defining populations (or “demes”) and their relationships via splits and gene flow. `demes` provides a standardized and accessible format for defining demographic models (Gower et al., 2022), and has been adopted by widely used software for population genetics simulation and inference (including `msprime` (Baumdicker et al., 2022), `mom` (Dilber and Terhorst, 2024), `fwddpy11` (Thornton, 2019), and `GADMA` (Noskova et al., 2023)). `demes` encourages interoperability and reuse of code across software, ease of model implementation and reduction of errors.

Demographic models are specified in YAML format (see Gower et al. (2022) and the associated documentation). To illustrate, the two-population split-with-migration model depicted in Figure 1A is specified as below:

```
description: split-with-migration model, loosely based on example 2
generation_time: 29
time_units: years
demes:
- name: ancestral
  epochs:
  - {start_size: 15000, end_time: 300000}
  - {start_size: 25000, end_time: 75000}
- name: popA
  ancestors: [ancestral]
  epochs:
  - {start_size: 30000}
- name: popB
  ancestors: [ancestral]
  epochs:
  - {start_size: 1200, end_size: 14000}
migrations:
- demes: [popA, popB]
  rate: 5e-5
```

In `moments`, a `demes`-specified demographic model can be directly used to compute either the SFS or multi-population LD statistics (Ragsdale and Gravel, 2019, 2020). For the SFS, we simply need to load the demographic model using `demes` and then specify the number of haploid samples to draw and from which populations.

```
import demes, moments
g = demes.load('model.yaml')
```

```
samples = {'popA': 60, 'popB': 60}
fs = moments.Demes.SFS(g, samples=samples)
```

Here, we have not specified a mutation rate. The default behavior sets  $u = 1$ , so that  $\theta = 4N_e u = 4N_e$ , where  $N_e$  is the ancestral population's initial size. With a known mutation rate, the SFS will be properly scaled by passing the mutation rate as a keyword argument: `moments.Demes.SFS(g, samples=samples, u=u)`. This is equivalent to scaling the output SFS above by multiplying by `u`, as `u*fs`.

## Examples: inference in moments using demes

Below, we show two examples of multi-population demographic inference using the joint SFS. The first example simulates data with known demographic parameters, which we try to reinfer using the both the original model and simpler misspecified models. In the second example, we infer a relatively simple human-Neanderthal history using data from two human populations and a high-coverage ancient sample from the Neanderthal lineage. For both of these examples, we briefly describe data processing and inference setup, emphasizing the high-level components of the analyses. For detailed information about each example, including managing data, specifying models, performing inference, computing confidence intervals and visualizing fits, we refer readers to the accompanying GitHub repository (<https://github.com/StatisticalPopulationGenomics-2ndEd/moments>), which is maintained with up-to-date versioning.

In order to optimize parameters in a `demes` model, we need a way to specify which parameters should be fit. For this, we use a separate YAML-formatted file to define `parameters`, each of which points to desired value(s) in the demographic model and for which lower and upper bounds may be set. This file also specifies any relative `constraints` between pairs of parameters, to ensure the optimization routine only explores valid model space. Below, we have included a truncated parameters file – the full example is available on GitHub (URL above).

```
parameters:
- name: T0
  values:
  - demes:
      ancestral:
        epochs:
          0: end_time
      lower_bound: 0
      upper_bound: 2e5
- name: T1
  values:
  - demes:
      ancestral:
        epochs:
          1: end_time
      lower_bound: 0
      upper_bound: 1e6
- name: Ne
  values:
  - demes:
      ancestral:
        epochs:
          0: start_size
      lower_bound: 1e2
      upper_bound: 1e6
```

```
...
constraints:
- params: [T0, T1]
  constraint: greater_than
```

Taking the demographic model, parameters file, and observed SFS together, we can perform inference using the `moments.Demes.Inference.optimize()` function. The initial parameter guesses are given by the input demographic model, which may be perturbed using a keyword argument. Any value in the demographic model that is not specified in the parameters file will remain unchanged, and is therefore treated as a fixed parameter. Here, we also assume we have an estimate for the total mutation rate,  $U$ .

```
import moments

data = moments.Spectrum.from_file('data.fs')
U = 1e-8 * 5e8 # per-base mutation rate times the total length

model_file = 'model.yaml'
params_file = 'params.yaml'
output = 'model.fit.yaml'

ret = moments.Demes.Inference.optimize(
    model_file, params_file, data,
    perturb=1, uL=U, output=output
)
```

Confidence intervals may also be computed using `moments.Demes.Inference.uncerts()`, once a (locally) optimal model has been found. The following will emit a tab-separated file containing parameter names, best-fit values and estimated standard errors.

```
log_file = 'log.txt'

uncerts = moments.Demes.Inference.uncerts(
    output, params_file, data,
    uL=U, output=log_file
)
```

Uncertainties may be computed using either the Fisher Information Matrix, which tends to underestimate confidence intervals due to nonindependence of linked sites, or using the Godambe Information Matrix, which uses a set of bootstrapped datasets to correct for this nonindependence in the data (Coffman et al., 2016). Example 1 in the associated GitHub repository demonstrates this using a simulated dataset.

## Example 1: Inferring parameters in a simulated split-with-migration model

Using the split-with-migration model defined above (illustrated in Figure 1A), we simulated data for 30 diploid individuals from both populations using `msprime` (Baumdicker et al., 2022). These simulations consisted of 500 regions, each of length 1 Mb, with per-base recombination and mutation rates of  $10^{-8}$ . The joint SFS was computed using `tskit`'s `ts.allele_frequency_spectrum()` function separately on each replicate region and summing across regions to find the total SFS across 500 Mb of data (Figure 1B–D).

We fit three demographic models to the simulated data. In addition to reinferring the parameters from the simulated model, we fit two simpler models to the same data. These had fewer parameters, both omitting the size change deeper in time in the ancestral population. This is meant to crudely mimic the scenario that

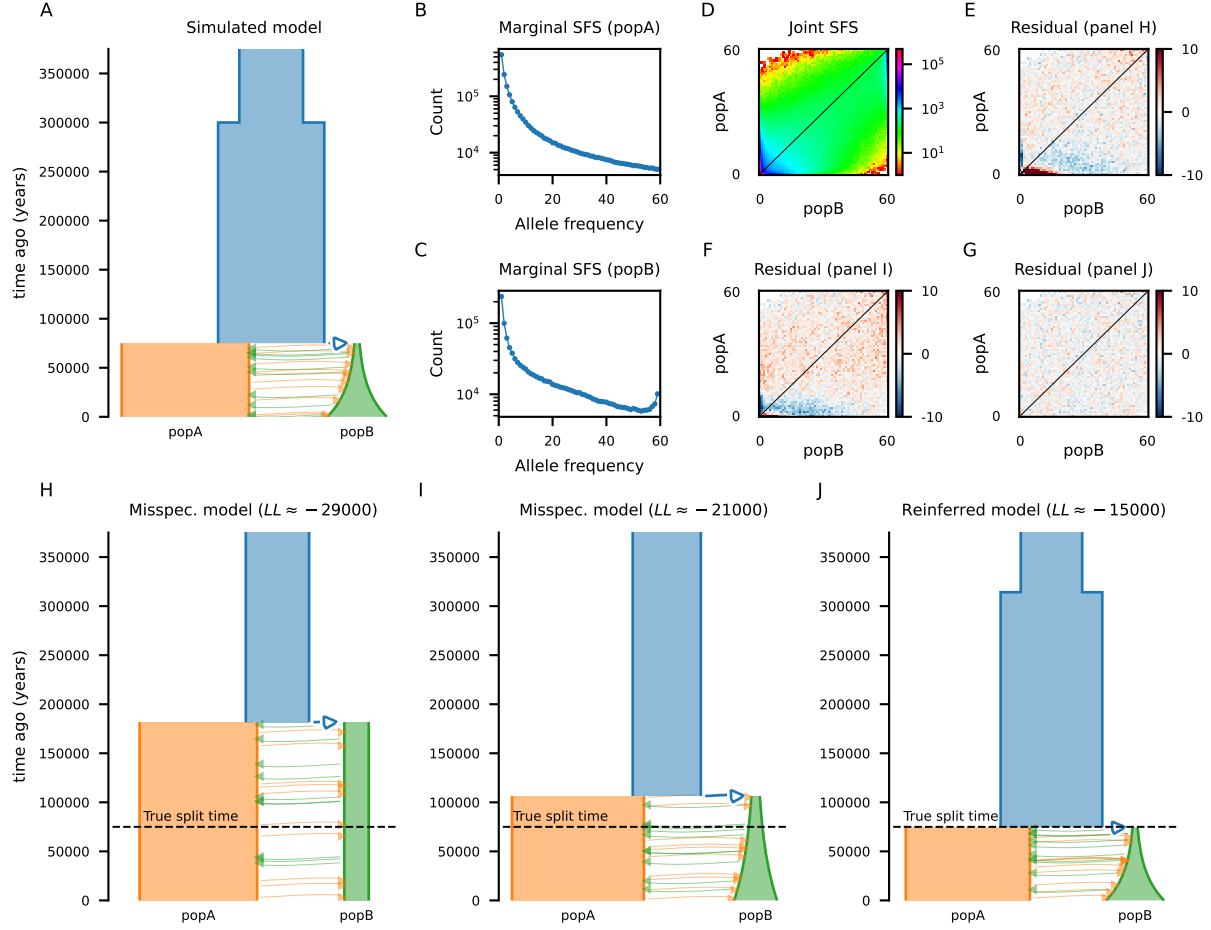


Figure 1: Inference using simulated data. (A) The true demographic model, including an expansion in the ancestral population and symmetric migration between the descendent populations. (B–D) The simulated data, showing both marginal and the joint SFS. (E–G) Residuals between the data SFS (in D) and the predicted SFS from the inferred demographic models in (H–J), resp. The two misspecified models (H and I) provide worse fits to the data and both overestimate the split time due to disallowing any size change in the ancestral population.

we often face, in which the true history is more complicated than the parameterized model. It also highlights biases in inferences that can arise when features of the true history are not included.

When fitting the two misspecified models that do not allow for size changes in the ancestral population, this split time is inferred to be substantially deeper in the past than the true split time. This effect, as demonstrated by Momigliano et al. (2021), is due to the decreased coalescence rate within the ancestral population between the time of the expansion and divergence of the descendent populations. These two misspecified models (Figure 1H,I) also provide a worse fit to the data, as seen by the lower log-likelihoods and the large residuals between model predictions and data (Figure 1E-G).

## Example 2: Inferring human-Neanderthal demographic parameters

This example uses publicly available data to infer a demographic model for two human groups (a northern European and western African population) and a Neanderthal individual. As shown below, even with just

a single sample from the Neanderthal lineage, we are able to recover key parameters such as the human-Neanderthal divergence and introgression proportion.

We obtained modern human genome sequences from a recent resequencing of the 1000 Genomes Project cohort (Byrska-Bishop et al., 2022), and used the high-coverage sequence of the Vindija Neandertal from Prüfer et al. (2017). From the 1000 Genomes cohort we took 85 MSL (Mende from Sierra Leone) and 91 GBR (British from England and Scotland) sequences as our modern human samples. We filtered out sites that lay outside the 1000 Genomes ‘strict’ mask, were not genotyped in the high-coverage archaic genome sequences, that lacked high-confidence ancestral state assignments, or fell within 10 kb of exonic or promoter regions. This left us with approximately 960 Mb of well-characterized and putatively neutrally-evolving sequence. We estimated the genome-wide SFS using built-in **moments** parsing functions. Details concerning data processing (including our liftover of the Vindija sequence to a more recent genome build) can be found in the linked GitHub repository.

The model that we wished to fit is relatively simple but still has many parameters (11 in the example we present), so we constructed it in stages by first fitting simpler one- and two-population models. Throughout, we used  $u = 1.5 \times 10^{-8}$  as an estimate of the mutation rate per generation and 29 years as the average generation time. When the goal is to infer a complex demographic history involving multiple populations, we typically start simple, with subsets of the full data, and sequentially add complexity. Here, we began by fitting one-population models with ancestral and recent size changes for MSL and GBR. For the MSL model, we allowed three epochs with constant effective size in each epoch. We attributed to GBR a sharp contraction  $\sim 60$  kya followed by exponential growth to represent the out-of-Africa bottleneck and subsequent rapid expansion. YAML files representing the models and parameters in this section can be found in the linked GitHub repository.

We next stitched these one-population models together and refitted the resulting demographic model to get a two-population model that included continuous symmetric migration between MSL and GBR and an ancestral size expansion. Following this, we added a Neandertal branch and fitted the Neandertal-modern human divergence time, Neandertal effective size, and the Neandertal-to-GBR admixture proportion. We also fitted the divergence time between the sampled Vindija Neandertal lineage and the Neandertal lineage which admixed with modern non-African humans. Finding that this parameter tended to converge to its lower bound (imposed by the estimated time of death of the Vindija Neandertal, 55 kya), we fixed it to a value supported by literature (90 kya, Prüfer et al., 2017) in a second round of optimization. Rather than fitting it, we fixed the time of the Neandertal-GBR pulse to 50 kya. Fixing parameters for which good estimates are available in the literature (see e.g., Sömer et al., 2025) may be reasonable in the first stages of model optimization, and we expected this value to be especially difficult to fit due to the small pulse proportion and Neandertal sample size. In the second round of optimization on the three-population model, we refit all parameters aside from the two event times mentioned above to refine the estimates.

After this refitting step, we were left with parameters in good agreement with prior authors, with a Neandertal-modern human divergence time of 547 kya, MSL-GBR split time of 77 kya, and Neandertal-GBR pulse proportion of 2.7 percent. We observe some large, systematic residuals in our best-fit model (Figure 2G-I), suggesting model misspecification that could be improved through further exploration of model space.

## Considerations and caveats

The site frequency spectrum is just one of many options for inferring evolutionary parameters from genetic data. It is straightforward to compute from data, and there are many methods for predicting the SFS, making it an appealing framework for demographic inference. As shown here in two examples, we have incorporated the **demes** format for specifying demographic models in **moments** to easily compute the SFS and perform demographic inference, including computing uncertainties.

**moments** is limited in the size of the dataset that it can consider. Sample sizes can be large (in the hundreds), especially when considering models with one or few populations, but will encounter memory issues when sample sizes are extreme or as the number of populations grows. We are limited to a maximum

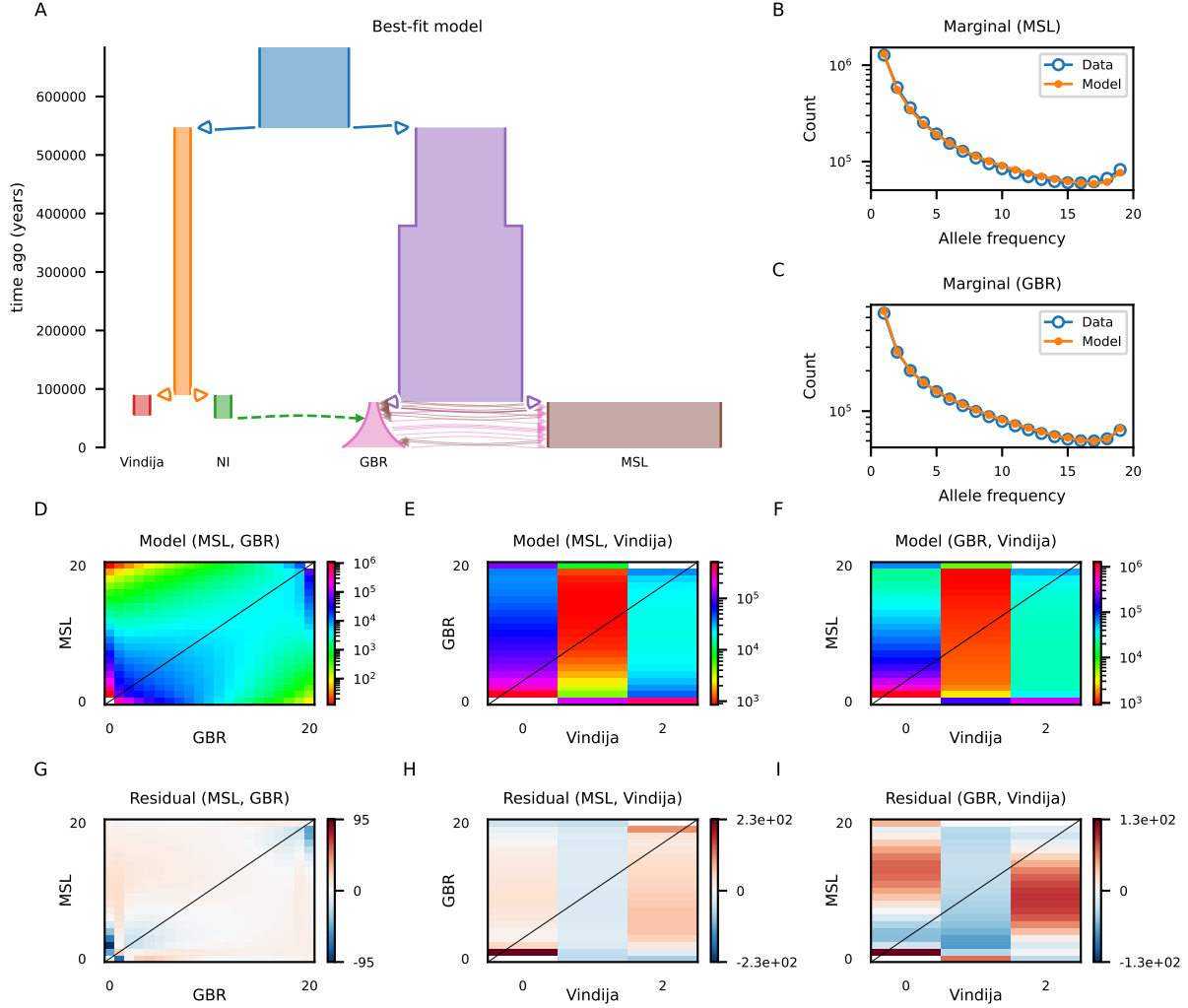


Figure 2: Inference using human and Neanderthal data. (A) The best-fit model including the Vindija Neanderthal and two human populations. (B, C) Marginal SFS fits between the model and empirical data are shown for MSL and GBR. (D–I) Marginal two-way spectra predicted by the model and their residuals against the data are plotted for the three pairs of populations MSL, GBR (D and G), MSL, Vindija (E and H), and GBR, Vindija (F, I).

of five populations existing at any one time, including ancient samples. Other SFS-based methods, such as *momi* (Kamm et al., 2020; Dilber and Terhorst, 2024), can include additional populations, typically with smaller sample sizes.

When inferring models with multiple populations, especially four or more, both the numbers of parameters and the space of plausible model topologies can grow rapidly. It can therefore become challenging for the optimization routines to find optima in parameter space, and overparameterization can become a problem. One hint that models are overparameterized (or models misspecified) is runaway behavior of parameter values, in which they converge to unreasonably small or large values (or approach the imposed bounds). Starting with simple parameterizations and sequentially adding complexity helps to alleviate this issue. The challenge of large model space is more difficult to address, and must be addressed through building and testing a wide range of plausible model topologies. While time-consuming, a broad exploration of

model space, especially when considering many populations, is crucial for evaluating robustness of particular features of inferred history.

Finally, natural selection distorts allele frequencies not only at sites directly under selection, but also in linked regions of the genome through background selection. This can have a sizeable impact on the SFS and lead to biases in demographic inference (Ewing and Jensen, 2016; Johri et al., 2021). In gene-sparse species, such as primates, considering data from regions away from functionally constrained elements is feasible, as we did in example 2 here. Alternatively, the genome may be partitioned by  $B$ -value (a measure of the local strength of background selection, McVicker et al., 2009) and regions experiencing low levels of background selection may be used (for an example, see Medina-Muñoz et al., 2023). In some species with gene-dense genomes, such as *Drosophila* species, this may be infeasible, and interspersed putatively neutral variation must be used, such as synonymous variation or short introns, which will still experience background selection. The appropriate strategy will depend on the genome architecture of the species and genomic resources available.

## Online resources

`moments` is available through `pypi` as `moments-popgen`, and at <https://github.com/MomentsLD/moments> with documentation at <https://momentsld.github.io/moments/>. The detailed online examples accompanying this chapter can be found at <https://github.com/StatisticalPopulationGenomics-2ndEd/moments/>.

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