Compressive Sensing as a tool for Video Analysis

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Motivation

"Big Brother is Watching You."
- George Orwell, 1984

What is compressive sensing?

Compressive sensing is a method of **reducing the amount of data collected** from a signal without compromising the ability to later **reconstruct the signal accurately.**

CS Methodology

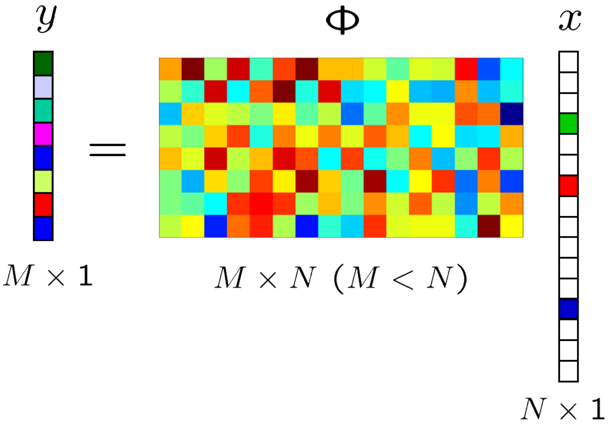

Figure: CS measurement process, courtesy of Volkan Cevher.

Restricted Isometry Property (RIP)

A matrix $\mathbf{\Phi}$ satisfies the Restricted Isometry Property (RIP) of order K if there exists a $\delta_K \in (0,1)$ such that

$$(1-\delta_k)||\boldsymbol{x}||_2^2 \leq ||\mathbf{\Phi}\boldsymbol{x}||_2^2 \leq (1+\delta_k)||\boldsymbol{x}||_2^2, \qquad (1)$$

for all $\boldsymbol{x} \in \sum_K = \boldsymbol{x} : ||\boldsymbol{x}||_0 \leq K$.

Recovery of sparse transforms

- $y = \Phi x$
- $\Delta(y, \Phi) = x$
- Infinitely many solutions!

$$\hat{x} = \arg min_{y=\phi x} ||x||_0$$

$$\hat{x} = \arg min_{y=\phi x} ||x||_1$$

Optimisation based on the l_1 norm can closely approximate compressible signals with high probability.

Orthogonal Matching Pursuit

We shall define the columns of Φ to be $\varphi_1, \varphi_2, \ldots, \varphi_N$ each of length M.

- Step 1: Find the index for the column of Φ which satisfies $\lambda_t = \text{argmax}_{\ j=1,\ldots,N} | < r_{t-1}, \varphi_j > |$
- Step 2: Keeps track of the columns used. $\Lambda_t = \Lambda_{t-1} \cup \lambda_t$, $\Phi_t = [\Phi_{t-1}, \psi_{\lambda_t}]$
- Step 3: Update the estimate of the signal. $x_t = \text{argmin}_x ||v - \Phi_t x||_2$.
- Step 4: Update the measurement residual. $r_t = y - \Phi_t x_t$.
- Output: Estimated sparse vector $\hat{x}$

Background Subtraction

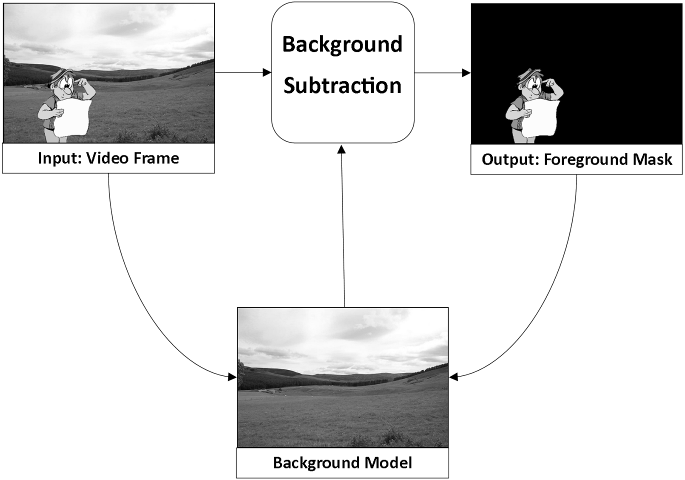

Figure: The background subtraction process

Foreground sparsity

(a) Original frame

(b) Background Model

(c) Foreground Mask

Background Subtraction with Compressive Sensing.

1. Initialise a compressed background y_0^b.
2. Compressively Sense $y_t = \Phi x_t$.
3. Reconstruct $\Delta(y_t - y_t^b)$
4. Update Background $y_{t+1}^b = \alpha y_i + (1-\alpha) y_i^b$

Experimentation

Figure: Sanfran test video courtesy of Seth Benton

Further Work

- Choice of Φ and Δ?
- More advanced methods of background subtraction.
- Adapting with varying sparsity.
- Knowing when to reconstruct.
- Exploiting the properties of natural images.

Any Questions?