Institute of Actuaries of India

Subject CS1-Actuarial Statistics (Paper B)

March 2022 Examination

EXAMINERS' REPORT

Introduction

The Examiners' Report is written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

A. General comments on the aims of this subject and how it is marked

The aim of the Actuarial Statistics subject is to provide a grounding in mathematical and statistical techniques that are of particular relevance to Actuarial work. Student should be able to demonstrate good understanding of the mathematical and statistical techniques and apply the same in solving practical problems.

B. Comments on candidates' performance in this session of the examination

Students are advice to practice a lot and try to solve problems under actual exam conditions. Read the question thoroughly. Students should start with the most prepared questions.

C. Pass Mark

The Pass Mark for this exam was 50.
216 candidates appeared and 34 passed.

Solution 1:

i)

```
x= c(5,10,15,20,25,30,35,40,45,50,55,60)
y=c(15,12,25,23,35,36,33,38,43,45,50,53)

> meanx = mean(x)
> meany=mean(y)
> meanx
[1] 32.5
> meany
[1] 34

> x_sq=x*x
> x_sq
 [1]   25  100  225  400  625  900 1225 1600 2025 2500 3025 3600
> y_sq=y*y
> xy=x*y
> xy
 [1]   75  120  375  460  875 1080 1155 1520 1935 2250 2750 3180



> sumx_sq=sum(x_sq)
> sumy_sq=sum(y_sq)
> sumxy=sum(xy)

> sumx_sq
[1] 16250
> sumy_sq
[1] 15760
> sumxy
[1] 15775


> Sxx=Sumx_sq-12*meanx^2
> Sxx
[1] 3575
> Sxy=sumxy-12*meanx*meany
> Sxy
[1] 2515
> Syy=sumy_sq-12*meany^2
> Syy
[1] 1888
```

(7)

ii)

```
> beta=Sxy/Sxx
> beta
[1] 0.7034965


> alpha=meany-beta*meanx
> alpha
[1] 11.13636
```

```
> sigmasq=(1/(12-2))*(Syy-Sxy^2/Sxx)
> sigmasq
[1] 11.87063
```

(3)

iii)

```
> expectedy=alpha+beta*x
> expectedy
 [1] 14.65385 18.17133 21.68881 25.20629 28.72378 32.24126 35.75874 39.27622 42.79371
46.31119 49.82867 53.34615
```

(1)

iv)

```
> e=y-alpha-beta*x
> e
 [1]  0.3461538 -6.1713287  3.3111888 -2.2062937  6.2762238  3.7587413
 [7] -2.7587413 -1.2762238  0.2062937 -1.3111888  0.1713287 -0.3461538

> meane=mean(e)
> meane
[1] -1.702344e-15

> var(e)
[1] 10.79148
```

Mean value of residuals is close to zero as expected as e~N(0,sigma^2)

(Otherwise, "e" could be calculated as e = y-expectedy)

Var of e is slightly lower than sigma square as calculated in part ii – as denominator is not adjusted
When denominator of 10 gets used instead of 11 we see that var of residuals = sigma^2

```
> var(e)*11/10
[1] 11.87063
```

(3)

v) <u>95% confidence interval for beta</u>

Ho: Beta is zero (i.e. no linear relationship between x and y)
H1: Beta is not equal to zero

(Beta_cap-0)/ sqrt(sigma^2_cap/ Sxx) ~ t_{10}

We use t distribution with n-2 i.e. 10 degrees of freedom

```
> qt(p=0.025, lower.tail = T, df=10)
[1] -2.228139
```

Being symmetric distribution, 97.5% point would be 2.228139

```
> sqrt(sigmasq/Sxx)
[1] 0.0576234
```

Hence, endpoints of CI would be

```
> end1=beta+sqrt(sigmasq/Sxx)*qt(p=0.025, lower.tail = T, df=10)
> end1
[1] 0.5751036
```

```
> end2=beta-sqrt(sigmasq/Sxx)*qt(p=0.025, lower.tail = T, df=10)
> end2
[1] 0.8318894
```

Hence 95% Confidence interval for beta is (0.5751, 0.8319)

As confidence interval for beta does not include zero, we can reject null hypothesis (viz. beta=0) and Hence, can conclude that beta is not equal to zero at 5% level.

95% CI for sigma^2

(n-2)sigmacap^2/ sigma^2 ~ Chi sq distribution with 10 degrees of freedom

Tabulated values of Chi square having 10 df can be obtained as

```
> chitenend1=qchisq(df=10, p=0.025)
> chitenend2=qchisq(df=10,p=0.975)
> chitenend1
[1] 3.246973
> chitenend2
[1] 20.48318
```

End points of CI would be

```
> sigmasqend1=(12-2)*sigmasq/chitenend1
> sigmasqend2=(12-2)*sigmasq/chitenend2
> sigmasqend1
[1] 36.55907
> sigmasqend2
[1] 5.795307
```

Hence 95% Confidence interval for sigma^2 is (5.795,36.559)

(7)

vi)

SS_{TOT} = Syy = 1888 (as calculated in part i)

SS_{REG} = Sxy^2/Sxx

```
> ss_reg=Sxy^2/Sxx
> ss_reg
[1] 1769.294
```

$SS_{RES} = SS_{TOT} - SS_{REG}$

```
> ss_res=Syy-ss_reg
> ss_res
[1] 118.7063
```

R^2 denotes the % of variability explained by the model

$R^2 = SS_{REG} / (SS_{REG} + SS_{RES})$

```
> Rsq = ss_reg/(ss_reg+ss_res)
> Rsq
[1] 0.9371259
```

Model is a good fit as 93.7% of the variability is explained by the model.

```
> adj_Rsq = 1-((12-1)/(12-1-1))*(1-Rsq)
> adj_Rsq
[1] 0.9308385
```

Adjusted R^2 (93.08%) is lower than R^2 (93.71%) as adjusted R square penalises for extra predictors and
hence is better suited to assess the adequacy of the model (or for comparison between models) compared to just using R^2 for model comparison as
R^2 cannot decrease on addition of more explanatory variables which can be undesirable (as it may promote too many explanatory variables though not adding significant improvement in the predicted value)

(5)

vii)
using results from earlier parts mean predicted response is calculated (using regression line equation)

```
> Emean52=alpha+beta*52
> Emean52
[1] 47.71818
```

Expected value of mean predicted response is 47.718 when x=52

```
varofmean52=((1/12)+(52-meanx)^2/Sxx)*sigmasq
> varofmean52
[1] 2.251822

> mean52end1=Emean52+qt(p=0.025, lower.tail = T, df=10)*sqrt(varofmean52)
> mean52end2=Emean52-qt(0.025,lower.tail = T, df=10)*sqrt(varofmean52)
> mean52end1
[1] 44.37462
> mean52end2
[1] 51.06174
```

Hence 95% confidence interval for the mean predicted response is (44.3746,51.0617)

(4)

[30 Marks]

Part (i), (ii), & (iii) Formula based questions. Most of the candidates scored full marks. Many used direct formulae and scored full marks.

Part (iv) Most of the candidates calculated residuals correctly along with mean and variance. Many candidates lost marks for the comment part.

Part (v) Many candidates struggled to calculate confidence interval for sigma^2.

Part (vi) Formula based question. But many students struggled to calculate adj R^2and on subsequent explanations.

Part (vii) Many candidates attempted well in first part. However, candidates found challenging to answer second part – CI for mean predicted response.

Solution 2:

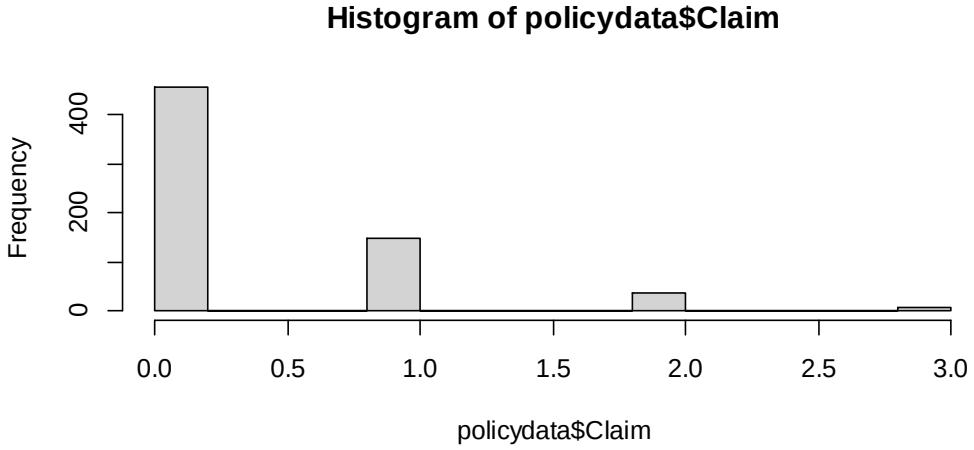
i)

```
library(dplyr)

> str(policydata)
'data.frame':           650 obs. of  4 variables:
 $ Policy  : int  1 2 3 4 5 6 7 8 9 10 ...
 $ Claim   : int  0 0 0 2 1 0 0 0 0 0 ...
 $ Cust_Exp: chr  "SA" "SA" "SA" "DS" ...
 $ Amount  : int  0 0 0 52601 56174 0 0 0 0 0 ...
>
> #a
> table(policydata$Claim)

  0   1   2   3
458 149  36   7
> #Alternative, if dplyr installed
> #count(policydata,Claim)

> print("458 Policies don't have any claim")
[1] "458 Policies don't have any claim"
```

(2)

ii)

```
> hist(policydata$Claim)
> #poisson and negative binomial distribution
```



(2)

iii)

```
> poisson.test(x=sum(policydata$Claim),T=length(policydata$Policy))

Exact Poisson test

data:  sum(policydata$Claim) time base: length(policydata$Policy)
number of events = 242, time base = 650, p-value < 2.2e-16
alternative hypothesis: true event rate is not equal to 1
95 percent confidence interval:
 0.3268739 0.4222903
sample estimates:
event rate
 0.3723077
```

```
> #0.35 is more suitable value of parameter since it lies between confidence interval.
```

(3)

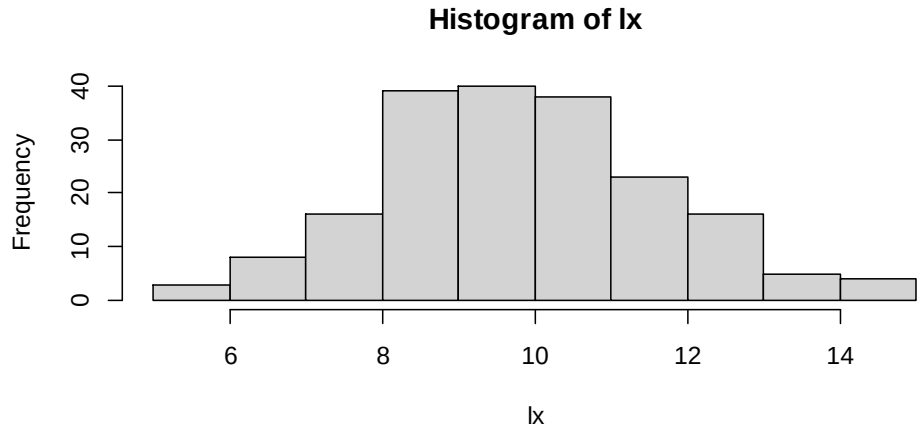
iv)

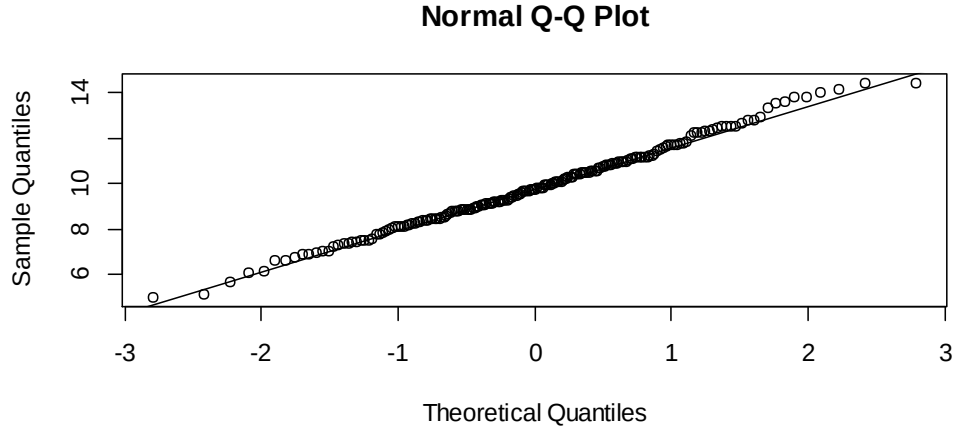
```
> lx=log(policydata$Amount[policydata$Amount>0])
> #Alternative, if dplyr installed
> #lx=log(filter(policydata,Amount >0)$Amount)

> mean(lx)
[1] 9.835205
> median(lx)
[1] 9.774659
> sd(lx)^2
[1] 3.425705
```

(4)

v)

```
> par(mfrow=c(2,1))
> hist(lx)
> qqnorm(lx)
> qqline(lx)
```





(3)

vi)

```
> # From Histogram and QQPlot it seems log amount closely follows normal distribution.
> # To add, the mean and median are very close indicating symmtery. One of the characterstics of Z.
> # Hence,Claim amount might be following log normal distribution.
```

(3)

vii)

```
> #Null Hypothesis : mu = 10 , alternate hypothesis mu >10
> t.test(lx,mu=10,alternative="greater", conf.level = .9)

        One Sample t-test

data:  lx
t = -1.2337, df = 191, p-value = 0.8906
alternative hypothesis: true mean is greater than 10
90 percent confidence interval:
 9.663428      Inf
sample estimates:
mean of x
 9.835205

> #Given p-value greater than 10% null hypothesis can not be rejected.
```

(4)

viii)

```
> ct=table(policydata$Claim,policydata$Cust_Exp)
> ct

   DS  SA  VD  VS
 0  63 306  20  69
 1  36  90   9  14
 2  16  14   6   0
 3   3   3   1   0
> # Null Hypothesis: No association between Policyholder's experience and Claim
> chisq.test(ct)

        Pearson's Chi-squared test

data:  ct
X-squared = 47.749, df = 9, p-value = 2.846e-07

Warning message:
In chisq.test(ct) : Chi-squared approximation may be incorrect
```

(3)

ix)

```
> #There are cells where the number of observations are less than 5.
```

(1)

x)

```
> policydata$Claim2=ifelse(policydata$Claim >2,2,policydata$Claim)
> policydata$Cust_Exp2=ifelse(policydata$Cust_Exp %in% c("DS","VD"),"DS","SA")
> ct2=table(policydata$Claim2,policydata$Cust_Exp2)
> ct2

   DS  SA
 0  83 375
 1  45 104
 2  26  17
```

```
> chisq.test(ct2)

        Pearson's Chi-squared test

data:  ct2
X-squared = 43.514, df = 2, p-value = 3.557e-10


> # There is a strong reason to reject null hypothesis.
> # Hence, it can concluded that policyholder's experience gets worse as
claim count increases
```

(5)

xi)

```
> summary(policydata$Amount)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
      0       0       0   29501    3232 1848069
> policydata$large= ifelse(policydata$Amount >100000,1,0)
> x = sum(policydata$large)

> n = length(policydata$Amount[policydata$Amount>0])
> #Alternative, if dplyr installed
> #n = length(filter(policydata,Amount >0)$Amount)

> binom.test(x,n)

        Exact binomial test

data:  x and n
number of successes = 35, number of trials = 192, p-value < 2.2e-16
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.1303796 0.2442928
sample estimates:
probability of success
          0.1822917

>
> # Since upper bound of c.i is less that .25, it is unlikely that more
that
> #25% claims are large
```

(5)

[35 Marks]

Part (i) Simple question answered well by most

Part (ii) Generally well answered. Many candidates could not specify correct distributions.

Part (iii) Not attempted by many candidates. Those who attempted it were able to reach at the right conclusion.

Part (iv) Generally well answered.

Part (v) Simple straightforward question, answered well

Part (vi) Many candidates incorrectly identified the distribution as normal distribution as against log normal distribution.

Part (vii) Many candidates didn't answered this, those who attempted answered well.

Part (viii) Most of the Candidates struggled on this question. Only few answered well.

Part (ix) Generally, those who answered previous part were able to answer this.

Part (x) Candidates struggled to manipulate the data and consequently answer this question. Only select few were able to answer this.

Part (xi) Most candidates failed to correctly filter the data and only a few could apply the binomial test. Thus, application side of the theoretical concepts not very satisfactory.

Solution 3:

```
# Sample mean and variance
Motorclaim = read.csv("Motorclaim.CSV")
Mean_Claim<-mean(Motorclaim$CLAIM)
Var_Claim<-var(Motorclaim$CLAIM)
```

i)

```
# Method of moments estimate

# Normal Distribution

Normal_mu <- Mean_Claim
Normal_sigma <- sqrt(Var_Claim)

Normal_mu
[1] 6357.314

Normal_sigma
[1] 6986.523

# Log Normal Distribution

LogNormal_sigma<- sqrt(log(1+Var_Claim/Mean_Claim^2))
LogNormal_mu<-log(Mean_Claim)-LogNormal_sigma^2/2

LogNormal_sigma
[1] 0.8899276

LogNormal_mu
[1] 8.361376

# Exponential Distribution

Exp_lamda <- 1/Mean_Claim

Exp_lamda
[1] 0.0001572991
```

 # Gamma Distribution

```
Gamma_lamda<-Mean_Claim/Var_Claim
Gamma_alpha<-Gamma_lamda*Mean_Claim

Gamma_lamda
[1] 0.0001302421

Gamma_alpha
[1] 0.82799
```

(8)

ii)
 # Histogram

```
hist(Motorclaim$CLAIM,breaks = 35,freq = FALSE)
```

#Superimpose Normal distribution

```
curve(dnorm(x,mean = Normal_mu,sd = Normal_sigma),from = min(Motorclaim$CLAIM), to =
max(Motorclaim$CLAIM), add = TRUE, col= "blue")
```

#Superimpose Log Normal distribution

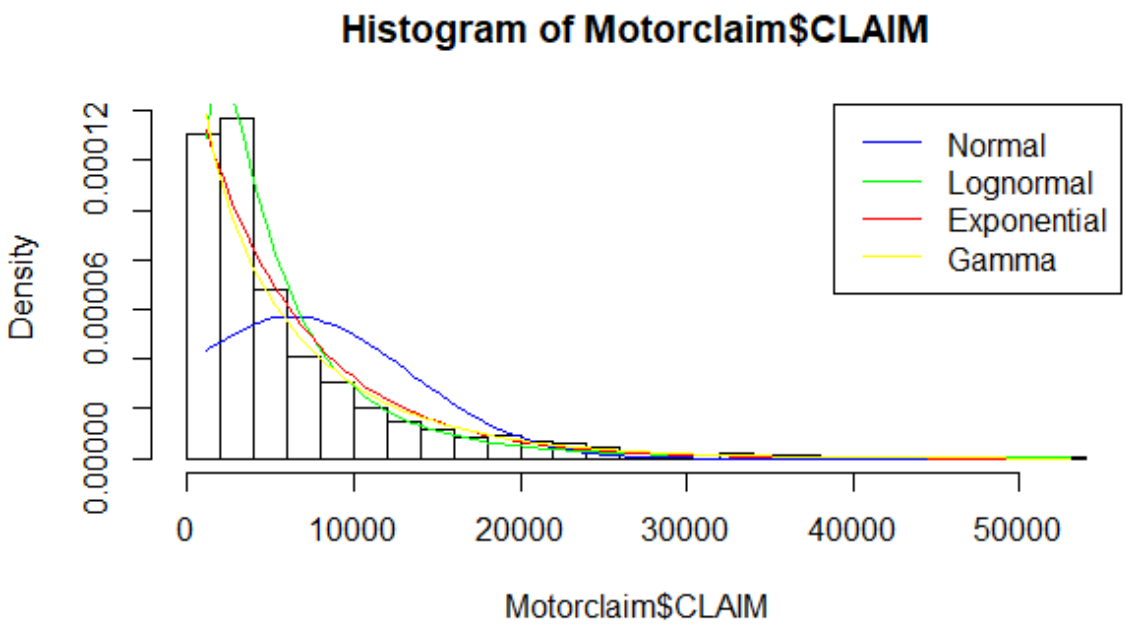
```
curve(dlnorm(x,meanlog = LogNormal_mu,sdlog = LogNormal_sigma),from =
min(Motorclaim$CLAIM), to = max(Motorclaim$CLAIM), add = TRUE, col= "green")
```

#Superimpose Exponential distribution

```
curve(dexp(x,rate = Exp_lamda),from = min(Motorclaim$CLAIM), to = max(Motorclaim$CLAIM), add
= TRUE, col= "red")
```

#Superimpose Gamma distribution

```
curve(dgamma(x,shape = Gamma_alpha,rate = Gamma_lamda),from = min(Motorclaim$CLAIM), to
= max(Motorclaim$CLAIM), add = TRUE, col= "yellow")

legend("topright",legend = c("Normal", "Lognormal", "Exponential", "Gamma"),lty = 1, col =
c("blue","green","red","yellow"))
```



(8)

iii)

 # Quantiles

 # Actual Claim Data

```
quantile(Motorclaim$CLAIM,c(0.05,0.25,0.5,0.75,0.95))

     5%       25%       50%       75%       95%
1324.561  1934.876  3631.070  7870.028 21246.913
```

 # Normal Distribution

```
qnorm(c(0.05,0.25,0.5,0.75,0.95),mean = Normal_mu,sd = Normal_sigma)

[1] -5134.494  1644.976  6357.314 11069.653 17849.123
```

 # Log Normal Distribution

```
qlnorm(c(0.05,0.25,0.5,0.75,0.95),meanlog = LogNormal_mu,sdlog = LogNormal_sigma)

[1]   989.8714  2347.5526  4278.5767  7798.0014 18493.5327
```

 # Exponential Distribution

```
qexp(c(0.05,0.25,0.5,0.75,0.95),rate = Exp_lamda)

[1]   326.0876  1828.8853  4406.5544  8813.1089 19044.8114
```

 # Gamma Distribution

```
qgamma(c(0.05,0.25,0.5,0.75,0.95),shape = Gamma_alpha,rate = Gamma_lamda)

[1]   193.6261  1479.4200  4053.4299  8797.0450 20369.6614
```

(5)

iv) From the histogram and superimposed plots it is clear that normal distribution is not good fit to the data.

The other three plots are getting superimposed more or less similar to the data. From the quantiles it is observed that lower value(5^{th} percentile) of lognormal is closed to actual value and higher values(95^{th} percentile) of gamma distribution is closed to actual value

The best fitting distribution among Lognormal, exponential & Gamma can not be decided basis of observations from (ii) & (iii). Further statistical tests need to be carried out to confirm best fit

(4)

v)

```
# Simulation from Gamma distribution

set.seed(2022)
Sim_samples <- rgamma(20000,Gamma_alpha,Gamma_lamda)

head(Sim_samples,10)
[1]  9505.735311  1376.831631   458.302589  3189.065594     5.340363  5821.017458
 [7] 11122.004509  5372.490004 43002.362493  3557.086406
```

(2)

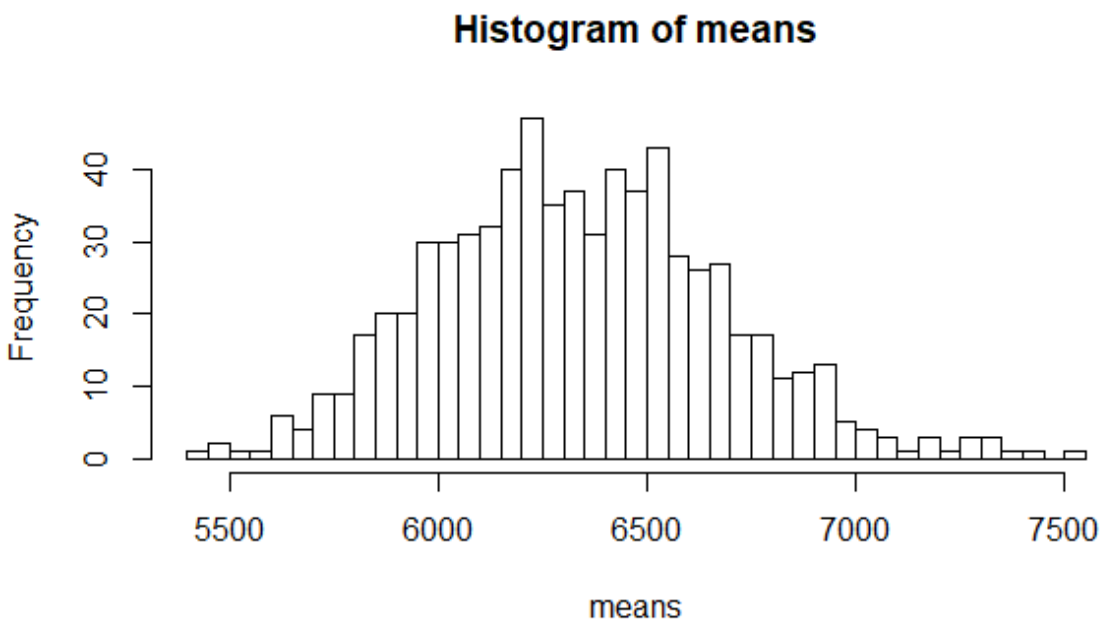
vi)

```
# Generating 700 random samples of size 400 and computing sample means

means<-c()
set.seed(2022)
for (i in 1:700){
selected_data_point<-sample(1:20000,400,FALSE)
random_sample<- Sim_samples[selected_data_point]
sample_mean<-mean(random_sample)
means<-c(means,sample_mean)
}
```

(5)

vii)

```
# Histogram of the sample means
hist(means,breaks = 40)
```



Comment:
The distribution of sample means tend to follow normal distribution however the actual data comes from gamma distribution. Central Limit Theorem states that the sample means tend to follow normal distribution as the sample size increases. The distribution of sample means will be closer to normal distribution by increasing the sample size from its current level of 400.

(3)

[35 Marks]

Part (i) Formula based simple question generally well answered

Part (ii) Most candidates only plotted the histogram. Only a few candidates correctly super-imposed the density functions.

Part (iii) Generally well answered.

Part (iv) Few candidates were able to provide suitable rationale. Only a few candidates were able to draw the comparison between lower and upper percentiles.

Part (v) Formula based simple question. Most scored full marks.

Part (vi) Candidates struggled for this part. Many candidates struggled with constructing the "for loop".

Part (vii) The comment on the histogram was partial. Most did not catch the application of central limit theorem here.

Additional Comments

- *Many lost easy marks on questions such as Adjusted R^2, appropriate distributions for claim counts, calculation of quantiles, manipulation of tables, parameter computations for distributions, chart plotting, simulations etc.*
- *These are few basic standard concepts and are expected from the candidates studying this paper.*

****