1 Notes on Laplacians

There are three major Graph Laplacians.

Don't assume eigenvectors of a matrix are normalised. (Normalised means the eigenvector has length one).

Assume eigenvectors ordered increasing (first k eigenvectors means the k smallest eigenvectors).

Unnormalized Laplacian is L = D - W

Normalized Laplacians

$L_{sym} := D^{-1/2}LD^{-1/2} = I - D^{-1/2}WD^{-1/2}$

This second bit follows since $D^{-1/2}DD^{-1/2}$

Note: How to calculate $D^{1/2}$ Let V be the vector of eigenvalues of D, and let Q be a matrix of the eigenvectors of D. Then $D^{1/2} = Qdiag(\frac{1}{\sqrt{V}})Q^T$

Note that L_{symm} is symmetric.

The other normalized Laplacian commonly used is $L_{rw} = D^{-1}L = I - D^{-1}W$