Confirmatory Factor Analysis

Statsomat.com

07 January 2021

Warning: The automatic computation and interpretation delivered by the Statsomat should not completely replace the classical, made by humans graphical exploratory data analysis and statistical analysis. There may be data cases for which the Statsomat does not deliver the most optimal solution or output interpretation.

Basic Information

Automatic statistics for the file:

File
case1.csv

Your selection for the encoding: Auto
Your selection for the decimal character: Auto
Observations (rows with at least one non-missing value): 301
Variables (columns with at least one non-missing value): 16
Variables considered continuous: 10

Variables considered continuous
V1
id
x1
x2
x3
x5
x6
x7
x8
x9

Numerical variables considered binary or ordinal: 5

Numerical variables considered binary or ordinal
sex
ageyr
agemo
grade
x4

Character variables considered binary: 1

Character variables considered binary
school

Warning: There exist model variable(s) with <5% missing values. We assume a random missing pattern and apply a missing values imputation technique.

Variable(s) with imputed missing values
grade

Warning: Based on an analysis of the variables considered continuous, we suspect outliers in the data. If observations are erroneous, you could drop them and restart the app. Outliers may affect negatively the execution or the results of the CFA. These are the suspected row numbers:

Table 6: Rows With Suspected Outliers

	V1	id	x1	x2	x3	x5	x6	x7	x8	x9
163	163	208	6	8.25	4.5	6.25	5.857143	5.608696	6.95	9.25

Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
visual	=~	x1	1	0
visual	=~	x2	1	1
visual	=~	x3	1	2
textual	=~	x4	1	0
textual	=~	x5	1	3
textual	=~	x6	1	4
speed	=~	x7	1	0
speed	=~	x8	1	5
speed	=~	x9	1	6
x1	~~	x1	0	7
x2	~~	x2	0	8
x3	~~	x3	0	9
x4	~~	x4	0	10
x5	~~	x5	0	11
x6	~~	x6	0	12
x7	~~	x7	0	13
x8	~~	x8	0	14
x9	~~	x9	0	15
visual	~~	visual	0	16
textual	~~	textual	0	17
speed	~~	speed	0	18
visual	~~	textual	0	19
visual	~~	speed	0	20
textual	~~	speed	0	21
x1	~1		0	22
x2	~1		0	23
x3	~1		0	24
x4	~1		0	25
x5	~1		0	26
x6	~1		0	27
x7	~1		0	28
x8	~1		0	29
x9	~1		0	30
visual	~1		0	0
textual	~1		0	0
speed	~1		0	0

Assumptions

Open issue

Model Settings

Outputs

Model Fit Summary

```
lavaan 0.6-7 ended normally after 35 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         30

  Number of observations                           301
  Number of missing patterns                         1

Model Test User Model:

  Test statistic                                85.535
  Degrees of freedom                                24
  P-value (Chi-square)                           0.000

Model Test Baseline Model:

  Test statistic                               918.242
  Degrees of freedom                                36
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.930
  Tucker-Lewis Index (TLI)                       0.895

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3737.933
  Loglikelihood unrestricted model (H1)      -3695.165

  Akaike (AIC)                                7535.865
  Bayesian (BIC)                              7647.079
  Sample-size adjusted Bayesian (BIC)         7551.936
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.092
  90 Percent confidence interval - lower         0.072
  90 Percent confidence interval - upper         0.114
  P-value RMSEA <= 0.05                          0.001

Standardized Root Mean Square Residual:

  SRMR                                           0.060

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Observed
  Observed information based on                Hessian

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual =~
    x1                1.000
    x2                0.554    0.109    5.061    0.000
    x3                0.730    0.118    6.210    0.000
  textual =~
    x4                1.000
    x5                1.113    0.065   17.128    0.000
    x6                0.926    0.056   16.481    0.000
  speed =~
    x7                1.000
    x8                1.174    0.149    7.858    0.000
    x9                1.068    0.192    5.578    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  visual ~~
    textual           0.408    0.080    5.119    0.000
    speed             0.262    0.056    4.719    0.000
  textual ~~
    speed             0.175    0.050    3.529    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                4.936    0.067   73.473    0.000
   .x2                6.088    0.068   89.855    0.000
   .x3                2.250    0.065   34.579    0.000
   .x4                3.061    0.067   45.694    0.000
   .x5                4.341    0.074   58.452    0.000
   .x6                2.186    0.063   34.667    0.000
```

```
   .x7              4.186    0.063   66.766    0.000
   .x8              5.525    0.058   94.899    0.000
   .x9              5.374    0.058   92.546    0.000
    visual          0.000
    textual         0.000
    speed           0.000

Variances:
                 Estimate  Std.Err  z-value  P(>|z|)
   .x1              0.550    0.119    4.608    0.000
   .x2              1.134    0.104   10.869    0.000
   .x3              0.844    0.095    8.870    0.000
   .x4              0.371    0.048    7.739    0.000
   .x5              0.446    0.058    7.704    0.000
   .x6              0.356    0.043    8.200    0.000
   .x7              0.795    0.087    9.118    0.000
   .x8              0.486    0.091    5.341    0.000
   .x9              0.573    0.090    6.370    0.000
    visual          0.809    0.150    5.396    0.000
    textual         0.980    0.112    8.729    0.000
    speed           0.388    0.092    4.205    0.000
```

Completely Standardized Parameter Estimates

```
Latent Variables:
                  est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  visual =~
    x1              0.772    0.058   13.384    0.000    0.659    0.885
    x2              0.424    0.063    6.751    0.000    0.301    0.547
    x3              0.581    0.059    9.933    0.000    0.467    0.696
  textual =~
    x4              0.852    0.023   37.614    0.000    0.807    0.896
    x5              0.855    0.022   38.530    0.000    0.812    0.899
    x6              0.838    0.024   35.598    0.000    0.792    0.884
  speed =~
    x7              0.572    0.058    9.880    0.000    0.459    0.686
    x8              0.724    0.062   11.686    0.000    0.602    0.845
    x9              0.660    0.066   10.021    0.000    0.531    0.789

Covariances:
                  est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  visual ~~
    textual         0.458    0.063    7.222    0.000    0.334    0.583
    speed           0.468    0.086    5.435    0.000    0.299    0.637
  textual ~~
    speed           0.284    0.071    3.974    0.000    0.144    0.424
```

```
Intercepts:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
   .x1              -0.000    0.058   -0.000    1.000   -0.113    0.113
   .x2              -0.000    0.058   -0.000    1.000   -0.113    0.113
   .x3               0.000    0.058    0.000    1.000   -0.113    0.113
   .x4              -0.000    0.058   -0.000    1.000   -0.113    0.113
   .x5               0.000    0.058    0.000    1.000   -0.113    0.113
   .x6              -0.000    0.058   -0.000    1.000   -0.113    0.113
   .x7              -0.000    0.058   -0.000    1.000   -0.113    0.113
   .x8              -0.000    0.058   -0.000    1.000   -0.113    0.113
   .x9              -0.000    0.058   -0.000    1.000   -0.113    0.113
    visual           0.000                                0.000    0.000
    textual          0.000                                0.000    0.000
    speed            0.000                                0.000    0.000

Variances:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
   .x1               0.405    0.089    4.547    0.000    0.230    0.579
   .x2               0.820    0.053   15.415    0.000    0.716    0.925
   .x3               0.662    0.068    9.733    0.000    0.529    0.795
   .x4               0.275    0.039    7.126    0.000    0.199    0.350
   .x5               0.269    0.038    7.085    0.000    0.194    0.343
   .x6               0.298    0.039    7.546    0.000    0.220    0.375
   .x7               0.672    0.066   10.133    0.000    0.542    0.802
   .x8               0.476    0.090    5.317    0.000    0.301    0.652
   .x9               0.564    0.087    6.483    0.000    0.394    0.735
    visual           1.000                                1.000    1.000
    textual          1.000                                1.000    1.000
    speed            1.000                                1.000    1.000
```

Communality

Table 8: Communality

Variable	Communality
x1	0.60
x2	0.18
x3	0.34
x4	0.73
x5	0.73
x6	0.70
x7	0.33
x8	0.52
x9	0.44

Factor Discriminant Validity

Table 9: Factor Discriminant Validity Test at Cutoff 0.85

			Factor Correlation	Chisq diff	Df diff	P-Value
visual	~~	textual	0.458	55.954	1	<0.001
visual	~~	speed	0.468	28.504	1	<0.001
textual	~~	speed	0.284	106.112	1	<0.001

Factor Reliability

Table 10: Factor Reliability

	visual	textual	speed	total
Omega (Bentler)	0.63	0.89	0.69	0.85
Omega (McDonald)	0.61	0.89	0.69	0.86
AVE	0.37	0.72	0.42	0.51

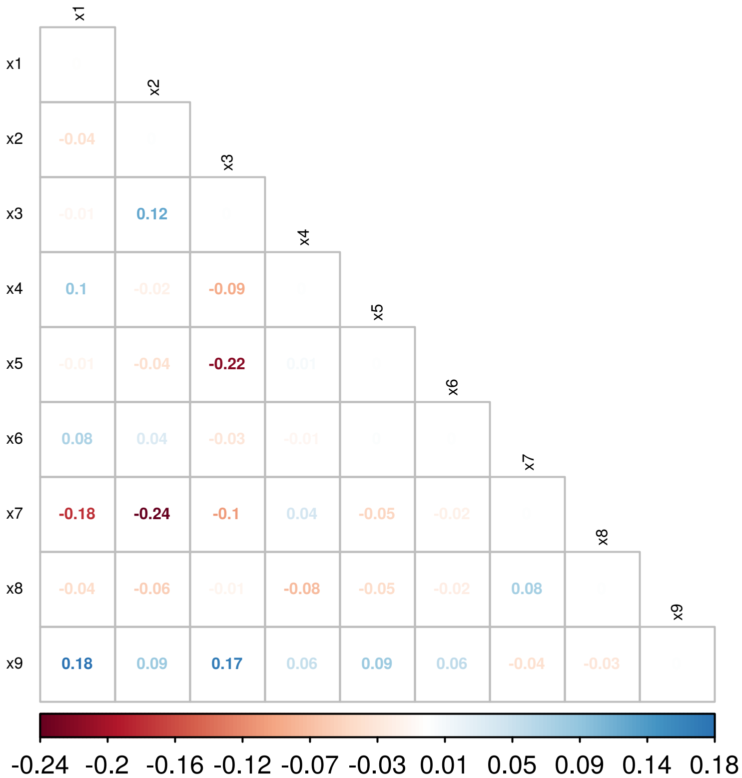
Observed Covariance Matrix

	x1	x2	x3	x4	x5	x6	x7	x8	x9
x1	1.36								
x2	0.41	1.38							
x3	0.58	0.45	1.27						
x4	0.5	0.21	0.21	1.35					
x5	0.44	0.21	0.11	1.1	1.66				
x6	0.45	0.25	0.24	0.9	1.01	1.2			
x7	0.08	-0.1	0.09	0.22	0.14	0.14	1.18		
x8	0.26	0.11	0.21	0.13	0.18	0.17	0.54	1.02	
x9	0.46	0.24	0.37	0.24	0.3	0.24	0.37	0.45	1.02

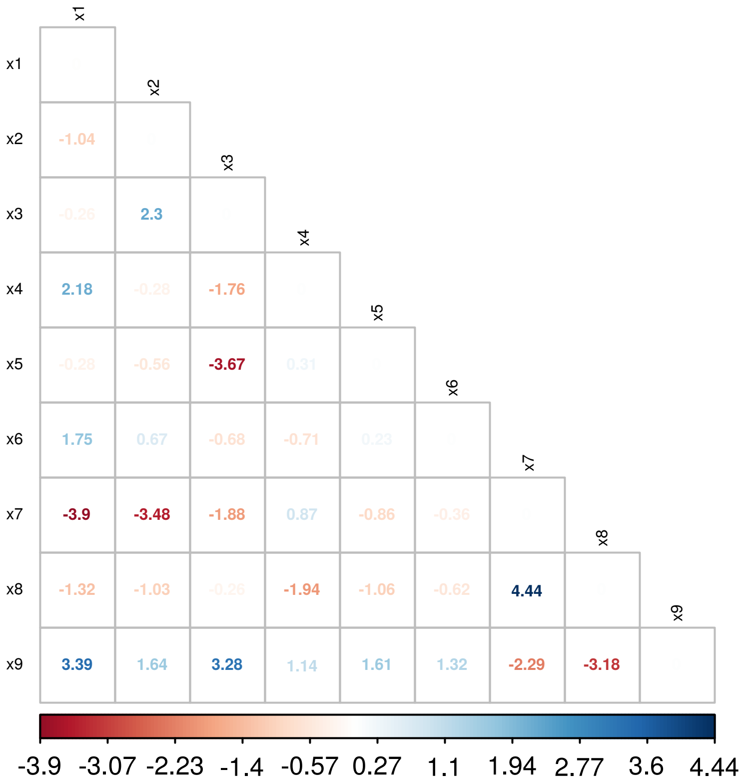
Model-Implied Covariance Matrix

	x1	x2	x3	x4	x5	x6	x7	x8	x9
x1	1.36								
x2	0.45	1.38							
x3	0.59	0.33	1.27						
x4	0.41	0.23	0.3	1.35					
x5	0.45	0.25	0.33	1.09	1.66				
x6	0.38	0.21	0.28	0.91	1.01	1.2			
x7	0.26	0.15	0.19	0.17	0.19	0.16	1.18		
x8	0.31	0.17	0.22	0.21	0.23	0.19	0.46	1.02	
x9	0.28	0.16	0.2	0.19	0.21	0.17	0.41	0.49	1.02

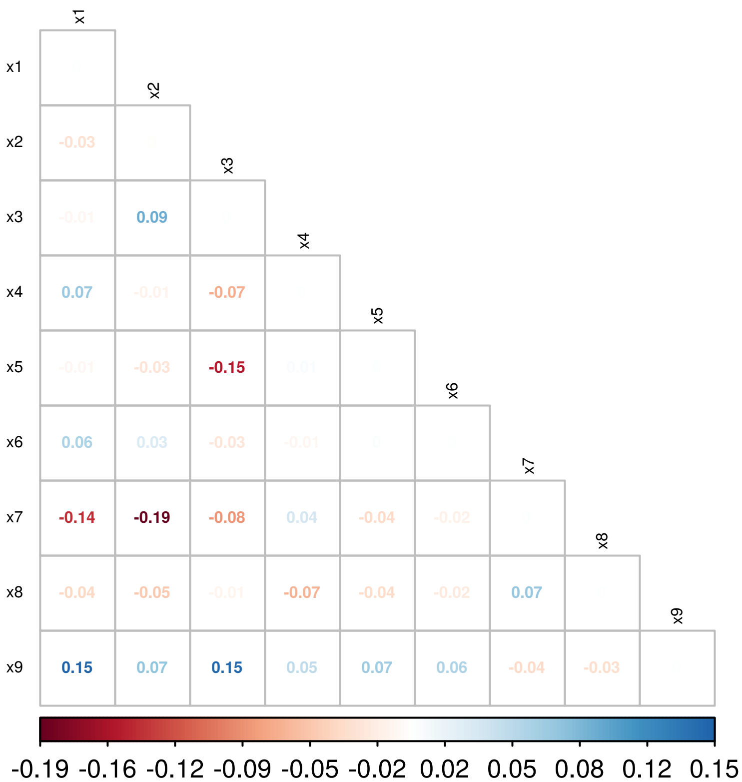
Residual Covariance Matrix



Standardized Residual Matrix



Residual Correlation Matrix



Modification Indices

Table 11: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
x7	~~	x8	34.132	0.543	0.1	0.190	**(m)**
x8	~~	x9	15.123	-0.422	0.1	0.151	**(m)**
x2	~~	x7	8.974	-0.183	0.1	0.373	**(m)**
x2	~~	x3	8.528	0.219	0.1	0.267	**(m)**
x3	~~	x5	7.856	-0.130	0.1	0.577	**(m)**
x1	~~	x9	7.498	0.139	0.1	0.501	**(m)**

Note:
Maximum 10 modification indices in descending order of their magnitude are listed.

Table 12: Modification Indices With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
visual	=~	x9	36.538	0.575	0.4	0.988	*epc:m*
visual	=~	x7	18.922	-0.425	0.4	0.984	epc:nm
textual	=~	x3	9.180	-0.272	0.4	0.994	epc:nm
textual	=~	x1	8.954	0.352	0.4	0.926	epc:nm

Note:

Table 12: Modification Indices With Respect To Factor Loadings (continued)

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision

Maximum 10 modification indices in descending order of their magnitude are listed.

Interpretation

Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [@brown], [@kline].

Model Test User Model
The degrees of freedom are calculated as the number of known parameters minus the number of free parameters: 54 - 30 = 24 . The 24 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 85.535 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.01 then the test is not statistically significant at 1 % error, the hypothesis cannot be rejected, which would be in favour of the model.
In our case, the p-value is <0.001 suggesting that the model may not be acceptable for the data. The Chi-square model fit index is based on a very stringent statistical hypothesis which may have no practical relevance. We will consider it only in connection with other model fit indices.

Model Test Baseline Model
The test statistic with the value 918.242 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is <0.001 suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

Root Mean Square Error of Approximation:
The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.
In our case, the RMSEA is 0.092. The upper bound of the 90% confidence interval of the RMSEA is 0.114 and greater or equal than the threshold value 0.1, suggesting a poor model fit.

Standardized Root Mean Square Residual:
The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.
In our case, the SRMR is 0.06 which is smaller than the threshold value 0.08 suggesting an acceptable model fit.

User Model versus Baseline Model
The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of

departure from the baseline model).
In our case, the CFI is 0.93 which is greater or equal than the threshold value 0.90, suggesting an acceptable model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.
In our case, the TLI is 0.895 which is smaller or equal than the threshold value 0.90, suggesting a poor model fit.

Summary of the Goodness of Fit Indices

The CFI and the SRMR tend to support the model but the Chi-square and the RSMSEA model fit indicate the opposite. The goodness-of-fit of the model is uncertain. We proceed by diagnosing the sources of possible misspecification.

Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.
A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.
Following variable pairs have standardized residuals which are larger or equal than the considered threshold 2.58 [@brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably underestimated:

Table 13: Pair(s) with Underestimated Covariance

Pair 1	x1	x9
Pair 2	x3	x9
Pair 3	x7	x8

Following variable pairs have standardized residuals which are smaller or equal than the considered threshold -2.58 [@brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably overestimated:

Table 14: Pair(s) with Overestimated Covariance

Pair 1	x1	x7
Pair 2	x2	x7
Pair 3	x3	x5
Pair 4	x8	x9
Pair 5	x9	x8

Depending on the sample size, the misspecification detected by the analysis of the residual covariance resp. correlation matrices can be statistically significant but not relevant and in practice negligible. This is matter of subject in the next section(s).

Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [@brown] and [@mi]. We cite from [@brown]: "The modification index reflects an approximation of how much the overall model Chi² will decrease if the fixed or constrained parameter is freely estimated." In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.
We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label "epc:m". For more information with regard to the labels of the decision column, please consult the Appendix.
We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.
Following modification indices with respect to factor loadings fulfill these requirements:

Table 15: Significant and Relevant MIs With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
visual	=~	x9	36.538	0.575	0.4	0.988	*epc:m*

We remark that there exist no modification indices with respect to factor covariances.

Parameter Estimates

Factor Loadings

We remark that the completely standardized factor loadings (section "Completely Standardized Parameter Estimates") are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [@brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 16: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading[1]	P-Value	Significant?[2]	Relevance[3]	Sign[4]	Check
visual	x1	0.77	<0.001	Yes	**	—	Ok
visual	x2	0.42	<0.001	Yes	*	—	Ok
visual	x3	0.58	<0.001	Yes	*	—	Ok
textual	x4	0.85	<0.001	Yes	***	—	Ok
textual	x5	0.86	<0.001	Yes	***	—	Ok
textual	x6	0.84	<0.001	Yes	***	—	Ok
speed	x7	0.57	<0.001	Yes	*	—	Ok
speed	x8	0.72	<0.001	Yes	**	—	Ok
speed	x9	0.66	<0.001	Yes	**	—	Ok

[1] The completely standardized factor loading can be interpreted as the correlation with the factor.
[2] Completely standardized factor loading significance at 5% type I error.

Table 16: Check Completely Standardized Factor Loadings (continued)

Latent Variable	Observed Variable	Loading[1]	P-Value	Significant?[2]	Relevance[3]	Sign[4]	Check

[3] Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.

[4] No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.

[5] _____

[6] _____

[7] _____

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section "Model Fit Summary", for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the "Model Fit Summary" section:

Table 17: Interpretation of Unstandardized Factor Loadings

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in visual leads to a 1.00 -unit increase in the x1
A 1-unit increase in visual leads to a 0.55 -unit increase in the x2
A 1-unit increase in visual leads to a 0.73 -unit increase in the x3
A 1-unit increase in textual leads to a 1.00 -unit increase in the x4
A 1-unit increase in textual leads to a 1.11 -unit increase in the x5
A 1-unit increase in textual leads to a 0.93 -unit increase in the x6
A 1-unit increase in speed leads to a 1.00 -unit increase in the x7
A 1-unit increase in speed leads to a 1.17 -unit increase in the x8
A 1-unit increase in speed leads to a 1.07 -unit increase in the x9

Factor Discriminant Validity

As noted by [@brown], "the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context." Though, the largest estimated factor intercorrelation within the section "Completely Standardized Parameter Estimates" is 0.47 which we regard as a proof of a reasonable discriminant validity. Moreover, the statistical test(s) for factor discriminant validity are statistically significant at 5% type I error.

Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 18: Completely Standardized Error Variances and Communality

Observed Variable	Error Variance[1]	Communality[23]	P-Value	Significant Error Variance?[4]
x1	0.40	0.60	<0.001	Yes
x2	0.82	0.18	<0.001	Yes
x3	0.66	0.34	<0.001	Yes
x4	0.27	0.73	<0.001	Yes
x5	0.27	0.73	<0.001	Yes
x6	0.30	0.70	<0.001	Yes

Table 18: Completely Standardized Error Variances and Communality (continued)

Observed Variable	Error Variance[1]	Communality[23]	P-Value	Significant Error Variance?[4]
x7	0.67	0.33	<0.001	Yes
x8	0.48	0.52	<0.001	Yes
x9	0.56	0.44	<0.001	Yes

[1] Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).
[2] Corresponds to the squared factor loading.
[3] Can be interpreted as proportion of explained variance by the latent factor(s) (%).
[4] 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

Intercepts

In case of missing values and estimation via FIML, a meanstructure i.e. the intercepts of the observed variables are added to the model. The means of the latent factors are fixed to zero. Therefore, the estimated intercepts within the section "Model Fit Summary" are just the means of the observed variables.

Factor Reliability

The table "Factor Reliability" contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretatibility of the reliability measures depend on the specific research context. Nevertheless, omega values below 0.6 or AVE values below 0.5 (at least one of these existent in your case) should be regarded with criticism. The factor reliability estimates are not further considered in the final summary.

Final Summary

In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.
Considering the goodness-of-fit indices, the quality of the model is uncertain. Moreover, there exist localized areas of ill fit. Therefore, we assume that the model is non-acceptable. Please reconsider your data and the theory behind. Only if supported by theory, you could try to respecify the model and improve the goodness-of-fit by applying (one of the) following recommendations or call for actions.

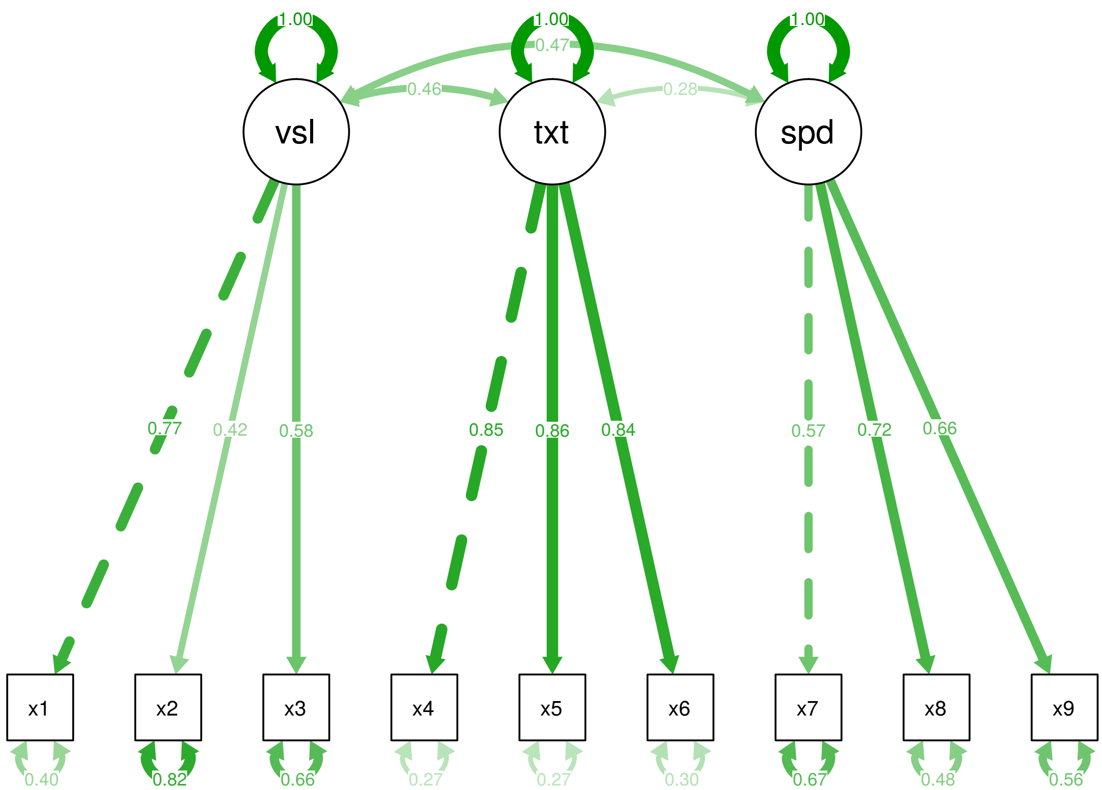
Addings to the Model

Observed Variables

Update the model by adding cross-loadings to the model indicated within the table "Significant and Relevant MIs With Respect To Factor Loadings".
Start updating the model by adding a parameter with the largest modification index (and EPC) and only if this parameter can be interpreted meaningfully. Other possible localized areas of misfit may be remedied by adding, i.e. freeing a single parameter.

Final comments: Please consider that this summary depends on hard-coded cutoff-values which may be too liberal or too conservative for your research area. If you have sound theory-based reasons and decide to respicify the model, we strongly recommend the replication of the CFA in an independent sample.

Path Diagram



APPENDIX

Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

References