Confirmatory Factor Analysis

Statsomat.com

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Warning: The automatic computation and interpretation delivered by the Statsomat should not completely replace the classical, made by humans graphical exploratory data analysis and statistical analysis. There may be data cases for which the Statsomat does not deliver the most optimal solution or output interpretation.

Basic Information

Automatic statistics for the file:

File
case21_female.csv

Your selection for the encoding: Auto
Your selection for the decimal character: Auto
Observations (rows with at least one non-missing value): 250
Variables (columns with at least one non-missing value): 9
Variables considered continuous: 9

Variables considered continuous
M1
M2
M3
M4
M5
M6
M7
M8
M9

Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
DEPRESS	=~	M1	1	0
DEPRESS	=~	M2	1	1
DEPRESS	=~	M3	1	2
DEPRESS	=~	M4	1	3
DEPRESS	=~	M5	1	4
DEPRESS	=~	M6	1	5
DEPRESS	=~	M7	1	6
DEPRESS	=~	M8	1	7
DEPRESS	=~	M9	1	8
M1	~~	M1	0	9
M2	~~	M2	0	10
M3	~~	M3	0	11
M4	~~	M4	0	12
M5	~~	M5	0	13
M6	~~	M6	0	14
M7	~~	M7	0	15
M8	~~	M8	0	16
M9	~~	M9	0	17
DEPRESS	~~	DEPRESS	0	18

Assumptions

Open issue

Model Settings

Outputs

Model Fit Summary

```
lavaan 0.6-7 ended normally after 31 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         18

  Number of observations                           250
```

```
Model Test User Model:

  Test statistic                                40.322
  Degrees of freedom                                27
  P-value (Chi-square)                           0.048

Model Test Baseline Model:

  Test statistic                               466.893
  Degrees of freedom                                36
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.969
  Tucker-Lewis Index (TLI)                       0.959

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -4550.451
  Loglikelihood unrestricted model (H1)      -4530.290

  Akaike (AIC)                                9136.902
  Bayesian (BIC)                              9200.289
  Sample-size adjusted Bayesian (BIC)         9143.227

Root Mean Square Error of Approximation:

  RMSEA                                          0.044
  90 Percent confidence interval - lower         0.005
  90 Percent confidence interval - upper         0.071
  P-value RMSEA <= 0.05                          0.600

Standardized Root Mean Square Residual:

  SRMR                                           0.046

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  DEPRESS =~
    M1                1.000
```

```
    M2                1.117    0.103   10.813    0.000
    M3                0.656    0.107    6.132    0.000
    M4                0.800    0.112    7.114    0.000
    M5                0.698    0.109    6.424    0.000
    M6                0.829    0.102    8.162    0.000
    M7                0.580    0.105    5.505    0.000
    M8                0.882    0.109    8.070    0.000
    M9                0.452    0.092    4.930    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .M1                1.128    0.154    7.321    0.000
   .M2                1.789    0.220    8.115    0.000
   .M3                3.599    0.337   10.675    0.000
   .M4                3.716    0.356   10.449    0.000
   .M5                3.647    0.344   10.615    0.000
   .M6                2.760    0.273   10.117    0.000
   .M7                3.627    0.336   10.789    0.000
   .M8                3.223    0.318   10.151    0.000
   .M9                2.826    0.260   10.876    0.000
    DEPRESS           1.808    0.269    6.722    0.000
```

Completely Standardized Parameter Estimates

```
Latent Variables:
                    est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  DEPRESS =~
    M1                0.785    0.035   22.380    0.000    0.716    0.853
    M2                0.747    0.038   19.869    0.000    0.673    0.820
    M3                0.421    0.059    7.181    0.000    0.306    0.536
    M4                0.487    0.055    8.845    0.000    0.379    0.595
    M5                0.441    0.058    7.648    0.000    0.328    0.554
    M6                0.557    0.051   10.977    0.000    0.458    0.657
    M7                0.379    0.061    6.241    0.000    0.260    0.498
    M8                0.551    0.051   10.772    0.000    0.451    0.652
    M9                0.340    0.062    5.446    0.000    0.218    0.462

Variances:
                    est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
   .M1                0.384    0.055    6.983    0.000    0.276    0.492
   .M2                0.442    0.056    7.879    0.000    0.332    0.552
   .M3                0.822    0.049   16.631    0.000    0.726    0.919
   .M4                0.763    0.054   14.203    0.000    0.657    0.868
   .M5                0.806    0.051   15.838    0.000    0.706    0.905
   .M6                0.689    0.057   12.180    0.000    0.578    0.800
   .M7                0.856    0.046   18.603    0.000    0.766    0.947
   .M8                0.696    0.056   12.340    0.000    0.586    0.807
```

```
.M9                    0.884    0.042   20.832    0.000    0.801    0.968
 DEPRESS               1.000                               1.000    1.000
```

Communality

Table 4: Communality

Variable	Communality
M1	0.62
M2	0.56
M3	0.18
M4	0.24
M5	0.19
M6	0.31
M7	0.14
M8	0.30
M9	0.12

Factor Reliability

Table 5: Factor Reliability

	DEPRESS
Omega (Bentler)	0.77
Omega (McDonald)	0.77
AVE	0.29

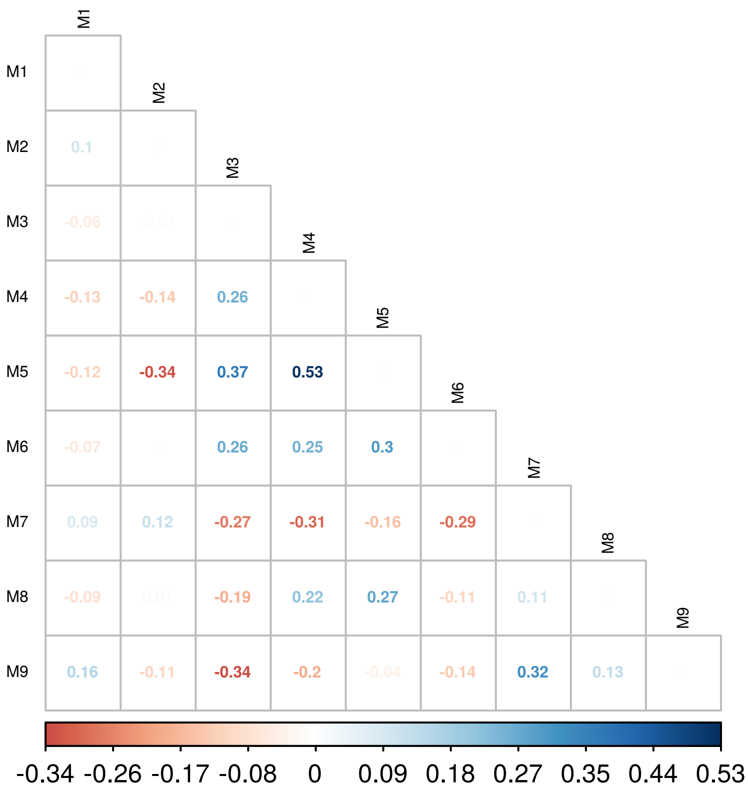
Observed Covariance Matrix

	M1	M2	M3	M4	M5	M6	M7	M8	M9
M1	2.94								
M2	2.12	4.04							
M3	1.13	1.32	4.38						
M4	1.32	1.47	1.21	4.87					
M5	1.14	1.07	1.2	1.54	4.53				
M6	1.43	1.67	1.25	1.45	1.35	4			
M7	1.14	1.3	0.41	0.53	0.57	0.58	4.23		
M8	1.51	1.8	0.85	1.49	1.39	1.21	1.03	4.63	
M9	0.97	0.8	0.19	0.45	0.53	0.54	0.8	0.85	3.19

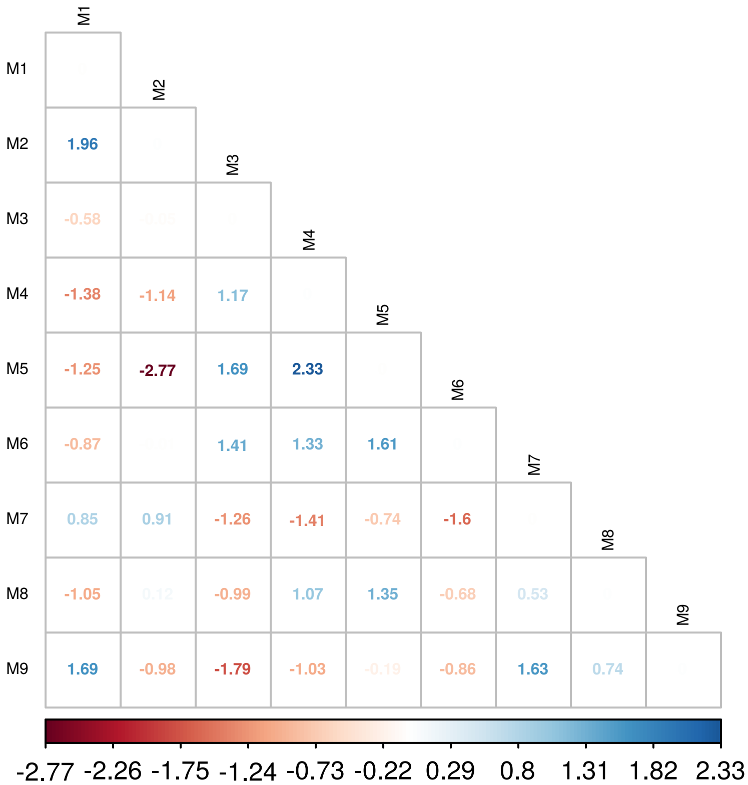
Model-Implied Covariance Matrix

	M1	M2	M3	M4	M5	M6	M7	M8	M9
M1	2.94								
M2	2.02	4.04							
M3	1.19	1.32	4.38						
M4	1.45	1.62	0.95	4.87					
M5	1.26	1.41	0.83	1.01	4.53				
M6	1.5	1.67	0.98	1.2	1.05	4			
M7	1.05	1.17	0.69	0.84	0.73	0.87	4.23		
M8	1.59	1.78	1.05	1.28	1.11	1.32	0.93	4.63	
M9	0.82	0.91	0.54	0.65	0.57	0.68	0.47	0.72	3.19

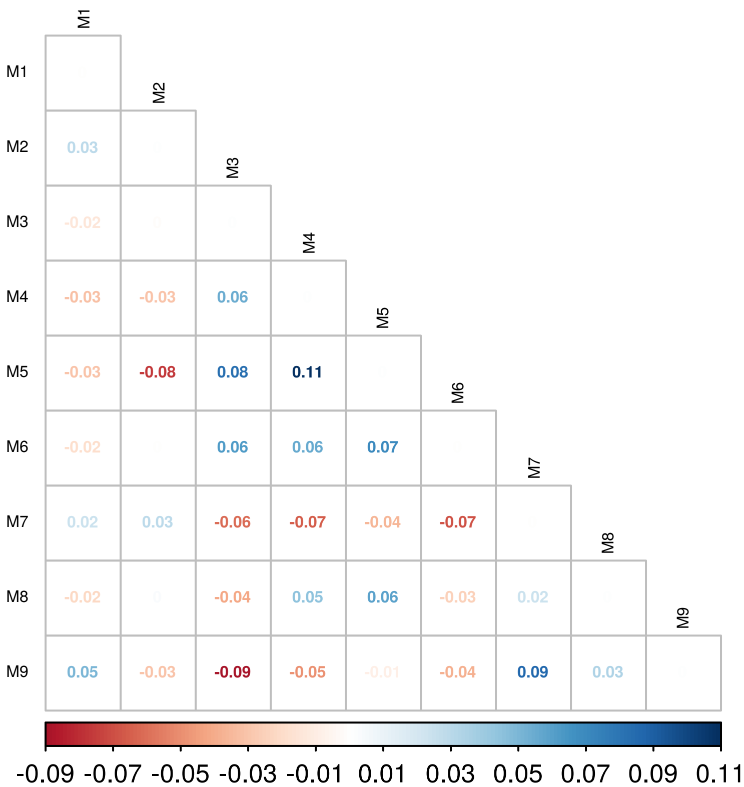
Residual Covariance Matrix



Standardized Residual Matrix



Residual Correlation Matrix



Modification Indices

Table 6: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
M2	~~	M5	6.811	-0.523	0.1	0.079	**(m)**
M4	~~	M5	5.838	0.601	0.1	0.069	**(m)**
M1	~~	M2	4.948	0.387	0.1	0.089	**(m)**
M3	~~	M9	3.164	-0.373	0.1	0.076	(i)
M1	~~	M9	3.129	0.254	0.1	0.107	(i)
M3	~~	M5	2.974	0.417	0.1	0.070	(i)
M7	~~	M9	2.741	0.347	0.1	0.077	(i)
M5	~~	M6	2.708	0.360	0.1	0.074	(i)
M6	~~	M7	2.492	-0.340	0.1	0.075	(i)
M3	~~	M6	2.070	0.311	0.1	0.075	(i)

Note:
Maximum 10 modification indices in descending order of their magnitude are listed.

Interpretation

Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [@brown], [@kline].

Model Test User Model

The degrees of freedom are calculated as the number of known parameters minus the number of free parameters: 45 - 18 = 27 . The 27 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 40.322 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.05 then the test is not statistically significant at 5 % error, the hypothesis cannot be rejected, which would be in favour of the model.
In our case, the p-value is 0.048 suggesting that the model may not be acceptable for the data. The Chi-square model fit index is based on a very stringent statistical hypothesis which may have no practical relevance. We will consider it only in connection with other model fit indices.

Model Test Baseline Model

The test statistic with the value 466.893 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is <0.001 suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

Root Mean Square Error of Approximation:

The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.
In our case, the RMSEA is 0.044. The upper bound of the 90% confidence interval of the RMSEA is 0.071 and smaller than the threshold value 0.08, suggesting a good model fit.

Standardized Root Mean Square Residual:

The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.
In our case, the SRMR is 0.05 which is smaller than the threshold value 0.1 suggesting an acceptable model fit.

User Model versus Baseline Model

The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of departure from the baseline model).
In our case, the CFI is 0.969 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.
In our case, the TLI is 0.959 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Summary of the Goodness of Fit Indices

The TLI model fit index suggests an acceptable model fit. Moreover the SRMR and the RMSEA model fit indices. The Chi-square model fit index suggests a poor model fit. The Chi-square model fit index is based on a very stringent statistical hypothesis. Therefore, we ignor it and tentatively assume an acceptable model fit. We verify this assertion by considering further metrics.

Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.
A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.
Following variable pairs have standardized residuals which are larger or equal than the considered threshold 2.58 [@brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably underestimated:

Table 7: Pair(s) with Underestimated Covariance

Pair 1	M4	M5

Following variable pairs have standardized residuals which are smaller or equal than the considered threshold -2.58 [@brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably overestimated:

Table 8: Pair(s) with Overestimated Covariance

Pair 1	M2	M5

Depending on the sample size, the misspecification detected by the analysis of the residual covariance resp. correlation matrices can be statistically significant but not relevant and in practice negligible. This is matter of subject in the next section(s).

Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [@brown] and [@mi]. We cite from [@brown]: "The modification index reflects an approximation of how much the overall model Chi² will decrease if the fixed or constrained parameter is freely estimated." In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.
We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label "epc:m". For more information with regard to the labels of the decision column,

please consult the Appendix.
We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.
We remark that there exist no modification indices with respect to factor loadings.
We remark that there exist no modification indices with respect to factor covariances.

Parameter Estimates

Factor Loadings

We remark that the completely standardized factor loadings (section "Completely Standardized Parameter Estimates") are all statistically significant at 5% type I error (i.e. p-value <= 0.05). Nevertheless, (some) completely standardized factor loadings are in absolute value smaller than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [@brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 9: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading[1]	P-Value	Significant?[2]	Relevance[3]	Sign[4]	Check
DEPRESS	M1	0.78	<0.001	Yes	**	—	Ok
DEPRESS	M2	0.75	<0.001	Yes	**	—	Ok
DEPRESS	M3	0.42	<0.001	Yes	*	—	Ok
DEPRESS	M4	0.49	<0.001	Yes	*	—	Ok
DEPRESS	M5	0.44	<0.001	Yes	*	—	Ok
DEPRESS	M6	0.56	<0.001	Yes	*	—	Ok
DEPRESS	M7	0.38	<0.001	Yes	!	—	Uncertain[7]
DEPRESS	M8	0.55	<0.001	Yes	*	—	Ok
DEPRESS	M9	0.34	<0.001	Yes	!	—	Uncertain[7]

[1] The completely standardized factor loading can be interpreted as the correlation with the factor.
[2] Completely standardized factor loading significance at 5% type I error.
[3] Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.
[4] No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.
[5] _____
[6] _____
[7] Uncertain. Significant but small(er) effect size. Further analysis is recommended.

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section "Model Fit Summary", for non-marker variables).

Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 10: Completely Standardized Error Variances and Communality

Observed Variable	Error Variance[1]	Communality[2,3]	P-Value	Significant Error Variance?[4]
M1	0.38	0.62	<0.001	Yes
M2	0.44	0.56	<0.001	Yes
M3	0.82	0.18	<0.001	Yes
M4	0.76	0.24	<0.001	Yes
M5	0.81	0.19	<0.001	Yes
M6	0.69	0.31	<0.001	Yes
M7	0.86	0.14	<0.001	Yes
M8	0.70	0.30	<0.001	Yes
M9	0.88	0.12	<0.001	Yes

[1] Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).
[2] Corresponds to the squared factor loading.
[3] Can be interpreted as proportion of explained variance by the latent factor(s) (%).
[4] 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

Factor Reliability

The table "Factor Reliability" contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretatibility of the reliability measures depend on the specific research context. Nevertheless, omega values below 0.6 or AVE values below 0.5 (at least one of these existent in your case) should be regarded with criticism. The factor reliability estimates are not further considered in the final summary.

Final Summary

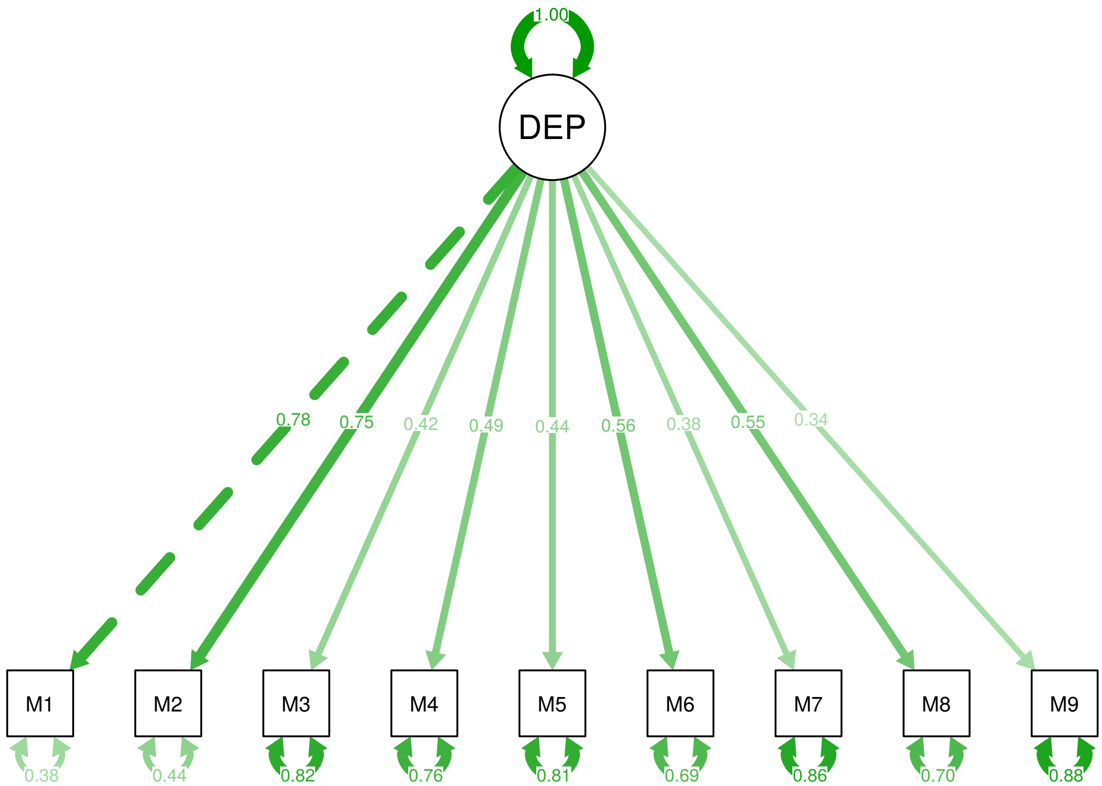
In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.
Considering the goodness-of-fit indices, the model is acceptable. Nevertheless, there exist localized areas of ill fit. Therefore, the quality of the model is uncertain. Only if supported by theory, you could try to respecify the model and improve the goodness-of-fit by applying (one of the) following recommendations or call for actions.

There exist factor loadings which are uncertain. To decide upon the statistical significance and relevance, further statistical tests and/or power and sample size analysis, not covered by this app are recommended. Only if supported by theory and further statistical analysis, you could drop the uncertain parameter(s) from the model.

Final Comments

Please consider that this summary depends on hard-coded cutoff-values which may be too liberal or too conservative for your research area. If you have sound theory-based reasons and decide to respicify the model, we strongly recommend the replication of the CFA in an independent sample.

Path Diagram



APPENDIX

Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

References