

Confirmatory Factor Analysis

Statsomat.com

08 January 2021

Warning: The automatic computation and interpretation delivered by the Statsomat should not completely replace the classical, made by humans graphical exploratory data analysis and statistical analysis. There may be data cases for which the Statsomat does not deliver the most optimal solution or output interpretation.

Basic Information

Automatic statistics for the file:

File
case23.csv

Your selection for the encoding: Auto

Your selection for the decimal character: Auto

Observations (rows with at least one non-missing value): 250

Variables (columns with at least one non-missing value): 12

Variables considered continuous: 12

Variables considered continuous
P1
P2
P3
C1
C2
C3
E1
E2
E3
S1
S2
S3

Warning: Based on an analysis of the variables considered continuous, we suspect outliers in the data. If observations are erroneous, you could drop them and restart the app. Outliers may affect negatively the execution or the results of the CFA. These are the suspected row numbers:

Table 3: Rows With Suspected Outliers

	P1	P2	P3	C1	C2	C3	E1	E2	E3	S1	S2	
193	2.542568	3.743492	1.972349	5.804081	7.306684	4.775175	4.926161	7.968632	5.167297	8.397417	6.210714	3.929

Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
PROBSLV	=~	P1	1	0
PROBSLV	=~	P2	1	1
PROBSLV	=~	P3	1	2
COGRES	=~	C1	1	0
COGRES	=~	C2	1	3
COGRES	=~	C3	1	4
EXPREMOT	=~	E1	1	0
EXPREMOT	=~	E2	1	5
EXPREMOT	=~	E3	1	6
SOCSP	=~	S1	1	0
SOCSP	=~	S2	1	7
SOCSP	=~	S3	1	8
PROBFOC	=~	PROBSLV	1	0
PROBFOC	=~	COGRES	1	9
EMOTFOC	=~	EXPREMOT	1	0
EMOTFOC	=~	SOCSP	1	10
P1	~~	P1	0	11
P2	~~	P2	0	12
P3	~~	P3	0	13
C1	~~	C1	0	14
C2	~~	C2	0	15
C3	~~	C3	0	16
E1	~~	E1	0	17
E2	~~	E2	0	18
E3	~~	E3	0	19
S1	~~	S1	0	20
S2	~~	S2	0	21
S3	~~	S3	0	22
PROBSLV	~~	PROBSLV	0	23
COGRES	~~	COGRES	0	24
EXPREMOT	~~	EXPREMOT	0	25
SOCSP	~~	SOCSP	0	26
PROBFOC	~~	PROBFOC	0	27
EMOTFOC	~~	EMOTFOC	0	28
PROBFOC	~~	EMOTFOC	0	29

Assumptions

Open issue

Model Settings

Outputs

Model Fit Summary

lavaan 0.6-7 ended normally after 64 iterations

Estimator	ML
Optimization method	NLMINB
Number of free parameters	29
Number of observations	250

Model Test User Model:

Test statistic	75.459
Degrees of freedom	49
P-value (Chi-square)	0.009

Model Test Baseline Model:

Test statistic	2510.468
Degrees of freedom	66
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.989
Tucker-Lewis Index (TLI)	0.985

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-5129.288
Loglikelihood unrestricted model (H1)	-5091.558
Akaike (AIC)	10316.575
Bayesian (BIC)	10418.697
Sample-size adjusted Bayesian (BIC)	10326.765

Root Mean Square Error of Approximation:

RMSEA	0.046
90 Percent confidence interval - lower	0.024
90 Percent confidence interval - upper	0.066
P-value RMSEA <= 0.05	0.590

Standardized Root Mean Square Residual:

SRMR 0.017

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
PROBSLV =~				
P1	1.000			
P2	1.424	0.074	19.235	0.000
P3	0.896	0.045	19.811	0.000
COGRES =~				
C1	1.000			
C2	1.028	0.054	19.037	0.000
C3	0.842	0.042	20.133	0.000
EXPREMOT =~				
E1	1.000			
E2	1.542	0.076	20.397	0.000
E3	1.133	0.062	18.161	0.000
SOCSPT =~				
S1	1.000			
S2	0.706	0.034	20.669	0.000
S3	0.381	0.019	20.242	0.000
PROBFOC =~				
PROBSLV	1.000			
COGRES	1.609	0.443	3.635	0.000
EMOTFOC =~				
EXPREMOT	1.000			
SOCSPT	1.572	0.427	3.685	0.000

Covariances:

	Estimate	Std.Err	z-value	P(> z)
PROBFOC ~~				
EMOTFOC	0.448	0.153	2.919	0.004

Variances:

	Estimate	Std.Err	z-value	P(> z)
.P1	0.333	0.053	6.221	0.000
.P2	1.111	0.136	8.162	0.000
.P3	0.384	0.050	7.669	0.000
.C1	1.118	0.150	7.449	0.000
.C2	1.355	0.170	7.961	0.000
.C3	0.654	0.098	6.685	0.000

.E1	0.946	0.118	8.031	0.000
.E2	1.156	0.215	5.382	0.000
.E3	1.371	0.162	8.447	0.000
.S1	1.805	0.254	7.102	0.000
.S2	0.946	0.130	7.299	0.000
.S3	0.308	0.040	7.725	0.000
.PROBSLV	0.552	0.292	1.890	0.059
.COGRES	1.388	0.756	1.835	0.066
.EXPREMOT	0.968	0.556	1.741	0.082
.SOCSPT	2.651	1.378	1.924	0.054
PROBFOC	1.067	0.323	3.305	0.001
EMOTFOC	2.071	0.622	3.329	0.001

Completely Standardized Parameter Estimates

Latent Variables:

	est.std	Std.Err	z-value	P(> z)	ci.lower	ci.upper
PROBSLV =~						
P1	0.911	0.017	55.179	0.000	0.878	0.943
P2	0.864	0.020	42.903	0.000	0.825	0.904
P3	0.879	0.019	46.366	0.000	0.842	0.916
COGRES =~						
C1	0.888	0.018	49.355	0.000	0.852	0.923
C2	0.874	0.019	45.749	0.000	0.837	0.911
C3	0.904	0.017	54.246	0.000	0.872	0.937
EXPREMOT =~						
E1	0.873	0.019	45.776	0.000	0.836	0.911
E2	0.928	0.015	62.189	0.000	0.899	0.958
E3	0.860	0.020	42.553	0.000	0.820	0.900
SOCSPT =~						
S1	0.901	0.016	54.639	0.000	0.868	0.933
S2	0.896	0.017	53.266	0.000	0.864	0.929
S3	0.886	0.018	50.153	0.000	0.852	0.921
PROBFOC =~						
PROBSLV	0.812	0.111	7.335	0.000	0.595	1.029
COGRES	0.816	0.111	7.344	0.000	0.598	1.034
EMOTFOC =~						
EXPREMOT	0.826	0.110	7.482	0.000	0.609	1.042
SOCSPT	0.812	0.109	7.455	0.000	0.598	1.025

Covariances:

	est.std	Std.Err	z-value	P(> z)	ci.lower	ci.upper
PROBFOC ~~						
EMOTFOC	0.301	0.078	3.866	0.000	0.148	0.454

Variances:

	est.std	Std.Err	z-value	P(> z)	ci.lower	ci.upper
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.P1	0.170	0.030	5.670	0.000	0.112	0.229
.P2	0.253	0.035	7.261	0.000	0.185	0.321
.P3	0.228	0.033	6.843	0.000	0.163	0.293
.C1	0.212	0.032	6.648	0.000	0.150	0.275
.C2	0.236	0.033	7.071	0.000	0.171	0.302
.C3	0.182	0.030	6.034	0.000	0.123	0.241
.E1	0.237	0.033	7.124	0.000	0.172	0.303
.E2	0.138	0.028	4.980	0.000	0.084	0.192
.E3	0.260	0.035	7.484	0.000	0.192	0.328
.S1	0.189	0.030	6.349	0.000	0.130	0.247
.S2	0.196	0.030	6.505	0.000	0.137	0.255
.S3	0.215	0.031	6.848	0.000	0.153	0.276
.PROBSLV	0.341	0.180	1.897	0.058	-0.011	0.693
.COGRES	0.334	0.181	1.844	0.065	-0.021	0.690
.EXPREMOT	0.319	0.182	1.749	0.080	-0.039	0.676
.SOCSPT	0.341	0.177	1.932	0.053	-0.005	0.688
PROBFOC	1.000				1.000	1.000
EMOTFOC	1.000				1.000	1.000

Communality

Table 5: Communality

Variable	Communality
C1	0.79
C2	0.76
C3	0.82
COGRES	0.67
E1	0.76
E2	0.86
E3	0.74
EXPREMOT	0.68
P1	0.83
P2	0.75
P3	0.77
PROBSLV	0.66
S1	0.81
S2	0.80
S3	0.79
SOCSPT	0.66

Factor Discriminant Validity

Table 6: Factor Discriminant Validity Test at Cutoff 0.85

			Factor Correlation	Chisq diff	Df diff	P-Value
PROBFOC	~~	EMOTFOC	0.301	81.719	1	<0.001

Factor Reliability

Table 7: Factor Reliability

	PROBSLV	COGRES	EXPREMOT	SOCSPPT	total
Omega (Bentler)	0.91	0.92	0.92	0.92	0.96
Omega (McDonald)	0.91	0.92	0.92	0.92	0.96
AVE	0.77	0.79	0.80	0.81	0.80

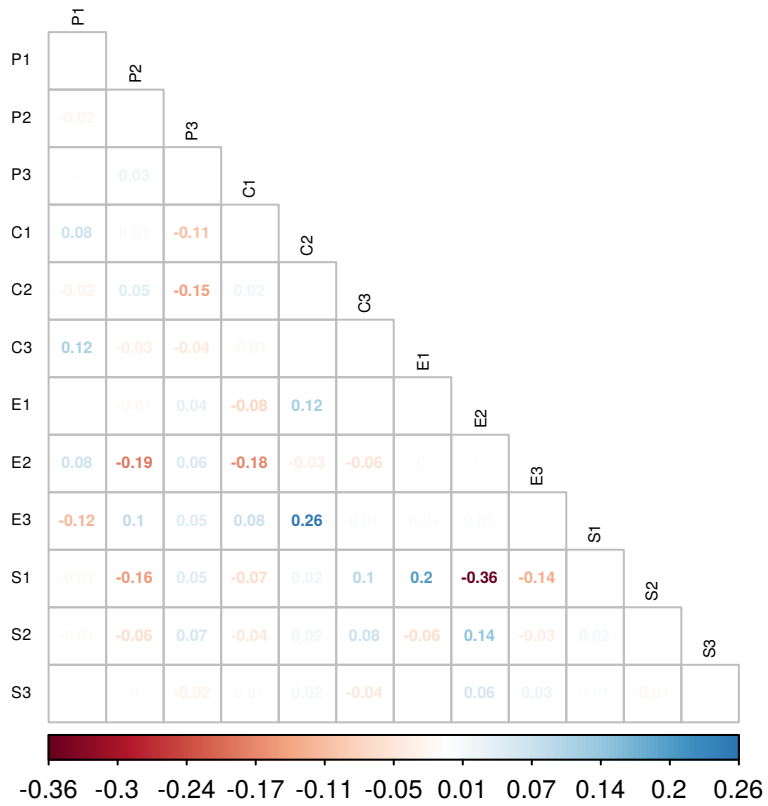
Observed Covariance Matrix

P1	1.95												
P2	2.28	4.39											
P3	1.45	2.09	1.68										
C1	1.8	2.45	1.43	5.27									
C2	1.74	2.56	1.43	4.29	5.74								
C3	1.56	2.03	1.25	3.48	3.59	3.6							
E1	0.45	0.63	0.44	0.64	0.86	0.61	3.98						
E2	0.77	0.79	0.68	0.93	1.11	0.88	4.68	8.38					
E3	0.38	0.82	0.51	0.9	1.1	0.7	3.44	5.31	5.27				
S1	0.69	0.84	0.68	1.07	1.19	1.06	3.46	4.66	3.55	9.57			
S2	0.49	0.64	0.51	0.76	0.84	0.75	2.24	3.69	2.57	5.5	4.82		
S3	0.27	0.38	0.22	0.44	0.46	0.32	1.24	1.98	1.43	2.96	2.08	1.43	

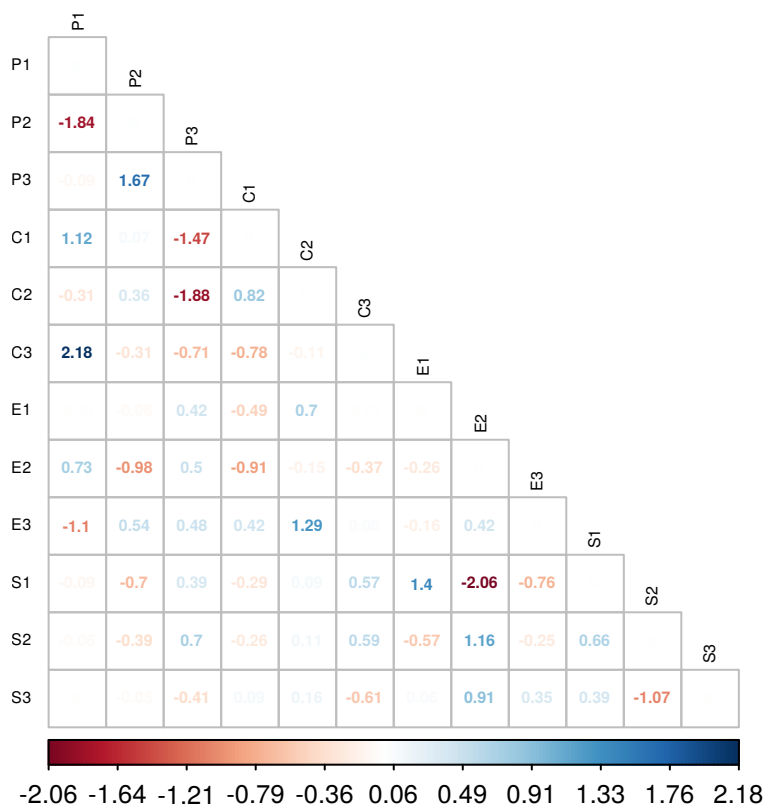
Model-Implied Covariance Matrix

P1	1.95											
P2	2.31	4.39										
P3	1.45	2.07	1.68									
C1	1.72	2.44	1.54	5.27								
C2	1.76	2.51	1.58	4.26	5.74							
C3	1.45	2.06	1.3	3.49	3.59	3.6						
E1	0.45	0.64	0.4	0.72	0.74	0.61	3.98					
E2	0.69	0.98	0.62	1.11	1.14	0.93	4.68	8.38				
E3	0.51	0.72	0.45	0.82	0.84	0.69	3.44	5.3	5.27			
S1	0.7	1	0.63	1.13	1.16	0.95	3.25	5.02	3.69	9.57		
S2	0.5	0.71	0.45	0.8	0.82	0.67	2.3	3.54	2.6	5.49	4.82	
S3	0.27	0.38	0.24	0.43	0.44	0.36	1.24	1.91	1.4	2.96	2.09	1.43

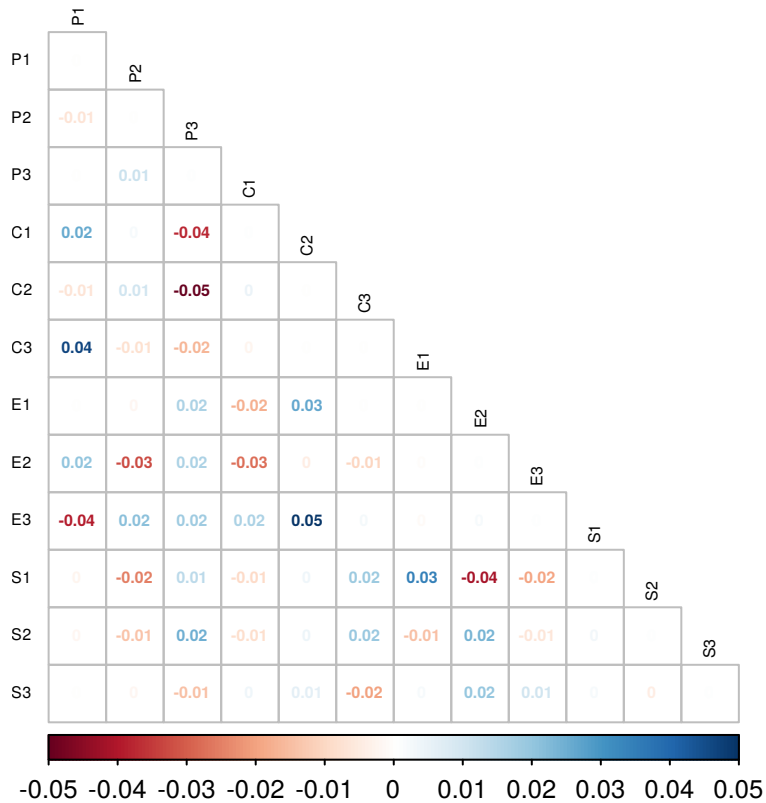
Residual Covariance Matrix



Standardized Residual Matrix



Residual Correlation Matrix



Modification Indices

Table 8: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
E1	~~	S1	16.880	0.469	0.1	0.141	** (m) **
E2	~~	S1	16.783	-0.615	0.1	0.102	** (m) **
P1	~~	E3	13.021	-0.219	0.1	0.378	** (m) **
P1	~~	E2	9.721	0.208	0.1	0.321	** (m) **
E2	~~	S2	8.104	0.306	0.1	0.154	** (m) **
P2	~~	E2	7.993	-0.310	0.1	0.149	** (m) **
P2	~~	E3	7.472	0.272	0.1	0.171	** (m) **
E1	~~	S2	6.532	-0.209	0.1	0.231	** (m) **
C3	~~	S3	4.820	-0.090	0.1	0.687	** (m) **
P1	~~	C3	3.868	0.092	0.1	0.571	** (m) **

Note:

Maximum 10 modification indices in descending order of their magnitude are listed.

Interpretation

Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [Brown], [Kline].

Model Test User Model

The degrees of freedom are calculated as the number of known parameters minus the number of free parameters: $78 - 29 = 49$. The 49 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 75.459 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.05 then the test is not statistically significant at 5 % error, the hypothesis cannot be rejected, which would be in favour of the model.

In our case, the p-value is 0.009 suggesting that the model may not be acceptable for the data. The Chi-square model fit index is based on a very stringent statistical hypothesis which may have no practical relevance. We will consider it only in connection with other model fit indices.

Model Test Baseline Model

The test statistic with the value 2510.468 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is <0.001 suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

Root Mean Square Error of Approximation:

The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.

In our case, the RMSEA is 0.046. The upper bound of the 90% confidence interval of the RMSEA is 0.066 and smaller than the threshold value 0.08, suggesting a good model fit.

Standardized Root Mean Square Residual:

The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.

In our case, the SRMR is 0.02 which is smaller than the threshold value 0.1 suggesting an acceptable model fit.

User Model versus Baseline Model

The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of departure from the baseline model).

In our case, the CFI is 0.989 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.

In our case, the TLI is 0.985 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Summary of the Goodness of Fit Indices

The TLI model fit index suggests an acceptable model fit. Moreover the SRMR and the RMSEA model fit indices. The Chi-square model fit index suggests a poor model fit. The Chi-square model fit index is based on a very stringent statistical hypothesis. Therefore, we ignore it and tentatively assume an acceptable model fit. We verify this assertion by considering further metrics.

Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.

A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.

There are no variable pairs with standardized residuals which are larger or equal than the considered threshold 2.58 [brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [kline]. Therefore, no relationships among the variables are substantially underestimated by the model. There are no variable pairs with standardized residuals which are smaller or equal than the considered threshold -2.58 [brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [kline]. Therefore, no relationships among the variables are substantially overestimated by the model.

Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [brown] and [mi]. We cite from [brown]: “The modification index reflects an approximation of how much the overall model χ^2 will decrease if the fixed or constrained parameter is freely estimated.” In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.

We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label “epc:m”. For more information with regard to the labels of the decision column, please consult the Appendix.

We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.

We remark that there exist no modification indices with respect to factor loadings.

We remark that there exist no modification indices with respect to factor covariances.

Parameter Estimates

Factor Loadings

We remark that the completely standardized factor loadings (section “Completely Standardized Parameter Estimates”) are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [brown]. Please con-

sider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 9: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading ¹	P-Value	Significant? ²	Relevance ³	Sign ⁴	Check
PROBSLV	P1	0.91	<0.001	Yes	***	—	Ok
PROBSLV	P2	0.86	<0.001	Yes	***	—	Ok
PROBSLV	P3	0.88	<0.001	Yes	***	—	Ok
COGRES	C1	0.89	<0.001	Yes	***	—	Ok
COGRES	C2	0.87	<0.001	Yes	***	—	Ok
COGRES	C3	0.90	<0.001	Yes	***	—	Ok
EXPREMOT	E1	0.87	<0.001	Yes	***	—	Ok
EXPREMOT	E2	0.93	<0.001	Yes	***	—	Ok
EXPREMOT	E3	0.86	<0.001	Yes	***	—	Ok
SOCST	S1	0.90	<0.001	Yes	***	—	Ok
SOCST	S2	0.90	<0.001	Yes	***	—	Ok
SOCST	S3	0.89	<0.001	Yes	***	—	Ok
PROBFOC	PROBSLV	0.81	<0.001	Yes	***	—	Ok
PROBFOC	COGRES	0.82	<0.001	Yes	***	—	Ok
EMOTFOC	EXPREMOT	0.83	<0.001	Yes	***	—	Ok
EMOTFOC	SOCST	0.81	<0.001	Yes	***	—	Ok

¹ The completely standardized factor loading can be interpreted as the correlation with the factor.

² Completely standardized factor loading significance at 5% type I error.

³ Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.

⁴ No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.

⁵ _____

⁶ _____

⁷ _____

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section “Model Fit Summary”, for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the “Model Fit Summary” section:

Table 10: Interpretation of Unstandardized Factor Loadings

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in PROBSLV leads to a 1.00 -unit increase in the P1
A 1-unit increase in PROBSLV leads to a 1.42 -unit increase in the P2
A 1-unit increase in PROBSLV leads to a 0.90 -unit increase in the P3
A 1-unit increase in COGRES leads to a 1.00 -unit increase in the C1
A 1-unit increase in COGRES leads to a 1.03 -unit increase in the C2
A 1-unit increase in COGRES leads to a 0.84 -unit increase in the C3
A 1-unit increase in EXPREMOT leads to a 1.00 -unit increase in the E1
A 1-unit increase in EXPREMOT leads to a 1.54 -unit increase in the E2
A 1-unit increase in EXPREMOT leads to a 1.13 -unit increase in the E3
A 1-unit increase in SOCST leads to a 1.00 -unit increase in the S1

Table 10: Interpretation of Unstandardized Factor Loadings (continued)

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in SOCSPT leads to a 0.71 -unit increase in the S2
A 1-unit increase in SOCSPT leads to a 0.38 -unit increase in the S3
A 1-unit increase in PROBFOC leads to a 1.00 -unit increase in the PROBSLV
A 1-unit increase in PROBFOC leads to a 1.61 -unit increase in the COGRES
A 1-unit increase in EMOTFOC leads to a 1.00 -unit increase in the EXPREMOT
A 1-unit increase in EMOTFOC leads to a 1.57 -unit increase in the SOCSPT

Factor Discriminant Validity

As noted by [Brown], “the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context.” Though, the largest (absolute) estimated factor intercorrelation within the section “Completely Standardized Parameter Estimates” is 0.3 which we regard as a proof of a reasonable discriminant validity. Moreover, the statistical test(s) for factor discriminant validity are statistically significant at 5% type I error.

Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 11: Completely Standardized Error Variances and Commuality

Observed Variable	Error Variance ¹	Communality ²³	P-Value	Significant Error Variance? ⁴
C1	0.21	0.79	<0.001	Yes
C2	0.24	0.76	<0.001	Yes
C3	0.18	0.82	<0.001	Yes
COGRES	0.33	0.67	0.065	No
E1	0.24	0.76	<0.001	Yes
E2	0.14	0.86	<0.001	Yes
E3	0.26	0.74	<0.001	Yes
EXPREMOT	0.32	0.68	0.08	No
P1	0.17	0.83	<0.001	Yes
P2	0.25	0.75	<0.001	Yes
P3	0.23	0.77	<0.001	Yes
PROBSLV	0.34	0.66	0.058	No
S1	0.19	0.81	<0.001	Yes
S2	0.20	0.80	<0.001	Yes
S3	0.21	0.79	<0.001	Yes
SOCSPT	0.34	0.66	0.053	No

¹ Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).

² Corresponds to the squared factor loading.

³ Can be interpreted as proportion of explained variance by the latent factor(s) (%).

⁴ 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

Factor Reliability

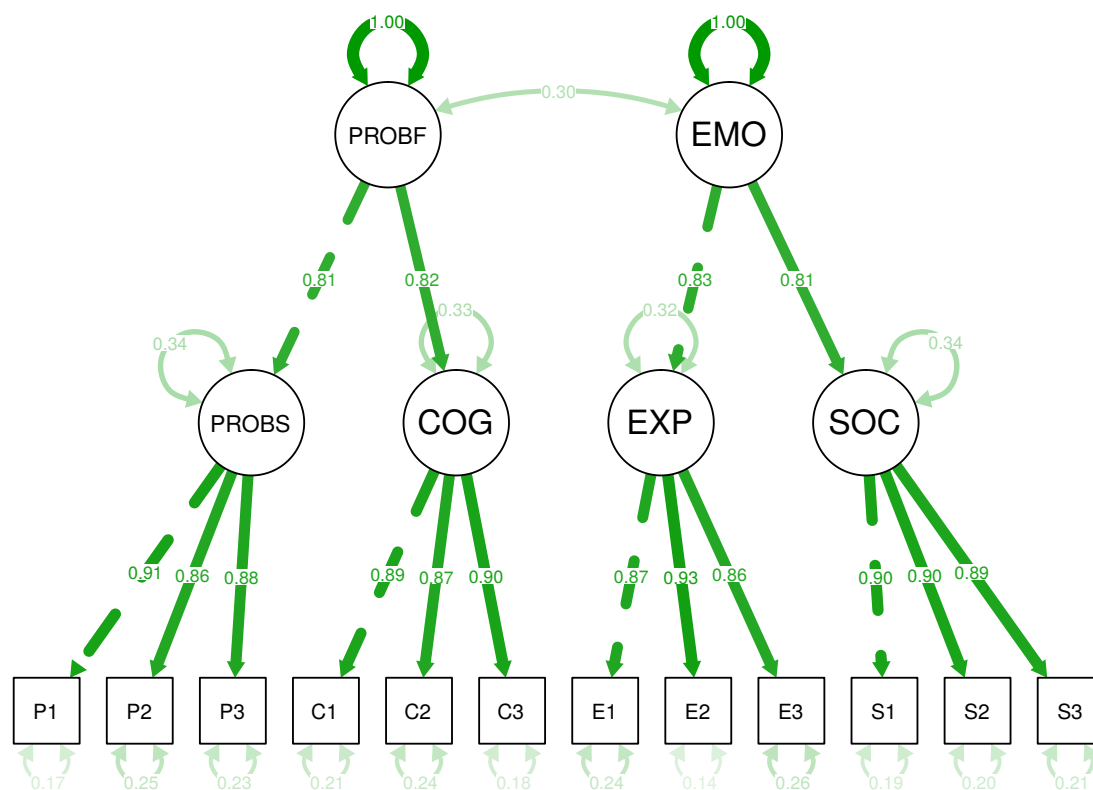
The table “Factor Reliability” contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretability of the reliability measures depend on the specific research context. In some fields of research, omega values greater or equal than 0.6 and AVE values greater or equal than 0.5 (fulfilled by and large in your case) could be sufficient.

Final Summary

In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.

Considering the goodness-of-fit indices, the model is acceptable. Moreover, we cannot identify localized areas of ill fit. We conclude that the model is acceptable.

Path Diagram



APPENDIX

Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

References