

Mathematics on T.V.

Rhian Davies

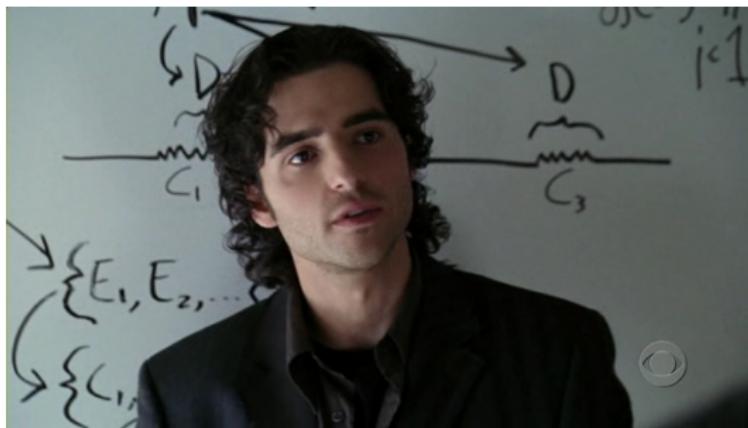
February 26, 2014

Mathematics
and Statistics



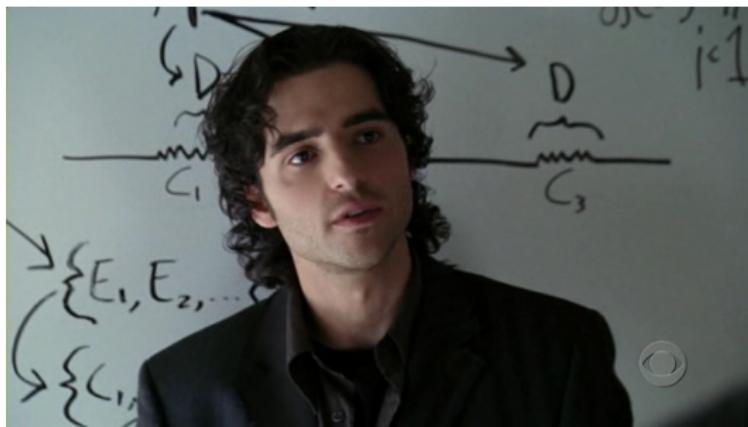
NUMB3RS

"We all use math every day; to predict weather, to tell time, to handle money. Math is more than formulas or equations; it's logic, it's rationality, it's using your mind to solve the biggest mysteries we know."

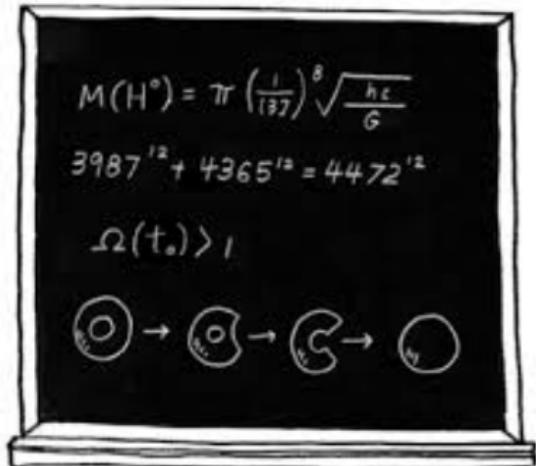


NUMB3RS

"We all use math every day; to predict weather, to tell time, to handle money. Math is more than formulas or equations; it's logic, it's rationality, it's using your mind to solve the biggest mysteries we know."



"Getting the math right and getting it to fit with the plot are not priorities for the NUMB3RS team"



- ▶ In 1637, Fermat stated that he can prove that the equation

$$x^n + y^n = z^n \quad \text{for} \quad (n > 2), \quad (1)$$

has no whole number solutions.

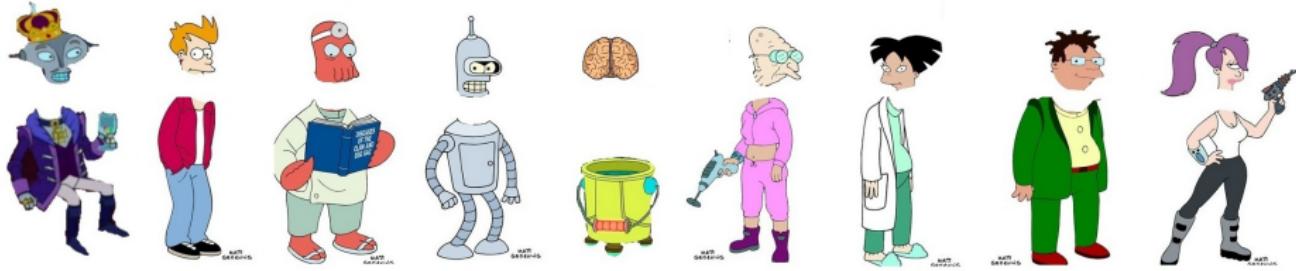
- ▶ He didn't ever write this "proof" down and died before telling anyone.
- ▶ Over 350 years later, in 1995 Andrew Wiles discovered a proof for Fermats Last Theorem. This was accepted by the mathematical community as being watertight.
- ▶ There are no whole number solutions to Equation (1).

The Prisoner of Bender

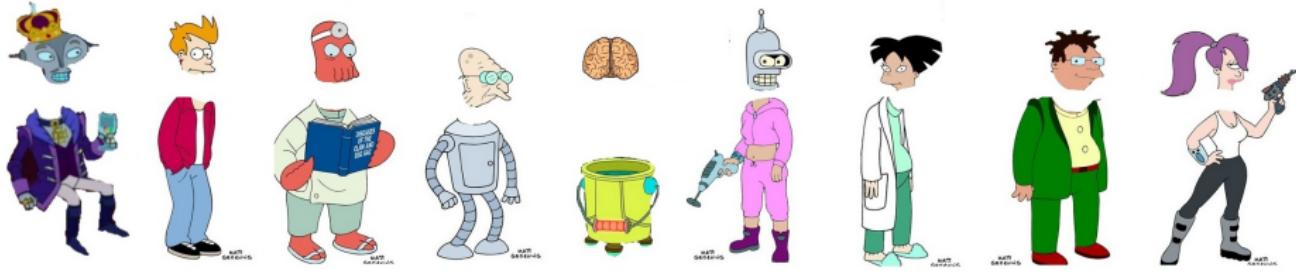
- ▶ In the Prisoner of Benda, the Professor and Amy have swap bodies.
- ▶ Once two bodies switch minds, they can never switch back.
- ▶ Maybe if they introduce another character they can play a little mental musical chairs.
- ▶ However, adding one extra body isn't enough to get everyone back.



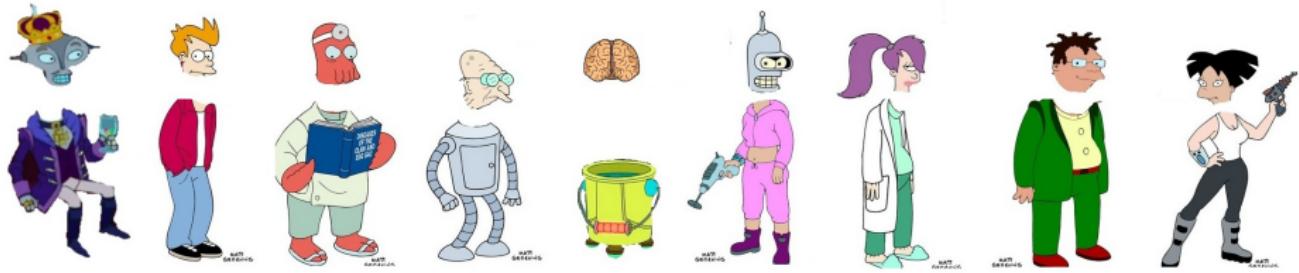




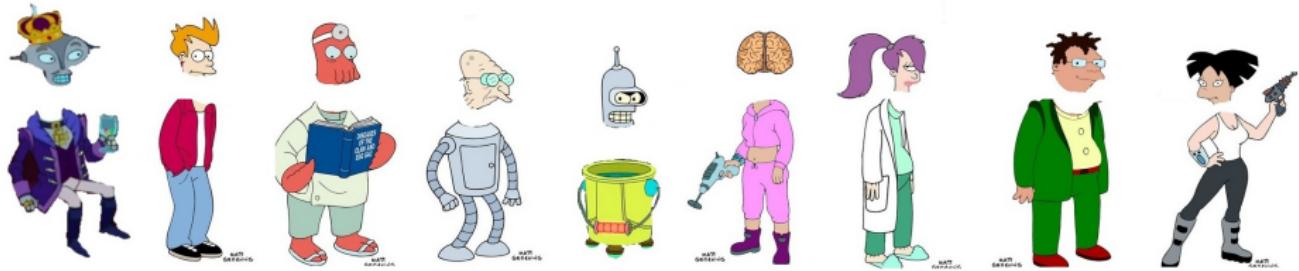
(a p)



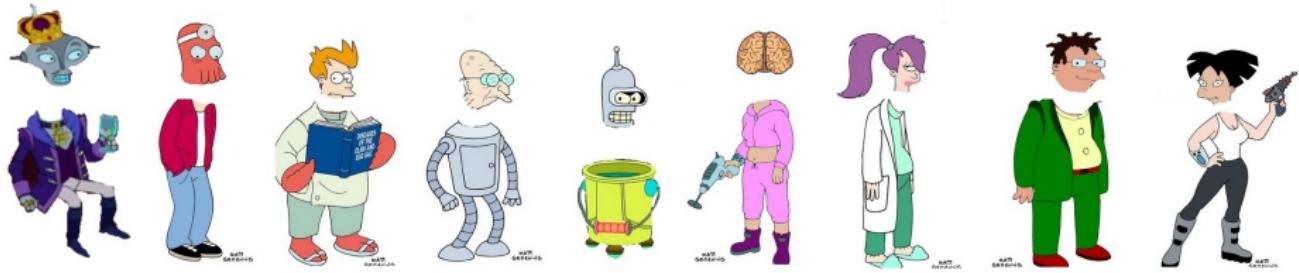
(*a p*)(*a b*)



$(a\ p)(a\ b)(p\ l)$



$(a\ p)(a\ b)(p\ l)(a\ w)$



(*a p*)(*a b*)(*p l*)(*a w*)(*f z*)



(*a p*)(*a b*)(*p l*)(*a w*)(*f z*)(*e w*)



(*a p*)(*a b*)(*p l*)(*a w*)(*f z*)(*e w*)(*h l*)

Ken Keeler



Ken Keeler



- ▶ To undo the mixing of minds, Ken Keeler used group theory in what must be one of the most mathematical resolutions to a plot on T.V - ever!

Keeler's Theorem

First, let π be some K -cycle on $[n] = \{1, \dots, n\}$; wlog write

$$\pi = \begin{pmatrix} 1 & 2 & \dots & K & K+1 & \dots & n \\ 2 & 3 & \dots & 1 & K+1 & \dots & n \end{pmatrix}$$

Let (a, b) represent the transposition that switches the contents of a and b .
By hypothesis π is generated by DISTINCT switches on $[n]$.

Introduce two "new bodies" $\{x, y\}$ and write $\pi^* = \begin{pmatrix} 1 & 2 & \dots & K & K+1 & \dots & n & x & y \\ 2 & 3 & \dots & 1 & K+1 & \dots & n & x & y \end{pmatrix}$

For any $i = 1, \dots, K$ let σ_i be the (L-to-R) series of switches

$$\sigma_i = ((x_i) (x, z) \dots (x_i)) ((y, i+1) (y, i+2) \dots (y, K)) ((x, i+1) (y, i)).$$

Note each switch exchanges an element of $[n]$ with one of $\{x, y\}$, so they're all distinct from the switches within $[n]$ that generated π , and also from (x, y) . By routine verification,

$$\pi^* \sigma_i = \begin{pmatrix} 1 & 2 & \dots & n & x & y \\ 1 & 2 & \dots & n & y & x \end{pmatrix} \text{ i.e., } \sigma_i \text{ inverts the } K\text{-cycle and leaves } x \text{ and } y \text{ switched (without performing } (x, y)\text{).}$$

NOW let π be an ARBITRARY permutation on $[n]$: it consists of disjoint (nontrivial) cycles, and each can be inverted as above in sequence, after which x and y can be switched if necessary via (x, y) , as was desired.



Keeler's Theorem (in plain English)

1. Have everybody who's messed up arrange themselves in circles.
2. Go get two "fresh" people. Let's call them Helper A and Helper B.
3. Fix the circles one by one as follows:
 - ▶ Start each time with Helper A and Helper B's minds in either their own or each other's bodies
 - ▶ Pick any circle of messed-up people you like and unwrap it into a line with whoever you like at the front
 - ▶ Swap the mind at the front of the line into Helper A's body
 - ▶ From back to front, have everybody in the line swap minds with Helper B's body in turn. (This moves each mind in the line, apart from the front one, forward into the right body)
 - ▶ Swap the mind in Helper A's body back where it belongs, into the body at the back of the line. Now the circle/line has been completely fixed.
4. After all the circles have been fixed, mind-swap the two Helpers if necessary.

