Design of a micro-randomized trial to optimize Just-In-Time-Adaptive Intervention for stress management during a smoking quit attempt: The Sense2Stop Trial

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Appendices

A Sense2Stop MRT Analysis

This supplementary appendix describes the analysis method used to address the primary aim of the Sense2Stop micro-randomized trial. The method is based on a generalization of Robins' multiplicative structural nested log-linear model [4] for use with data arising from a micro-randomized trial. It is an extension of the approaches described in [1, 2, 3] with the use of a log-link function to accommodate the primary outcome in this study, which is a vector of trivariate outcomes.

The primary analysis considers a causal excursion effect [3] of the intervention prompt to perform a stress management behavior versus no intervention prompt. These causal effects are marginal over all but a subset of the individual's prior data and are "excursions" in that the excursion effect involves rules for how further intervention prompts, if any, would occur during the 120 minutes following randomization. In particular the excursion effect addresses "what is the effect of delivering an intervention prompt now then not delivering an intervention prompt for the next 120 minutes as compared to *not* delivering an intervention prompt now then not delivering an intervention prompt for the next 120 minutes?" These causal effects are marginal over previous intervention prompt assignments.

A.1 Notation and Study Set-up

- Each minute within a user's 12-hour day is a decision point. Over the course of 10 days $t = 1, \ldots, T$ where $T = 720 \times 10 = 7200$.

- Primary Outcome: $Y_t = 1$ if at minute t, the user is within a *detected stressed* episode; $Y_t = 2$ if at minute t, the user is within a *physically active* episode; $Y_t = 3$ if at minute t, the user is within a *not detected stressed* episode. In these analyses, we will assess the causal excursion effect of the intervention prompt at a decision point, t on the primary outcome in the subsequent 120 minutes, $Y_{t+1}, \ldots, Y_{t+120}$.

- Missing Data Indicator: M_t is set to 1 if at minute t, the primary outcome, Y_t is observed and $M_t = 0$ if Y_t is missing.

- Availability: $I_t = 1$ if user is available at time t and is 0 otherwise. At each of the 720 decision times per day, a user is considered available if they: *(i)* have not received a random EMA in the last 10 minutes; *(ii)* have not received a smoking EMA in the last 10 minutes; *(iii)* have not received an EMI in the last hour; *(iv)* are at the peak of either a detected stressed or not detected stressed episode; *(v)* satisfy other conditions: data quality in the last 5 minutes is good[1], they are not driving, phone battery level is greater than 10%, not currently physically active[2], e.g., walking or moving. An available decision time is a decision time when $I_t = 1$. If $M_t = 0$ then $I_t = 0$.

- Intervention prompt randomization: $A_t = 1$ if user is randomized to receive an intervention prompt and is 0 otherwise. Randomization occurs only if an individual is available: $I_t = 1$. If $I_t = 0$ then there is no randomization.

- Control covariates: L_t. All covariates/summaries in L_t must be observed prior to A_t. Control covariates include both variables that are used to reduce noise as well as variables that are used to justify the missing at random assumption (see below):
  - Previous episode type is not detected stressed (Binary with values in $0, 1$). Note that this refers to the episode prior to the episode that contains time t.
  - Previous episode type is missing (Binary with values in $0, 1$). Note that this refers to the episode prior to the episode that contains time t.
  - Body Mass Index (BMI) on day 1 of study
  - Age
  - Age started smoking

- Moderators: The analysis will include one moderator - For decision point t, $X_t = 1$ if $Y_t = 1$ and $X_t = 0$ if $Y_t = 3$. Note that for a decision point to be available, the current

[1] Good quality data corresponds to not having more than 33% of a minute's corresponding data missing due to sensor detachment or sensor off the body, low phone or sensor battery, momentary wireless data loss or software crash.

[2] Physical activity was determined by using activity recognition algorithms that automatically analyze data from the AutoSense-based accelerometer to classify participants' current activity.

detection is either *detected stressed* ($Y_t = 1$) or *not detected stressed* ($Y_t = 3$). In our analysis we include X_t in L_t.

- Temporal ordering of the variables (those in {} are observed at the same time):

$$\ldots, \{M_t, M_tY_t, L_t, X_t, I_t\}, A_t, \{M_{t+1}, M_{t+1}Y_{t+1}, L_{t+1}, X_{t+1}, I_{t+1}\}, A_{t+1}, \ldots$$

- History (covariate data): H_t is all of the data we have observed on the user, up to and including time t, including M_t, I_t and current stress classification, Y_t, but excluding randomizations, A_t. $H_t = \{L_1, I_1, A_1, M_2, M_2Y_2, \ldots, M_t, M_tY_t, L_t, I_t\}$.

- Randomization probability: $p_t(H_t) = P(A_t = 1 \mid H_t)$.

- The proximal outcome following decision point t is: $Y_{t+1}, Y_{t+2}, \ldots, Y_{t+120}$.

- We use O to denote the data observed for a generic individual.

- **(Missing at Random.)** Z_t is a subset of H_t such that Y_{t+j} is independent of M_{t+j}, conditional

- Let s index the number of episodes. C_s is the classification of the s-th episode. $C_s = 1$ refers to episode s being classified as not detected stressed, $C_s = 2$ refers to episode s being classified as detected stressed and $C_s = 3$ refers to episode s being classified as an unknown episode. An unknown episode is one in which there was either *(i)* less than 50% of minutes from the start to the peak of the episode corresponding to good quality data; or *(ii)* more than 50% of minutes from the start to the peak of an episode corresponding to physical activity minutes.

A.2 Randomization Probabilities

At each available decision time (i.e., at t where $I_t = 1$) within a block (there are three four-hour blocks per day), the randomization algorithm calculates the number of remaining available decision times to be sampled in the block and divides this by the expected number of available decision times remaining in the block.

The inputs of the randomization algorithm are the tuning parameters $N_{C_s} \in \mathbb{R}$, $\lambda \in (0, 1)$, together with a function $g(i, r)$ with $i = 1, 2$, $r > 0$, the latter of which is used to predict the number of available detected stressed (or not detected stressed) episodes that would occur in the remaining time r. At the peak of the s-th episode, suppose $T_s \leq c$. The Sense2Stop MRT intervention prompt vs no intervention prompt randomization probabilities (if available, i.e., $I_t = 1$) are given as follows:

$$p_t(H_t) = \frac{N_{C_s} - \sum_{\varsigma=1}^{s-1} \left[\lambda_\varsigma A_\varsigma + (1 - \lambda_\varsigma) p(C_\varsigma)\right] \mathbb{1}_{\{C_\varsigma = C_s\}}}{1 + g(C_s, c - T_s)}, \quad \text{for} \quad C_s = 1, 2,$$

t is a minute within the sth episode, $\lambda_{\varsigma} = \lambda^{T_s - T_{\varsigma}}$, c is the end time of episode s and T_s is the time after the peak of an episode at which an intervention prompt will be randomized given availability. Note that when $s = 1$ the sum in the numerator is defined as 0, e.g., $p(C_1 = N_{C_1}/(1 + g(C_1, c - T_1))$. In addition, the randomization probability is restriction within the interval $[0.05, 0.95]$ for detected stressed episodes and $[0, 1]$ for not detected stressed episodes.

The values of the tuning parameters are $\lambda = 0.4$,

$$N_1 = \begin{cases} 1.6, & \text{if during pre-lapse} \\ 1.65, & \text{if during post-lapse} \end{cases}, \quad N_2 = \begin{cases} 2.25, & \text{if during pre-lapse} \\ 3, & \text{if during post-lapse} \end{cases},$$

and the function g depends on another tuning parameter η_{C_s} which takes on values

$$\eta_1 = \begin{cases} 1.1, & \text{if during pre-lapse} \\ 1.2, & \text{if during post-lapse} \end{cases}, \quad \eta_2 = 0.5.$$

A.3 The Causal Effects for Primary Analysis

As mentioned in Section A.1, we assume that $Z_t \subset H_t$ is a set of variables such that missing at random holds. More precisely, we assume that for all m and $a \in \{0, 1\}$,

$$Y_{t+m}(\bar{A}_{t-1}, a, \bar{0}_{m-1}) \perp M_{t+m}(\bar{A}_{t-1}, a, \bar{0}_{m-1}) \mid Z_t, I_t = 1$$

The causal effect we are interested in is, for $k = 1, 2$ and for $m = 1, 2, \ldots, 120$,

$$\frac{P\left\{Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}_{m-1}) = k \mid I_t = 1, X_t\right\}}{P\left\{Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}_{m-1}) = k \mid I_t = 1, X_t\right\}}. \tag{1}$$

For a given $k \in \{1, 2\}$, display (1) is the relative risk between the probabilities of $Y_{t+m} = k$ for two excursions (i.e., potential treatment assignments):

- Excursion 1: Following the MRT treatment protocol up to $t-1$ then receiving treatment at t and receiving no treatment for the next $m - 1$ minutes.

- Excursion 2: Following the MRT treatment protocol up to $t - 1$ then receiving no treatment at t and receiving no treatment for the next $m - 1$ minutes.

The relative risk in (1) is conditional on the individual being available at decision point t because we are only interested in the causal effect at available moments. The relative risk is conditional on whether the individual is currently within a detected stressed episode (X_t), because we are interested in how the causal effect differs depending on the individual's current stress level. This causal effect (1) is on the relative risk scale; a value greater than 1 indicates that sending a push notification increases the probability of that the proximal outcome belongs to category k in the m-th minute following the decision point. (Recall from the definition of the proximal outcome Y that category 1 is a *detected stressed* episode, category 2 is a *physically active* episode and category 3 is a *not detected stressed* episode.)

In the analysis, we model the causal relative risk in (1) using a log-linear model

$$\frac{P\left\{Y_{t+m}(\bar{A}_{t-1},1,\bar{0}_{m-1})=k \mid I_t=1,X_t\right\}}{P\left\{Y_{t+m}(\bar{A}_{t-1},0,\bar{0}_{m-1})=k \mid I_t=1,X_t\right\}}=e^{\beta_{k1}X_t+\beta_{k0}(1-X_t)}, \tag{2}$$

where (β_{k1},β_{k0}) for $k=1,2$ are unknown. For example, $\exp(\beta_{11})$, which corresponds to $k=1$ and $X_t=1$, captures the causal relative risk between the probabilities of the individual being in a detected stressed episode at time $t+m$ under the aforementioned two excursions, when the individual is within a detected stressed episode at decision point t. This model (2), assumes that the causal relative risks, β_{kj}'s ($k\in\{1,2,3\}$, $j\in\{0,1\}$), are the same for all $1\le t\le T$ and $1\le m\le 120$. It is possible that the true causal relative risk is different at different t and for different m; in such cases the estimated β_{kx}'s from the primary analysis serves as an average over $1\le t\le T$ and $1\le m\le 120$.

We further assume a log linear model on the missingness mechanism:

$$E\left\{M_{t+m}(\bar{A}_{t-1},a_t,\bar{0}_{m-1}) \mid I_t=1,H_t\right\}=e^{Z_t^T\xi+a_tZ_t^T\eta}. \tag{3}$$

This model is auxiliary in the sense that the parameters ξ and η are not of interest for our primary analysis, and its purpose is solely to facilitate estimation of the parameters of interest, β_{kx}.

A.4 Estimation

We describe a three-step procedure for estimating parameters in the causal effect of interest, β_{kx} for $k\in\{1,2\}$ and $x\in\{0,1\}$.

A.4.1 Step 1: Form the marginalization weights

For $x\in\{0,1\}$, we calculate the numerator probabilities as follows. For $x=0,1$, let

$$\hat{p}(x)=\frac{\mathbb{P}_n\sum_{t=1}^T I_t1(X_t=x)p_t(H_t)}{\mathbb{P}_n\sum_{t=1}^T I_t1(X_t=x)}. \tag{4}$$

Let the marginalization weight be

$$\hat{W}_t=\left\{\frac{\hat{p}(x)}{p_t(H_t)}\right\}^{A_t}\left\{\frac{1-\hat{p}(x)}{1-p_t(H_t)}\right\}^{1-A_t}\times\prod_{m=1}^{119}\frac{1(A_{t+m}=0)}{1-p_{t+m}(H_{t+m})}.$$

A.4.2 Step 2: Estimating the Missingness Mechanism

We compute $(\hat{\xi},\hat{\eta})$ that solves the following estimating equation:

$$\mathbb{P}_n\sum_{t=1}^T\sum_{m=1}^{120}I_t\hat{W}_t\left\{e^{-A_tZ_t^T\eta}M_{t+m}-e^{Z_t^T\xi}\right\}\begin{bmatrix}Z_t\\ \{A_t-\hat{p}(X_t)\}Z_t\end{bmatrix}=0. \tag{5}$$

A.4.3 Step 3: Estimating β, the Parameter of Interest

Let $e^{L_t^T \alpha_k}$ be a working model for

$$P\{Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}_{m-1}) = k \mid H_t, I_t = 1\},$$

where L_t are the control variables chosen as described in section **??**. This model does not need to be correct for the resulting estimator for β to be consistent; rather, this model is used to reduce estimation variance.

Define the blipped-down residuals for $k = 1, 2$, $1 \le t \le T$, $1 \le m \le 120$ as follows:

$$R_{ktm}(\alpha, \beta) = e^{-A_t\{X_t \beta_{k1} + (1-X_t)\beta_{k0}\}} 1(Y_{t+m} = k) - e^{L_t^T \alpha_k}. \tag{6}$$

We compute $(\alpha_1, \alpha_2, \beta_{10}, \beta_{11}, \beta_{20}, \beta_{21})$ that solves the following estimating equation:

$$\mathbb{P}_n \sum_{t=1}^{T} \sum_{m=1}^{120} I_t \hat{W}_t e^{-Z_t^T \hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} R_{1tm}(\alpha, \beta) L_t \\ R_{2tm}(\alpha, \beta) L_t \\ R_{1tm}(\alpha, \beta)\{A_t - \hat{p}(X_t)\} X_t \\ R_{1tm}(\alpha, \beta)\{A_t - \hat{p}(X_t)\}(1 - X_t) \\ R_{2tm}(\alpha, \beta)\{A_t - \hat{p}(X_t)\} X_t \\ R_{2tm}(\alpha, \beta)\{A_t - \hat{p}(X_t)\}(1 - X_t) \end{bmatrix} = 0. \tag{7}$$

The estimators for the parameters in (2) are $(\hat{\beta}_{10}, \hat{\beta}_{11}, \hat{\beta}_{20}, \hat{\beta}_{21})$.

A.4.4 Variance-Covariance Estimation

The variance-covariance matrix of $(\hat{\beta}_{10}, \hat{\beta}_{11}, \hat{\beta}_{20}, \hat{\beta}_{21})$ can be estimated by the lower-right 4×4 submatrix of $n^{-1} V_n^{-1} \Sigma_n (V_n^{-1})^T$. Σ_n and V_n are defined as follows.

Let O be the observed data for a generic individual.

Define

$$U_N(O; \rho) = \begin{bmatrix} \sum_{t=1}^T I_t 1_{X_t=0} \{p_t(H_t) - \rho_0\} \\ \sum_{t=1}^T I_t 1_{X_t=1} \{p_t(H_t) - \rho_1\} \end{bmatrix},$$

$$U_M(O; \eta, \xi, \rho) = \sum_{t=1}^{T} \sum_{m=1}^{120} I_t W_t(\rho) \left\{ e^{-A_t Z_t^T \eta} M_{t+m} - e^{g_M(Z_t)^T \xi} \right\} \begin{bmatrix} g_M(Z_t) \\ \{A_t - \tilde{p}(X_t; \rho)\} Z_t \end{bmatrix},$$

$$U_1(O; \alpha, \beta, \xi, \eta, \rho) = \sum_{t=1}^{T} \sum_{m=1}^{120} I_t W_t(\rho) e^{-g_M(Z_t)^T \xi - A_t Z_t^T \eta} M_{t+m} D_t(\rho) \begin{bmatrix} R_{1tm}(\alpha, \beta) \\ R_{2tm}(\alpha, \beta) \end{bmatrix}.$$

For ease of sanity check, we consider for now that $\dim(\rho) = 2$, $\dim(\eta) = \dim(\xi) = q$, $\dim(\alpha) = 2q$, $\dim(\beta) = 4$.

Let

$$U(O;\rho,\eta,\xi,\alpha,\beta)=\begin{bmatrix} U_N(O;\rho) \\ U_M(O;\eta,\xi,\rho) \\ U_1(O;\alpha,\beta,\xi,\eta,\rho)\end{bmatrix}_{(4q+6)\times 1}.$$

Define Σ_n by

$$\Sigma_n=\mathbb{P}_n\left\{U(O;\hat{\alpha},\hat{\beta},\hat{\xi},\hat{\eta},\hat{\rho})U(O;\hat{\alpha},\hat{\beta},\hat{\xi},\hat{\eta},\hat{\rho})^T\right\}.$$

To define V_n, we first introduce a few additional definition. Let q be the dimension of Z_t. Let $\vec{X}_t=(X_t,1-X_t)^T$. Let $\beta_1=(\beta_{10},\beta_{11})^T$ and $\beta_2=(\beta_{20},\beta_{21})^T$. Recall that D_t is

$$D_t=\begin{bmatrix} g_Y(Z_t) & 0_{q\times 1} \\ 0_{q\times 1} & g_Y(Z_t) \\ \{A_t-\hat{p}(X_t)\}\vec{X}_t & 0_{2\times 1} \\ 0_{2\times 1} & \{A_t-\hat{p}(X_t)\}\vec{X}_t \end{bmatrix}_{(2q+4)\times 2}.$$

Define V_n by $V_n=\mathbb{P}_n(V)$, with

$$V=\begin{bmatrix} V_{11} & 0 & 0 & 0 & 0 \\ V_{21} & V_{22} & V_{23} & 0 & 0 \\ V_{31} & V_{32} & V_{33} & V_{34} & V_{35}\end{bmatrix}_{(4q+6)\times(4q+6)},$$

where

$$(\dim = 2\times 2)\quad V_{11}=\begin{bmatrix} -\sum_{t=1}^T I_t 1_{X_t=0} & 0 \\ 0 & -\sum_{t=1}^T I_t 1_{X_t=1}\end{bmatrix}$$

$$(\dim = 2q\times 2)\quad V_{21}=\sum_{t=1}^T\sum_{m=1}^{120} I_t\hat{W}_t\left\{e^{-A_tZ_t^T\hat{\eta}}M_{t+m}-e^{g_M(Z_t)^T\hat{\xi}}\right\}$$
$$\times\begin{bmatrix} \frac{2A_t-1}{\hat{p}(X_t)^{A_t}\{1-\hat{p}(X_t)\}^{1-A_t}}g_M(Z_t) \\ \left\{\frac{2A_t-1}{\hat{p}(X_t)^{A_t}\{1-\hat{p}(X_t)\}^{1-A_t}}(A_t-\hat{p}(X_t))-1\right\}Z_t\end{bmatrix}\vec{X}_t^T$$

$$(\dim = 2q\times 2q)\quad V_{22}=\sum_{t=1}^T\sum_{m=1}^{120} I_t\hat{W}_t\left\{-e^{g_M(Z_t)^T\hat{\xi}}\right\}\begin{bmatrix} g_M(Z_t) \\ (A_t-\hat{p}(X_t))Z_t\end{bmatrix}g_M(Z_t)^T$$

$$(\dim = 2q\times q)\quad V_{23}=\sum_{t=1}^T\sum_{m=1}^{120} I_t\hat{W}_t\left\{e^{-A_tZ_t^T\hat{\eta}}M_{t+m}\right\}\begin{bmatrix} g_M(Z_t) \\ (A_t-\hat{p}(X_t))Z_t\end{bmatrix}(-A_tZ_t^T)$$

$$(\dim = (2q+4)\times 2)\quad V_{31} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-g_M(Z_t)^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \frac{2A_t - 1}{\hat{p}(X_t)^{A_t}\left\{1-\hat{p}(X_t)\right\}^{1-A_t}} D_t$$

$$\times \begin{bmatrix} R_{1tm}(\hat{\alpha},\hat{\beta}) \\ R_{2tm}(\hat{\alpha},\hat{\beta}) \end{bmatrix} \vec{X}_t^T$$

$$+ \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-g_M(Z_t)^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} 0_{q\times 2} \\ 0_{q\times 2} \\ -R_{1tm}(\hat{\alpha},\hat{\beta})\vec{X}_t\vec{X}_t^T \\ -R_{2tm}(\hat{\alpha},\hat{\beta})\vec{X}_t\vec{X}_t^T \end{bmatrix}$$

$$(\dim = (2q+4)\times q)\quad V_{32} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-g_M(Z_t)^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} R_{1tm}(\hat{\alpha},\hat{\beta}) g_Y(Z_t) \\ R_{2tm}(\hat{\alpha},\hat{\beta}) g_Y(Z_t) \\ R_{1tm}(\hat{\alpha},\hat{\beta})\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t \\ R_{2tm}(\hat{\alpha},\hat{\beta})\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t \end{bmatrix}$$

$$\times\left(-g_M(Z_t)^T\right)$$

$$(\dim = (2q+4)\times q)\quad V_{33} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-g_M(Z_t)^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m} \begin{bmatrix} R_{1tm}(\hat{\alpha},\hat{\beta}) g_Y(Z_t) \\ R_{2tm}(\hat{\alpha},\hat{\beta}) g_Y(Z_t) \\ R_{1tm}(\hat{\alpha},\hat{\beta})\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t \\ R_{2tm}(\hat{\alpha},\hat{\beta})\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t \end{bmatrix} \left(-A_t Z_t^T\right)$$

$$(\dim = (2q+4)\times 2q)\quad V_{34} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-g_M(Z_t)^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m}$$

$$\times \begin{bmatrix} -e^{g_Y(Z_t)^T\hat{\alpha}_1} g_Y(Z_t) g_Y(Z_t)^T & 0_{q\times q} \\ 0_{q\times q} & -e^{g_Y(Z_t)^T\hat{\alpha}_2} g_Y(Z_t) g_Y(Z_t)^T \\ -e^{g_Y(Z_t)^T\hat{\alpha}_1}\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t g_Y(Z_t)^T & 0_{q\times q} \\ 0_{q\times q} & -e^{g_Y(Z_t)^T\hat{\alpha}_2}\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t g_Y(Z_t)^T \end{bmatrix}$$

$$(\dim = (2q+4)\times 4)\quad V_{35} = \sum_{t=1}^{T}\sum_{m=1}^{120} I_t \hat{W}_t e^{-g_M(Z_t)^T\hat{\xi} - A_t Z_t^T \hat{\eta}} M_{t+m}$$

$$\times \begin{bmatrix} -e^{-A_t\vec{X}_t^T\hat{\beta}_1} 1(Y_{t+m}=1) A_t g_Y(Z_t)\vec{X}_t^T & 0_{q\times 2} \\ 0_{q\times 2} & -e^{-A_t\vec{X}_t^T\hat{\beta}_2} 1(Y_{t+m}=2) A_t g_Y(Z_t)\vec{X}_t^T \\ -e^{-A_t\vec{X}_t^T\hat{\beta}_1} 1(Y_{t+m}=1) A_t\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t\vec{X}_t^T & 0_{q\times 2} \\ 0_{q\times 2} & -e^{-A_t\vec{X}_t^T\hat{\beta}_2} 1(Y_{t+m}=2) A_t\left\{A_t - \hat{p}(X_t)\right\}\vec{X}_t\vec{X}_t^T \end{bmatrix}$$

A.5 Projection of Estimand

Here we provide the form of the probability limit of the estimator $\hat{\beta}$ when the treatment effect model on the proximal outcome (2) might be incorrect. This probability limit is sometimes called "projection of the estimand", and it is how we would interpret the causal effect of interest.

Note: The following probability limit β'_{k0}, β'_{k1} for $k \in \{1, 2\}$ holds under the assumption that the missingness mechanism (3) is correct.

The probability limit β'_{k0}, β'_{k1} is (for $k \in \{1, 2\}$)

$$\beta'_{k0} = \log \frac{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}) = k) \mid I_t = 1, X_t = 0\right\} P(I_t = 1, X_t = 0)}{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}) = k) \mid I_t = 1, X_t = 0\right\} P(I_t = 1, X_t = 0)}, \tag{8}$$

$$\beta'_{k1} = \log \frac{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}) = k) \mid I_t = 1, X_t = 1\right\} P(I_t = 1, X_t = 1)}{\sum_t^T \sum_{m=1}^{120} E\left\{1(Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}) = k) \mid I_t = 1, X_t = 1\right\} P(I_t = 1, X_t = 1)}. \tag{9}$$

Consider β'_{10} (so $k = 1$, indicating minutes classified as stressed. Then

$$\beta'_{10} = \log \frac{\sum_t^T E\left\{\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}) = 1) \mid I_t = 1, X_t = 0\right\} P(I_t = 1, X_t = 0)}{\sum_t^T E\left\{\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1}, 0, \bar{0}) = 1) \mid I_t = 1, X_t = 0\right\} P(I_t = 1, X_t = 0)},$$

Now $\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}) = 1)$ is the number of minutes classified as stressed after receiving a message at time t.

So $E\left\{\sum_{m=1}^{120} 1(Y_{t+m}(\bar{A}_{t-1}, 1, \bar{0}) = 1) \mid I_t = 1, X_t = 0\right\}$ is the conditional mean of the number of minutes classified as stressed after receiving a message at time t among individuals who are available and not classified as stressed at time t. (I can't recall so I'm pretending that $X_t = 0$ means individual is currently not classified as stressed $X_t = 1_{Y_t=1}$ on set $I_t = 1$)..

So $e^{\beta'_{10}}$ is the ratio of two weighted averages. The numerator is the weighted average, across time, of the conditional mean of the number of minutes classified as stressed after receiving a message among individuals who are currently available and not classified as stressed. The denominator is the weighted average, across time, of the conditional mean of the number of minutes classified as stressed after **not** receiving a message among individuals who are currently available and not classified as stressed. The weight at each decision time is the probability of being available and not classified as stressed.

We could alter the estimator to make the weights all equal to 1. However I am not sure that is a good idea as $I_t = 1$ may decrease with time t. Also distribution of $X_t = 1_{Y_t=1}$ given $I_t = 1$ may vary with time. If we make the weights equal then we will overweight times at which most people have $I_t = 0$.

A.6 Determination of Z_t and L_t

A.6.1 Choosing Z_t

In order to justify the MAR assumption for the missing data in our proximal outcome, we investigated which covariates prior to an available decision point are predictive of missing minutes within the proximal outcome. Note that missing minutes within the proximal outcome are either due to bad quality stress episode data or completely missing episode data.

We consider the following for candidate covariates that may predict missing episodes:

- x_1: ID (Integer with values in $\{1, \ldots, 48\}$)
- x_2: Day in MRT (Integer with values in $\{1, \ldots, 10\}$)
- x_3: Detected stressed episode at available decision time (Binary with values in $\{1, 0\}$)
- x_4: Episode length in minutes (from start to peak of episode). Note that this episode includes an available decision time.
- x_5: Previous Episode is Missing (Binary with values in $\{1, 0\}$). Note that this refers to the episode prior to the episode that contains the available decision time.
- x_6: Previous Episode is Detected Stressed (Binary with values in $\{1, 0\}$). Note that this refers to the episode prior to the episode that contains the available decision time.
- x_7: Previous Episode is Not Detected Stressed (Binary with values in $\{1, 0\}$). Note that this refers to the episode prior to the episode that contains the available decision time.
- x_8: Previous Episode Length in minutes (from start to end of episode). Note that this refers to the episode prior to the episode that contains the available decision time.
- x_9: Previous Day's Proportion of Activity (Proportion of minutes within 12 hour day that correspond to physical activity minutes)
- x_{10}: Previous Day's Proportion of Bad Quality REP Data (Proportion of minutes within 12 hour day that correspond to bad quality REP data)
- x_{11}: Previous Day's Proportion of Bad Quality ECG Data (Proportion of minutes within 12 hour day that correspond to bad quality ECG data)
- x_{12}: Number of Interventions Sent Previous Day
- x_{13}: BMI on Day 1 of study (continuous variable from 18 to 46)
- x_{14}: Gender (0 = Female, 1 = Male)
- x_{15}: Age (Integer from 20 to 63)
- x_{16}: Age Started Smoking (Integer from 20 to 63)
- x_{17}: Total Fagerstrom Score (Integer from 0 to 9)
- x_{18}: Weekday (1 = Weekday, 0 = Weekend)
- x_{19}: Hour of Day (Integer with values in $\{1, \ldots, 23\}$)

- x_{20}: Is Morning (1 = Morning, 0 = Other)
- x_{21}: Is Afternoon (1 = Afternoon, 0 = Other)
- x_{22}: Is Night (1 = Night, 0 = Other)

We use a logistic regression model for the binary outcome: missing or not missing episode. We trained and tested the predictive performance of a logistic regression model with cross validation (i.e., we train on one portion of data and test on another that the model has not yet seen and we do this multiple times). For predictive performance, we use the recall score, which is defined as the ratio: (true positives)/(true positives + false negatives). The best model achieved a weighted[3] recall score of 0.59 and a weighted F1 score of 0.65. The five most influential covariates in descending order are (we use the magnitude and sign of the coefficients to detect the influence of the covariates given that the data was standardised prior to the model fit):

- (−) Previous Episode Type Is Not Detected Stressed
- (+) Previous Episode Type Is Missing
- (+) BMI on Day 1 of study
- (−) Age
- (+) Age Started Smoking

The sign in parentheses above is the direction of influence these 5 variables had on the missing episodes.

Finally, we have $Z_t = \{$ Previous Episode Type Is Not Detected Stressed, Previous Episode Type Is Missing, BMI on Day 1 of study, Age, Age Started Smoking $\}$.

A.6.2 Choosing L_t

In addition to the covariates in Z_t, we also investigate which covariates, if introduced into the analysis, will likely increase the chance that if there is an effect of the notification on the minute level outcome, we will detect this. The natural control covariates to consider are pre-decision point measures of the outcome. This would include:

- Number of minutes in a detected stressed episode in the prior 120 minutes from the available decision time

[3] Calculate metrics for each label, and find their average weighted by support (the number of true instances for each label).

- Number of minutes in a physically active episode in the prior 120 minutes from the available decision time

In addition, we include the following identifying covariates: s

- ID (Integer with values in 1, . . . , 48)
- Day in MRT (Integer with values in 1, . . . , 10)
- Detected stressed episode at available decision time (Binary with values in $\{1, 0\}$)
- Minute number after available decision time (Integer with values in $\{1, \ldots, 120\}$)
- Weekday (1 = Weekday, 0 = Weekend)
- Hour of Day (Integer with values in $\{1, \ldots, 23\}$)
- Is Morning (1 = Morning, 0 = Other)
- Is Afternoon (1 = Afternoon, 0 = Other)
- Is Night (1 = Night, 0 = Other)

All numerical covariates were converted to their standard scores. We note that each row within the data set used for this analysis corresponds to a minute's outcome within the 120 minutes following an available decision time (i.e., there are up to 120 minutes to an available decision time).

To learn which of these variables explain the minute level outcomes we use a multi-class logistic regression model for the multi-class outcome: physically-active-minute, detected-stressed-minute, not-detected-stressed-minute.

We train and test the predictive performance of a multi-class logistic regression model with cross validation (i.e., we train on one portion of data and test on another that the model has not yet seen and we do this multiple times). For predictive performance, we use the recall score, which is defined as the ratio: (true positives)/(true positives + false negatives). The best model achieved an overall weighted recall score of 0.57 and an overall weighted F1 score of 0.63. The two most influential covariates are (here, we can think of the influence of a covariate as the usability of a single covariate to distinguish two classes "one-vs-rest").

- Detected stressed episode at available decision time (Binary with values in $\{1, 0\}$)
- Is Night (1 = Night, 0 = Other)

A classification report is shown in Table 1.

Due to the best achieved F1 score of 0.63 being relatively low, we opt to include only the covariates in Z_t in L_t. That is, for our primary analysis, we let $L_t = Z_t$.

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	precision	recall	f1-score	support
detected stressed	0.14	0.40	0.21	7227
not detected stressed	0.89	0.60	0.71	109325
physically active	0.18	0.44	0.25	15383
weighted avg	0.76	0.57	0.63	131935

Table 1: Classification report.

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