Institute of Actuaries of India

Subject CS2B – Risk Modelling and Survival Analysis (Paper B)

March 2022 Examination

EXAMINER'S REPORT

Introduction

The Examiners' Report is written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

A. General comments on the *aims of this subject.*

The objective of Risk Modelling and Survival Analysis is to give a foundation in mathematical and statistical modelling approaches, including stochastic processes and survival models, that are particularly relevant to actuarial work.

The candidates are reminded that they must include the R code used to construct their answers together with the primary R output in their response script. Where the R code was absent from a particular question section, only partial credit was given, even if the output (e.g., a graph) was there. When just the R code was submitted, but not the R output, partial credit was provided.

Below is a summary of probable R code solutions for each question. Other eligible R code solutions were awarded full credit unless a particular technique was specifically asked in the exam question paper.

In instances when the same inaccuracy was repeated in subsequent portions of a response, candidates were awarded full credit for the subsequent portions.

In problems requiring higher-order abilities where comments were requested, well-reasoned remarks that varied from those offered in the answers were also awarded credit when appropriate.

B. Comments on candidates' performance in this session of the examination

Overall, performance fell short of expectations. In general, candidates proved their ability to utilize R to undertake analysis, but not their capacity to analyze the findings.

C. Pass Mark

The Pass Mark was 50.
106 candidates appeared and 48 passed.

Solution 1:

```
library(moments)

# i)

f=function(x){
    ifelse(x[1]>150 & x[2]>50,ifelse(x[3]>300,1,0),0)
}
```

```
#OR

f=function(x){
 ifelse(x[1]>150 & x[2]>50,0,ifelse(x[3]>300,1,0))
}

#OR

f=function(x){
 ifelse(x[1]>150 & x[2]>50,ifelse(x[3]>300,0,1),0)
}

#OR

f=function(x){
 ifelse(x[1]>150 & x[2]>50,0,ifelse(x[3]>300,0,1))
}
```

(3)

```
# (ii)

f(c(180,75,350))

## [1] 1

## [1] 0

## [1] 0

## [1] 0
```

(1)

```
# (iii)

decisionData=read.csv("D:\\IAI Question Paper\\Mar22 Diet\\CS2B_Mar22_Dataset
1.csv")
model=apply(decisionData,1,f)
model

##  [1] 1 0 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 1 0 1 1 1 0 0 0
0 0 1
## [39] 0 0 1 0 0 0 0 0 0 1 0 0

##[1] 0 1 0 1 1 0 1 0 0 0 0 1 1 1 1 0 1 0 0 0 0 1 1 1 1 0 1 0 0 0 0 0 1 1 1 1
1 0 0 0 0 1 1 0 1 0 1 0 0 0
```

```
##[1] 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 1 1 0 0 0 0 0 0 0 0 1 1

## [1] 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 1 0 0 0 0
```

(3)

```
# (iv)

TP=sum(model == 1 & decisionData$OperationStatus == 1)
print(paste("TP = ", TP))

## [1] "TP =  8"

FP=sum(model == 1 & decisionData$OperationStatus == 0)
print(paste("FP = ", FP))

## [1] "FP =  5"

TN=sum(model == 0 & decisionData$OperationStatus == 0)
print(paste("TN = ", TN))

## [1] "TN =  22"

FN=sum(model == 0 & decisionData$OperationStatus == 1)
print(paste("FN = ", FN))

## [1] "FN =  15"

precision=(TP/(TP+FP))
precision

## [1] 0.6153846

recall=(TP/(TP+FN))
recall

## [1] 0.3478261

f1=(2*precision*recall)/(precision+recall)
f1

## [1] 0.4444444
```

(6)

```
# (v)

# Lower precision value indicates the presence of larger number of False posi
tives
# Lower recall value indicates the presence of larger number of False negativ
es
# This test is not really effective at correctly identifying individuals who
require Surgery
# F1 tells how precise your classifier is (how many instances it classifies c
orrectly), as well as how robust it is (it does not miss a significant number
of instances)
```

```
# Low F1 score indicates that the model can be modified to increase its perfo
rmance
```

(2)
[15 Marks]

Part (i) There was an error in the printing of this question where Yes / No markers were omitted from t he decision tree. However most of the candidates made assumptions and moved ahead with it. Marks have been given for all the possible scenarios. Most of the candidates were clear with the concept of cr eating a function in R & applied it well

Part (ii) Standard extension of the previous part. Candidates did well. Those who answered i) also answ ered this part and most of them got it correct.

Part (iii) Poorly answered. Most Codes were not precise and comprehensive. Many did not paste the ou tput and were penalized.

Part (iv) This part was one of the most poorly answered questions of the exam. Candidates were not cl ear with the concepts of True / False Positives / Negatives and their part in the precision, recall and F1 calculation. Majority of the candidates did not attempt this part.

Part (v) As an extension of the previous, this remains one of the most poorly answered questions as we ll with majority not attempting.

Solution 2:

```
# i)

covid_data=read.csv("D:\\IAI Question Paper\\Mar22 Diet\\CS2B_Mar22_Dataset2.
csv")
covid_data$StateFrom = as.factor(covid_data$StateFrom)
covid_data$State.To = as.factor(covid_data$State.To)
States_From = levels(covid_data$StateFrom)
States_To = levels(covid_data$State.To)
States = unique(c(States_From,States_To))

# Transition Probabilities for each pair

transitions_Master = c()
for (i in States) {
    for (j in States) {

        transition = sum(covid_data$StateFrom == i & covid_data$State.To == j
)/sum(covid_data$StateFrom == i)
        transition = ifelse(sum(covid_data$StateFrom == i) ==0, ifelse(i ==j,
1,0),transition)
        transitions_Master = c(transitions_Master,transition)

    }

}

transition_matrix = matrix(transitions_Master, nrow=5, byrow = T, dimnames=li
```

```
st(States, States))
transition_matrix
```

```
##     H I   Q   D   R
## H 0.2 0 0.0 0.2 0.6
## I 0.4 0 0.6 0.0 0.0
## Q 0.2 0 0.3 0.0 0.5
## D 0.0 0 0.0 1.0 0.0
## R 0.0 0 0.0 0.0 1.0
```

(6)

```
#(ii)

# The two absorbing states are "Recovered" and "Death"
```

(2)

```
# (iii)

library(markovchain)
```

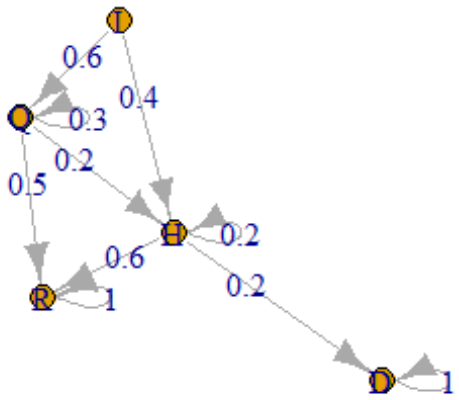
```
## Warning: package 'markovchain' was built under R version 4.0.5
```

```
## Package:  markovchain
## Version:  0.8.5-4
## Date:     2021-01-07
## BugReport: https://github.com/spedygiorgio/markovchain/issues
```

```
covid19=new("markovchain",states=c("H","I","Q", "D","R"),
                        transitionMatrix=transition_matrix)

covid19
```

```
## Unnamed Markov chain
##  A  5 - dimensional discrete Markov Chain defined by the following states:
##  H, I, Q, D, R
##  The transition matrix  (by rows)  is defined as follows:
##     H I   Q   D   R
## H 0.2 0 0.0 0.2 0.6
## I 0.4 0 0.6 0.0 0.0
## Q 0.2 0 0.3 0.0 0.5
## D 0.0 0 0.0 1.0 0.0
## R 0.0 0 0.0 0.0 1.0
```

```
plot(covid19)
```



(4)

```
# (iv)

# (a)
initialstate=c(0,1,0,0,0)
afterweek=initialstate*(covid19)
afterweek
```

```
##        H I   Q D R
## [1,] 0.4 0 0.6 0 0
```

```
## [1] 0.6
```

(1)

```
# (b)

aftertwoweeks=initialstate*(covid19*covid19)
aftertwoweeks
```

```
##        H I    Q    D    R
## [1,] 0.2 0 0.18 0.08 0.54
```

```
# Probability of either hospital or quarantine after 2 weeks
p1 = aftertwoweeks[1]+aftertwoweeks[3]
p1
```

```
## [1] 0.38
```

(2)

```
# (c)
afterthreeweeks=initialstate*(covid19*covid19*covid19)
afterthreeweeks

##          H I     Q    D    R
## [1,] 0.076 0 0.054 0.12 0.75

#Probability of recovery after 3 weeks
p2 = afterthreeweeks[5]
p2

## [1] 0.75
```

(2)

```
# (v)

# Long term probability (after 13 weeks)

# Quarantined People
initial_Q = c(0,0,1,0,0)
after_quarter = initial_Q*(covid19^13)
initial_H = c(1,0,0,0,0)
after_quarter1 = initial_H*(covid19^13)


Recovered = 20000*after_quarter[5]+10000*after_quarter1[5]
Died = 20000*after_quarter[4]+10000*after_quarter1[4]
Quarantined = 20000*after_quarter[3]+10000*after_quarter1[3]
Hospitalized = 20000*after_quarter[1]+10000*after_quarter1[1]

round(Recovered,0)

## [1] 26071

round(Died,0)

## [1] 3929

round(Hospitalized,0)

## [1] 0

round(Quarantined,0)

## [1] 0
```

(5)

```
# (vi)

# In the long run, everyone needs to reach a steady state probabilities.
# As there are two absorbing states, one of the two is the destination
# The other states are completely zeroes
```

(3)

[25 Marks]

Part (i) There were variety of methods candidates used to calculate transition probabilities (using R, by standard arithmetic counting). Most candidates found it difficult to use Codes to derive the transition p

robabilities. Many got incorrect probabilities. Mixed performance by candidates where the well prepar ed candidates were able to score well by getting all the probabilities right.

Part (ii) Standard bookwork question. Well answered by most of the candidates

Part (iii) Standard R functions. Well answered by most of the candidates with some candidates forgetti ng to plot the model.

Part (iv) (a),(b) & (c) Well answered by most of the candidates

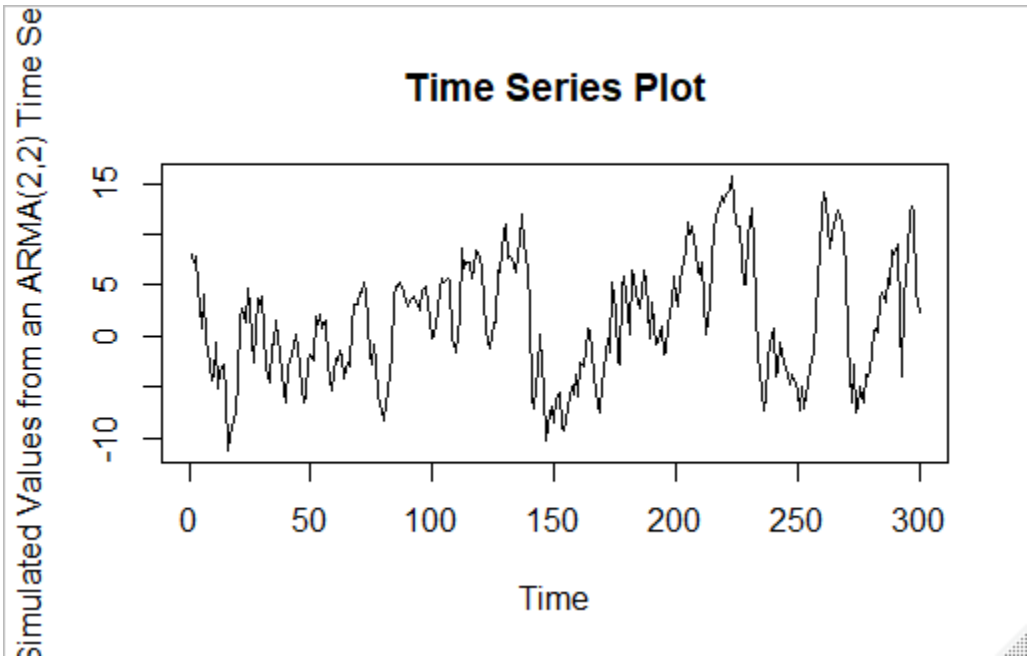
Part (v) This is also one of the not so well answered questions of the paper with only the best candidate s being able to produce sensible answers

Part (vi) Most candidates missed the part where the population has reached a "steady" state but revol ved around the absorbing state

Solution 3:

(i)

```
set.seed(100)
z=2+arima.sim(n=300,list(ar=c(0.8,0.1),ma=c(0.4,0.1),sd=7^0.5))
plot(z, xlab = "Time", ylab = "Simulated Values from an ARMA(2,2) Time Series
", main = "Time Series Plot")
```



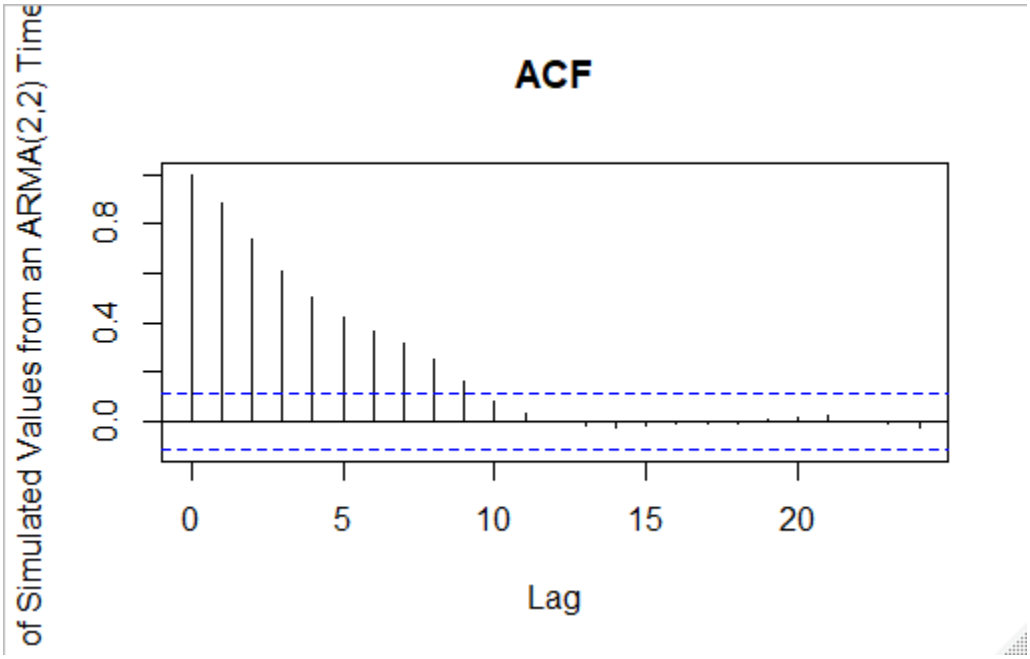
(4)

```
# (ii)

# (a)
mean (z)

## [1] 1.520674
```

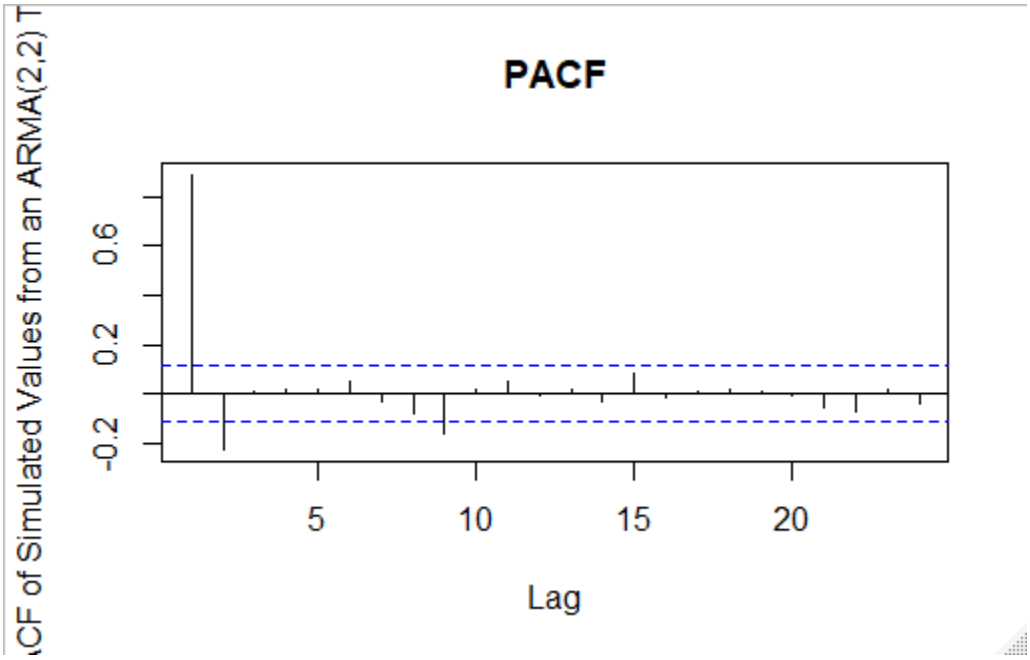
```
sd(z)

## [1] 5.865453

# (b)
mean(z[1:150])

## [1] 0.678429

sd(z[1:150])

## [1] 4.997272

# (c)
mean(z[151:300])

## [1] 2.362919

sd(z[151:300])

## [1] 6.529698
```

(6)

```
# (iii)

acf(z, xlab = "Lag", ylab = "ACF of Simulated Values from an ARMA(2,2) Time Series", main = "ACF")
```




```
pacf(z, xlab = "Lag", ylab = "Partial ACF of Simulated Values from an ARMA(2,2) Time Series", main = "PACF")
```



(4)

```
# (iv)

# The series appears stationary as there are no obvious trends or cycles in t
he graph of the series and it appears to have constant mean.
# However, from (ii), it appears that mean of the two subsets of the data is
very different.
# Working with a larger subset takes the mean values closer to constant thus
revealing in stationarity.
# For a stationary time series, the ACF will drop to zero relatively quickly,
while the ACF of non-stationary data decreases slowly
# Hence, we confirm that the series is stationary
```

(3)

```
# (v)

# PACF becomes insignificant after two lags and ACF goes gradually to zero in
dicating the strong presence of AR(2)process compared to an ARMA (2,2) models
# In ARMA(2,2) model, we should observe, ACF and PACF to gradually go down to
zero after a few lags
```

(3)

[20 Marks]

Part (i) Well answered by most of the candidates. However the most common mistake seen was forget ting to add 2 to ARIMA Simulation and not specifying the correct SD for simulation process i.e. setting i t to 7 instead of sqrt(7)

Part (ii) Straightforward bookwork question with standard R functions. Well answered by most of the c andidates

Part (iii) Straightforward bookwork question with standard R functions. Well answered by most of the c andidates, but labels were not appropriate.

Part (iv) Comments were not up to the mark in general. This showed lack of in depth knowledge

Part (v) Not answered by manu with the majority not being able to conclude the ARMA(2,2) process wi th a sensible explanation

Solution 4:

```
# i)
# a)

mu = 10
var = 4
dlnorm(5000,mu,sqrt(var))

## [1] 3.030741e-05

# The likelihood that the claim will be of 5000 is 0.000030
```

(2)

```
# b)

1-plnorm(5000,mu,sqrt(var))

## [1] 0.7707756

# probability that the claim payout will be greater than 5000 is 0.77
```

(2)

```
# c)
qlnorm(0.9,mu,sqrt(var))

## [1] 285815.9

qlnorm(0.99,mu,sqrt(var))

## [1] 2309856

# maximum claim payout in the confidence interval [0.9,0.99] is [285815.9, 23
09856]
```

(3)

```
# d)
qlnorm(0.5,mu,sqrt(var))

## [1] 22026.47
#Median of Z is 22026.47
```

(2)

```
# e)
qlnorm(0.75,mu,sqrt(var))-qlnorm(0.25,mu,sqrt(var))

## [1] 79162.81

# Interquartile range for Z is 79162.81
```

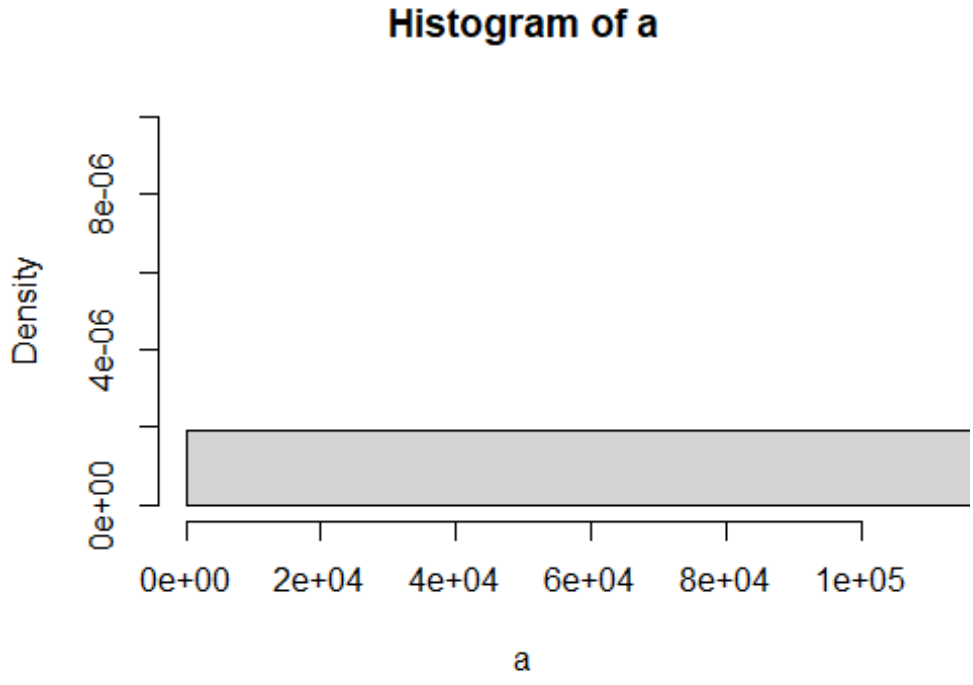
(2)

```
# ii)
```

```
a)
set.seed(50)
a=rlnorm(100,mu,sqrt(var))
mean(a)

## [1] 121227.3

median(a)

## [1] 22785.05

sd(a)

## [1] 481295.3

skewness(a)

## [1] 8.163985

sk = function(x) sum((x-mean(x))^3)/(100*(sd(x))^3)
sk(a)

## [1] 8.041831
```

(4)

```
# (b)
hist(a, freq = FALSE,xlim = c(0,100000),ylim = c(0,0.00001))
```




```
#hist(a,freq = FALSE)
```

(5)

[20 Marks]

Part (i) (a) Standard Bookwork. Most common mistake made by candidates was the use of ROUND function instead of SIGNIF

Part (i) (b) Standard Bookwork. Most common mistake made by candidates was the use of ROUND function instead of SIGNIF

Part (i) (c) A good number of candidates did not attempt this part

Part (i) (d) & (e) Standard Bookwork question. Well answered by most of the candidates

Part (ii) (a) Standard Bookwork question, however most of the candidates were not able to secure full marks due to not being well versed with the formula for skewness, hence not attempting to calculate it

Part (ii) (b) Varied answers by candidates where some produced a lineplot, some a histogram. Some showed frequency and some calculated density. Mixed performance by candidates

Solution 5:

 # (i)

For the Nelson-Aalen model the estimated integrated hazard is calculated as:

$$\hat{\Lambda}(t) = \sum_{t_j \le t} \frac{d_j}{n_j}$$

The estimated variance of the estimator of the integrated hazard is calculated as:

$$\text{var}\left[\hat{\Lambda}(t)\right] = \sum_{t_j \le t} \frac{d_j(n_j - d_j)}{n_j^3}$$

(2)

 # (ii)

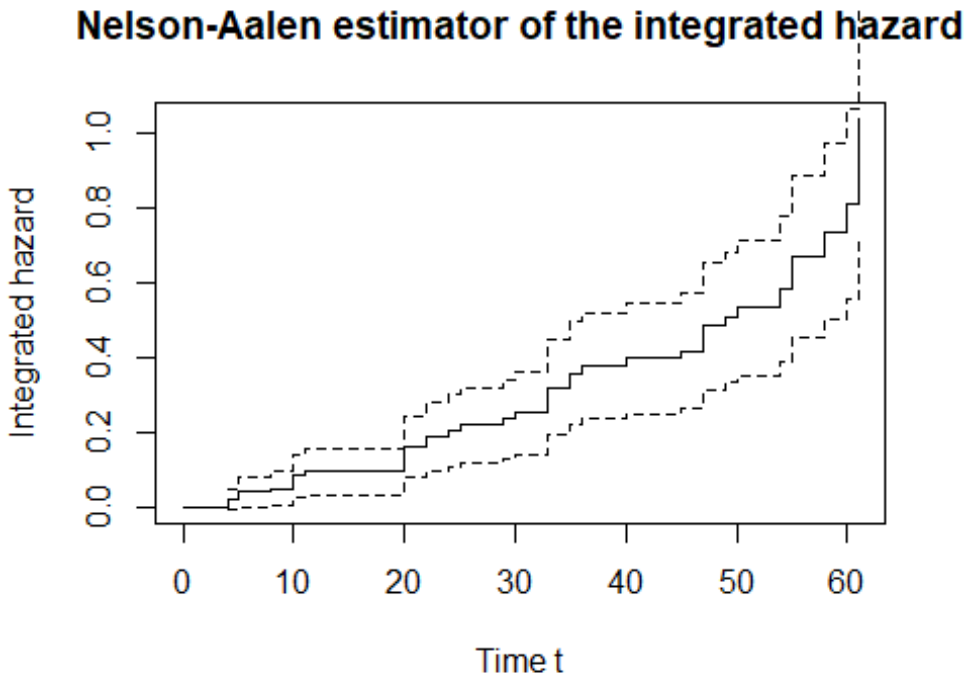
```
data = read.csv("D:\\IAI Question Paper\\Mar22 Diet\\CS2B_Mar22_Dataset3.csv"
)
names(data) = c("j","tj","nj","dj")
data$lambda = cumsum(data$dj/data$nj)
data$sdlambda=sqrt(cumsum(data$dj*(data$nj-data$dj)/data$nj^3))
data

data$varlambda=cumsum(data$dj*(data$nj-data$dj)/data$nj^3)
data
```

```
##     j tj  nj dj     lambda    varlambda
## 1   1  4 100  2 0.02000000 0.0001960000
## 2   2  5  98  2 0.04040816 0.0003999966
## 3   3  8  96  1 0.05082483 0.0005073733
## 4   4 10  90  3 0.08415816 0.0008653980
## 5   5 11  87  1 0.09565242 0.0009959972
## 6   6 15  86  0 0.09565242 0.0009959972
## 7   7 20  76  5 0.16144189 0.0018046975
```

```
## 8   8 22  71  2 0.18961090 0.0021902682
## 9   9 24  67  1 0.20453628 0.0024097101
## 10 10 25  66  1 0.21968779 0.0026358002
## 11 11 29  61  1 0.23608123 0.0029001395
## 12 12 30  60  1 0.25274790 0.0031732876
## 13 13 33  58  4 0.32171342 0.0042803441
## 14 14 35  54  2 0.35875046 0.0049408125
## 15 15 36  52  1 0.37798122 0.0053035230
## 16 16 40  50  1 0.39798122 0.0056955230
## 17 17 45  49  1 0.41838939 0.0061035163
## 18 18 47  45  3 0.48505605 0.0074862324
## 19 19 49  42  1 0.50886558 0.0080396283
## 20 20 50  40  1 0.53386558 0.0086490033
## 21 21 54  38  2 0.58649716 0.0099611480
## 22 22 55  35  3 0.67221144 0.0122002150
## 23 23 58  30  2 0.73887811 0.0142742891
## 24 24 60  27  2 0.81295218 0.0168145523
## 25 25 61  22  5 1.04022491 0.0247972720
```

(6)

```
# (iii)
plot(c(0,data$tj),c(0,data$lambda),type="s",main="Nelson-Aalen estimator of t
he integrated hazard",xlab="Time t",ylab="Integrated hazard")
lines(data$tj,data$lambda-1.96*data$sdlambda,type="s",lty=2)
lines(data$tj,data$lambda+1.96*data$sdlambda,type="s",lty=2)
```



(6)

```
# (iv)
```

```
data$Survival_KM = cumprod(1-data$dj/data$nj)
data$Survival_KM
```

```
##  [1] 0.9800000 0.9600000 0.9500000 0.9183333 0.9077778 0.9077778 0.8480556
##  [8] 0.8241667 0.8118657 0.7995647 0.7864571 0.7733494 0.7200150 0.6933478
## [15] 0.6800142 0.6664139 0.6528136 0.6092927 0.5947857 0.5799161 0.5493942
## [22] 0.5023033 0.4688164 0.4340892 0.3354326
```

(3)

```
#(v)

# The inequality states that:

# Survival function of Kaplan Meir SKM(t) < survival function of Nelson Aalen SNA(t)

# To demonstrate the inequality:

data$Survival_NA = exp(-data$lambda)
sum(data$Survival_KM<data$Survival_NA)
```

```
## [1] 25
```

```
# Since all the values are true, the inequality is proved
```

(3)

[20 Marks]

Part (i) Standard Bookwork question but most candidates did not mention the summation ranges, instead mentioned cumsum function of R. Terms in the formula in this part of the question were not explained by most of the candidates.

Part (ii) Standard bookwork question, answered well by most of the candidates

Part (iii) Varied answers by candidates where some plotted the hazard, some the survival function. Both were given marks. Answered well by prepared candidates

Part (iv) Well answered by candidates with some using the function survfit (they were not given credit for that)

Part (v) Most of the candidates did not attempt this, looks like they were running out of time. Those who answered iv) correctly were able to produce correct results

Additional Comments:

- *To get credit, candidates MUST submit the R code used to get their answers along with the primary R output generated in the Word document. Please note that inability to provide the R code will result in a heavy penalty.*
- *The paper, in general, was not well answered*
- *Lack of adequate background knowledge was revealed by most of the candidates.*

- *Use of proper R Codes was lagging.*
- *Not all R Codes used and output obtained were pasted in the answer scripts.*

****