Confirmatory Factor Analysis

Statsomat.com

07 January 2021

Warning: The automatic computation and interpretation delivered by the Statsomat should not completely replace the classical, made by humans graphical exploratory data analysis and statistical analysis. There may be data cases for which the Statsomat does not deliver the most optimal solution or output interpretation.

Basic Information

Automatic statistics for the file:

File
case14.csv

Your selection for the encoding: Auto
Your selection for the decimal character: Auto
Observations (rows with at least one non-missing value): 75
Variables (columns with at least one non-missing value): 11
Variables considered continuous: 5

Variables considered continuous
y1
y8
x1
x2
x3

Numerical variables considered binary or ordinal: 6

Numerical variables considered binary or ordinal
y2
y3
y4
y5
y6
y7

Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
ind60	=~	x1	1	0
ind60	=~	x2	1	1
ind60	=~	x3	1	2
dem60	=~	y1	1	0
dem60	=~	y2	1	3
dem60	=~	y3	1	4
dem60	=~	y4	1	5
dem65	=~	y5	1	0
dem65	=~	y6	1	6
dem65	=~	y7	1	7
dem65	=~	y8	1	8
x1	~~	x1	0	9
x2	~~	x2	0	10
x3	~~	x3	0	11
y1	~~	y1	0	12
y2	~~	y2	0	13
y3	~~	y3	0	14
y4	~~	y4	0	15
y5	~~	y5	0	16
y6	~~	y6	0	17
y7	~~	y7	0	18
y8	~~	y8	0	19
ind60	~~	ind60	0	20
dem60	~~	dem60	0	21
dem65	~~	dem65	0	22
ind60	~~	dem60	0	23
ind60	~~	dem65	0	24
dem60	~~	dem65	0	25

Assumptions

Open issue

Model Settings

Outputs

Model Fit Summary

```
lavaan 0.6-7 ended normally after 47 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         25

  Number of observations                            75

Model Test User Model:

  Test statistic                                72.462
  Degrees of freedom                                41
  P-value (Chi-square)                           0.002

Model Test Baseline Model:

  Test statistic                               730.654
  Degrees of freedom                                55
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.953
  Tucker-Lewis Index (TLI)                       0.938

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1564.959
  Loglikelihood unrestricted model (H1)      -1528.728

  Akaike (AIC)                                3179.918
  Bayesian (BIC)                              3237.855
  Sample-size adjusted Bayesian (BIC)         3159.062

Root Mean Square Error of Approximation:

  RMSEA                                          0.101
  90 Percent confidence interval - lower         0.061
  90 Percent confidence interval - upper         0.139
  P-value RMSEA <= 0.05                          0.021
```

```
Standardized Root Mean Square Residual:

  SRMR                                           0.055

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  ind60 =~
    x1                1.000
    x2                2.182    0.139   15.714    0.000
    x3                1.819    0.152   11.956    0.000
  dem60 =~
    y1                1.000
    y2                1.354    0.175    7.755    0.000
    y3                1.044    0.150    6.961    0.000
    y4                1.300    0.138    9.412    0.000
  dem65 =~
    y5                1.000
    y6                1.258    0.164    7.651    0.000
    y7                1.282    0.158    8.137    0.000
    y8                1.310    0.154    8.529    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  ind60 ~~
    dem60             0.660    0.206    3.202    0.001
    dem65             0.774    0.208    3.715    0.000
  dem60 ~~
    dem65             4.487    0.911    4.924    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                0.082    0.020    4.180    0.000
   .x2                0.118    0.070    1.689    0.091
   .x3                0.467    0.090    5.174    0.000
   .y1                1.942    0.395    4.910    0.000
   .y2                6.490    1.185    5.479    0.000
   .y3                5.340    0.943    5.662    0.000
   .y4                2.887    0.610    4.731    0.000
   .y5                2.390    0.447    5.351    0.000
   .y6                4.343    0.796    5.456    0.000
   .y7                3.510    0.668    5.252    0.000
   .y8                2.940    0.586    5.019    0.000
```

```
    ind60             0.448    0.087    5.169    0.000
    dem60             4.845    1.088    4.453    0.000
    dem65             4.345    1.051    4.134    0.000
```

Completely Standardized Parameter Estimates

```
Latent Variables:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  ind60 =~
    x1               0.920    0.023   39.823    0.000    0.874    0.965
    x2               0.973    0.017   58.858    0.000    0.941    1.006
    x3               0.872    0.031   28.034    0.000    0.811    0.933
  dem60 =~
    y1               0.845    0.039   21.698    0.000    0.769    0.921
    y2               0.760    0.054   14.142    0.000    0.655    0.866
    y3               0.705    0.063   11.225    0.000    0.582    0.828
    y4               0.860    0.036   23.650    0.000    0.789    0.931
  dem65 =~
    y5               0.803    0.046   17.602    0.000    0.714    0.893
    y6               0.783    0.049   15.918    0.000    0.687    0.879
    y7               0.819    0.043   19.122    0.000    0.735    0.903
    y8               0.847    0.038   22.389    0.000    0.773    0.921

Covariances:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  ind60 ~~
    dem60            0.448    0.102    4.393    0.000    0.248    0.648
    dem65            0.555    0.090    6.195    0.000    0.379    0.730
  dem60 ~~
    dem65            0.978    0.026   37.483    0.000    0.927    1.029

Variances:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
   .x1               0.154    0.042    3.636    0.000    0.071    0.238
   .x2               0.053    0.032    1.634    0.102   -0.010    0.116
   .x3               0.240    0.054    4.417    0.000    0.133    0.346
   .y1               0.286    0.066    4.348    0.000    0.157    0.415
   .y2               0.422    0.082    5.166    0.000    0.262    0.582
   .y3               0.503    0.089    5.676    0.000    0.329    0.676
   .y4               0.261    0.063    4.173    0.000    0.138    0.383
   .y5               0.355    0.073    4.842    0.000    0.211    0.499
   .y6               0.387    0.077    5.024    0.000    0.236    0.538
   .y7               0.329    0.070    4.696    0.000    0.192    0.467
   .y8               0.283    0.064    4.416    0.000    0.157    0.408
    ind60            1.000                               1.000    1.000
    dem60            1.000                               1.000    1.000
    dem65            1.000                               1.000    1.000
```

Communality

Table 5: Communality

Variable	Communality
x1	0.85
x2	0.95
x3	0.76
y1	0.71
y2	0.58
y3	0.50
y4	0.74
y5	0.65
y6	0.61
y7	0.67
y8	0.72

Factor Discriminant Validity

Table 6: Factor Discriminant Validity Test at Cutoff 0.85

			Factor Correlation	Chisq diff	Df diff	P-Value
ind60	~~	dem60	0.448	33.676	1	<0.001
ind60	~~	dem65	0.555	21.897	1	<0.001
dem60	~~	dem65	0.978	0.000	1	1

Factor Reliability

Table 7: Factor Reliability

	ind60	dem60	dem65	total
Omega (Bentler)	0.94	0.87	0.89	0.94
Omega (McDonald)	0.94	0.87	0.89	0.95
AVE	0.86	0.62	0.66	0.65

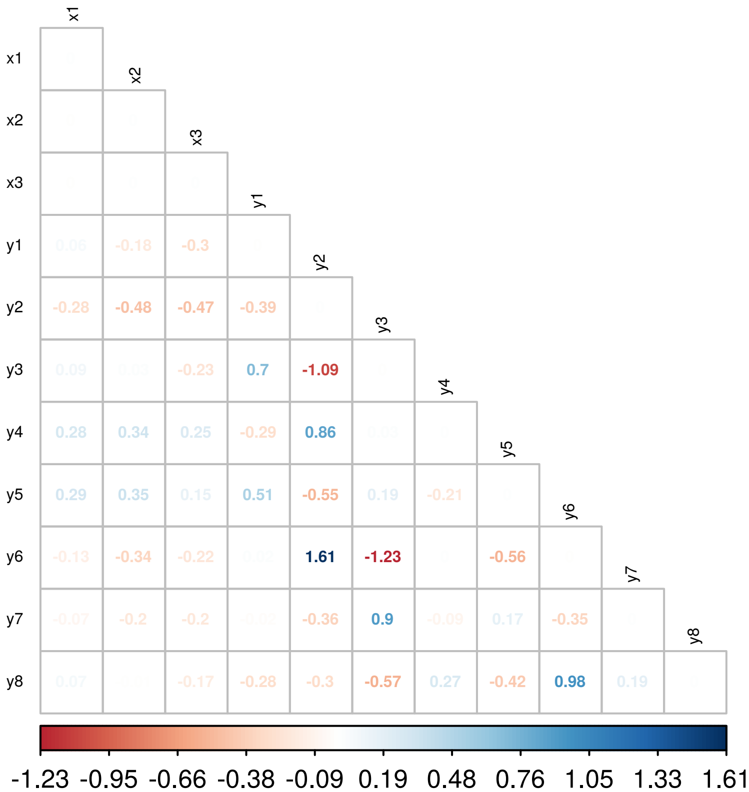
Observed Covariance Matrix

	x1	x2	x3	y1	y2	y3	y4	y5	y6	y7	y8
x1	0.53										
x2	0.98	2.25									
x3	0.81	1.78	1.95								
y1	0.72	1.26	0.9	6.79							
y2	0.61	1.47	1.15	6.17	15.37						
y3	0.78	1.53	1.03	5.76	5.76	10.62					
y4	1.14	2.21	1.81	6.01	9.38	6.6	11.07				
y5	1.07	2.04	1.56	5	5.53	4.87	5.63	6.73			
y6	0.84	1.78	1.55	5.67	9.26	4.66	7.34	4.91	11.22		
y7	0.92	1.97	1.6	5.73	7.43	6.91	7.39	5.74	6.66	10.66	
y8	1.09	2.2	1.67	5.6	7.65	5.56	7.91	5.27	8.14	7.49	10.39

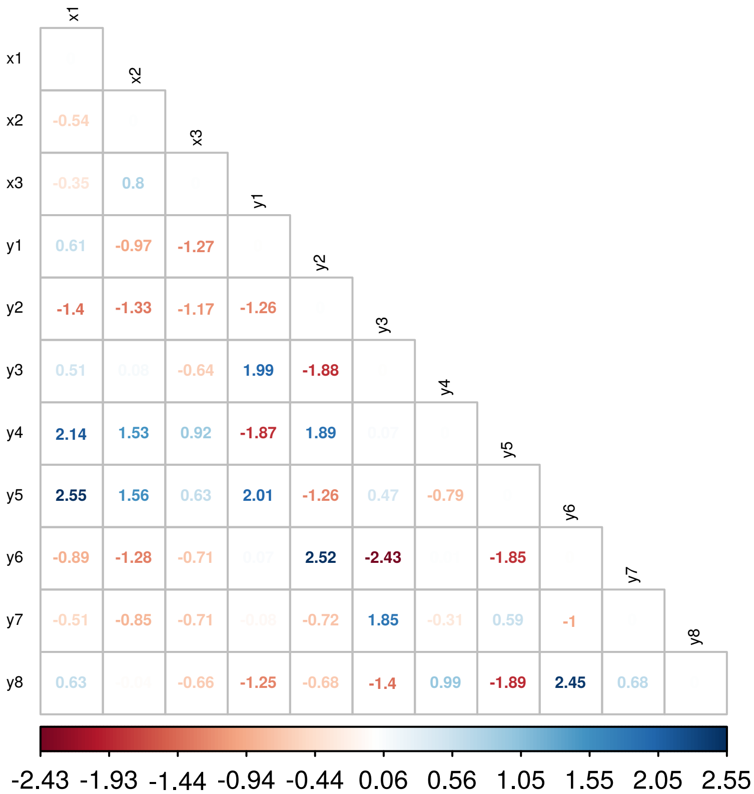
Model-Implied Covariance Matrix

	x1	x2	x3	y1	y2	y3	y4	y5	y6	y7	y8
x1	0.53										
x2	0.98	2.25									
x3	0.82	1.78	1.95								
y1	0.66	1.44	1.2	6.79							
y2	0.89	1.95	1.63	6.56	15.37						
y3	0.69	1.5	1.25	5.06	6.85	10.62					
y4	0.86	1.87	1.56	6.3	8.53	6.57	11.07				
y5	0.77	1.69	1.41	4.49	6.08	4.68	5.83	6.73			
y6	0.97	2.13	1.77	5.65	7.65	5.9	7.34	5.47	11.22		
y7	0.99	2.17	1.81	5.75	7.79	6.01	7.48	5.57	7.01	10.66	
y8	1.01	2.21	1.84	5.88	7.96	6.14	7.64	5.69	7.16	7.3	10.39

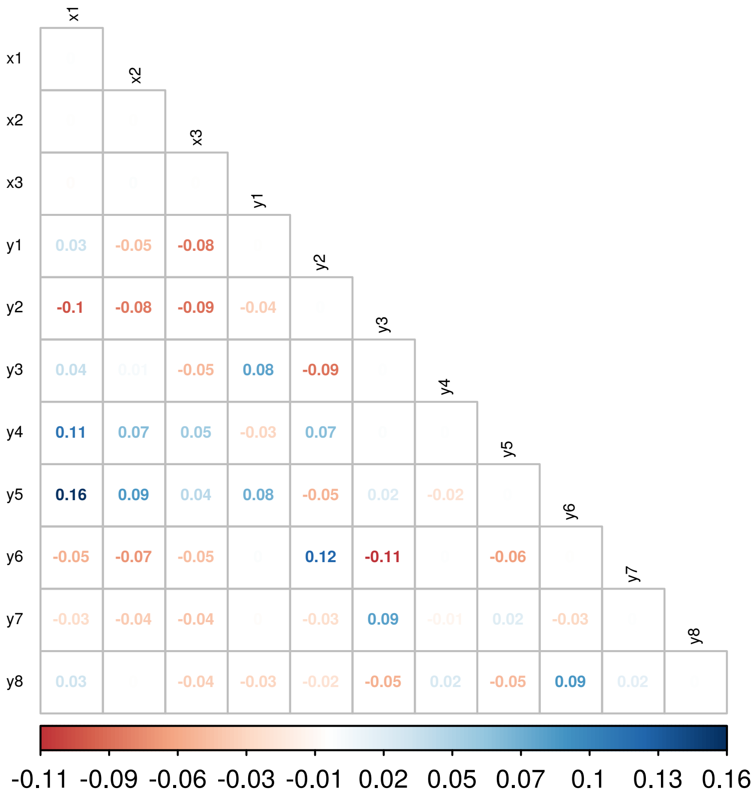
Residual Covariance Matrix



Standardized Residual Matrix



Residual Correlation Matrix



Modification Indices

Table 8: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
y2	~~	y6	9.279	2.129	0.1	0.052	**(m)**
y6	~~	y8	8.668	1.513	0.1	0.054	**(m)**
y1	~~	y5	8.183	0.884	0.1	0.062	**(m)**
y3	~~	y6	6.574	-1.590	0.1	0.053	**(m)**
y1	~~	y3	5.204	1.024	0.1	0.056	**(m)**
y2	~~	y4	4.911	1.432	0.1	0.053	**(m)**
y3	~~	y7	4.088	1.152	0.1	0.054	**(m)**
x1	~~	y2	3.785	-0.192	0.1	0.173	(i)

Note:
Maximum 10 modification indices in descending order of their magnitude are listed.

Table 9: Modification Indices With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
ind60	=~	y5	4.007	0.762	0.4	0.183	**(m)**
ind60	=~	y4	3.568	0.811	0.4	0.154	(i)

Note:

Table 9: Modification Indices With Respect To Factor Loadings (continued)

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision

Maximum 10 modification indices in descending order of their magnitude are listed.

Interpretation

Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [@brown], [@kline].

Model Test User Model

The degrees of freedom are calculated as the number of known parameters minus the number of free parameters: 66 - 25 = 41 . The 41 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 72.462 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.05 then the test is not statistically significant at 5 % error, the hypothesis cannot be rejected, which would be in favour of the model.
In our case, the p-value is 0.002 suggesting that the model may not be acceptable for the data. The Chi-square model fit index is based on a very stringent statistical hypothesis which may have no practical relevance. We will consider it only in connection with other model fit indices.

Model Test Baseline Model

The test statistic with the value 730.654 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is <0.001 suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

Root Mean Square Error of Approximation:

The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.
In our case, the RMSEA is 0.101. The upper bound of the 90% confidence interval of the RMSEA is 0.139 and greater or equal than the threshold value 0.1, suggesting a poor model fit.

Standardized Root Mean Square Residual:

The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.
In our case, the SRMR is 0.05 which is smaller than the threshold value 0.1 suggesting an acceptable model fit.

User Model versus Baseline Model

The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of

departure from the baseline model).
In our case, the CFI is 0.953 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.
In our case, the TLI is 0.938 which is greater or equal than the threshold value 0.90, suggesting an acceptable model fit.

Summary of the Goodness of Fit Indices

The TLI model fit index suggests an acceptable model fit. The SRMR model fit index suggests an acceptable model fit. The Chi-square model fit index and the RMSEA suggest a poor model fit. The goodness-of-fit of the model is uncertain. We proceed by diagnosing the sources of possible misspecification.

Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.
A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.
Following variable pairs have standardized residuals which are larger or equal than the considered threshold 2.58 [@brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably underestimated:

Table 10: Pair(s) with Underestimated Covariance

Pair 1	x1	y4
Pair 2	x1	y5
Pair 3	y2	y6

Following variable pairs have standardized residuals which are smaller or equal than the considered threshold -2.58 [@brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably overestimated:

Table 11: Pair(s) with Overestimated Covariance

Pair 1	y3	y6

Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [@brown] and [@mi]. We cite from [@brown]: “The modification index reflects an approximation of how much the overall model Chi² will decrease if the fixed or constrained parameter is freely estimated.” In other words, if adding a line with a high modification index

to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.
We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label "epc:m". For more information with regard to the labels of the decision column, please consult the Appendix.
We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.
We remark that these conditions are not fulfilled for modification indices with respect to factor loadings. Therefore, there exist no significant and relevant modification indices with respect to factor loadings.
We remark that there exist no modification indices with respect to factor covariances.

Parameter Estimates

Factor Loadings

We remark that the completely standardized factor loadings (section "Completely Standardized Parameter Estimates") are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [@brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 12: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading[1]	P-Value	Significant?[2]	Relevance[3]	Sign[4]	Check
ind60	x1	0.92	<0.001	Yes	***	—	Ok
ind60	x2	0.97	<0.001	Yes	***	—	Ok
ind60	x3	0.87	<0.001	Yes	***	—	Ok
dem60	y1	0.84	<0.001	Yes	***	—	Ok
dem60	y2	0.76	<0.001	Yes	**	—	Ok
dem60	y3	0.71	<0.001	Yes	**	—	Ok
dem60	y4	0.86	<0.001	Yes	***	—	Ok
dem65	y5	0.80	<0.001	Yes	***	—	Ok
dem65	y6	0.78	<0.001	Yes	**	—	Ok
dem65	y7	0.82	<0.001	Yes	***	—	Ok
dem65	y8	0.85	<0.001	Yes	***	—	Ok

[1] The completely standardized factor loading can be interpreted as the correlation with the factor.
[2] Completely standardized factor loading significance at 5% type I error.
[3] Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.
[4] No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.
[5] _____
[6] _____
[7] _____

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section "Model Fit Summary", for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the "Model Fit Summary" section:

Table 13: Interpretation of Unstandardized Factor Loadings

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in ind60 leads to a 1.00 -unit increase in the x1
A 1-unit increase in ind60 leads to a 2.18 -unit increase in the x2
A 1-unit increase in ind60 leads to a 1.82 -unit increase in the x3
A 1-unit increase in dem60 leads to a 1.00 -unit increase in the y1
A 1-unit increase in dem60 leads to a 1.35 -unit increase in the y2
A 1-unit increase in dem60 leads to a 1.04 -unit increase in the y3
A 1-unit increase in dem60 leads to a 1.30 -unit increase in the y4
A 1-unit increase in dem65 leads to a 1.00 -unit increase in the y5
A 1-unit increase in dem65 leads to a 1.26 -unit increase in the y6
A 1-unit increase in dem65 leads to a 1.28 -unit increase in the y7
A 1-unit increase in dem65 leads to a 1.31 -unit increase in the y8

Factor Discriminant Validity

As noted by [@brown], "the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context." Still, large or statistically significant factor covariances resp. correlations are questionable and provide evidence of poor discriminant validity. There is evidence to question the distinctness of the following factor pair(s), since their correlation approaches in absolute value 1.0 and their test of discriminant validity is non-statistically significant at 5% type I error:

Table 14: Factor with Poor Discriminant Validity

				Factor Correlation	Chisq diff	Df diff	P-Value
Pair 1	dem60	~~	dem65	0.978	0	1	1

Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 15: Completely Standardized Error Variances and Communality

Observed Variable	Error Variance[1]	Communality[23]	P-Value	Significant Error Variance?[4]
x1	0.15	0.85	<0.001	Yes
x2	0.05	0.95	0.102	No
x3	0.24	0.76	<0.001	Yes
y1	0.29	0.71	<0.001	Yes
y2	0.42	0.58	<0.001	Yes
y3	0.50	0.50	<0.001	Yes
y4	0.26	0.74	<0.001	Yes
y5	0.35	0.65	<0.001	Yes
y6	0.39	0.61	<0.001	Yes

Table 15: Completely Standardized Error Variances and Communality (continued)

Observed Variable	Error Variance[1]	Communality[23]	P-Value	Significant Error Variance?[4]
y7	0.33	0.67	<0.001	Yes
y8	0.28	0.72	<0.001	Yes

[1] Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).

[2] Corresponds to the squared factor loading.

[3] Can be interpreted as proportion of explained variance by the latent factor(s) (%).

[4] 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

Factor Reliability

The table "Factor Reliability" contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretatibility of the reliability measures depend on the specific research context. In some fields of research, omega values greater or equal than 0.6 and AVE values greater or equal than 0.5 (fulfilled by and large in your case) could be sufficient.

Final Summary

In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.
Considering the goodness-of-fit indices, the quality of the model is uncertain. Moreover, there exist localized areas of ill fit. Therefore, we assume that the model is non-acceptable. Please reconsider your data and the theory behind. Only if supported by theory, you could try to respecify the model and improve the goodness-of-fit by applying (one of the) following recommendations or call for actions.

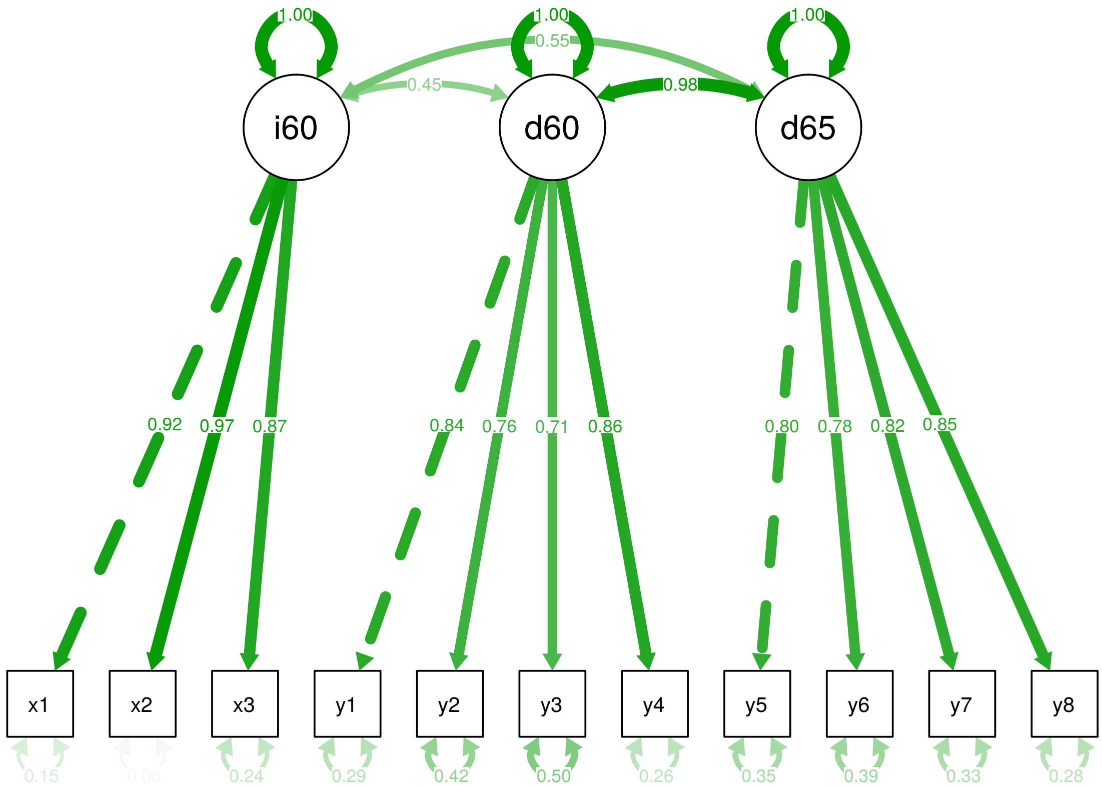
Eliminations from the Model

Factors*
Only if supported by theory, you could try to collapse the factors which were identified as probably non-distinguishable.

Final Comments

Please consider that this summary depends on hard-coded cutoff-values which may be too liberal or too conservative for your research area. If you have sound theory-based reasons and decide to respicify the model, we strongly recommend the replication of the CFA in an independent sample.

Path Diagram



APPENDIX

Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

References