Confirmatory Factor Analysis

Statsomat.com

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Warning: The automatic computation and interpretation delivered by the Statsomat should not completely replace the classical, made by humans graphical exploratory data analysis and statistical analysis. There may be data cases for which the Statsomat does not deliver the most optimal solution or output interpretation.

Basic Information

Automatic statistics for the file:

File
case19.csv

Your selection for the encoding: Auto
Your selection for the decimal character: Auto
Observations (rows with at least one non-missing value): 250
Variables (columns with at least one non-missing value): 6
Variables considered continuous: 6

Variables considered continuous
X1
X2
X3
X4
X5
X6

Warning: Based on an analysis of the variables considered continuous, we suspect outliers in the data. If observations are erroneous, you could drop them and restart the app. Outliers may affect negatively the execution or the results of the CFA. These are the suspected row numbers:

Table 3: Rows With Suspected Outliers

	X1	X2	X3	X4	X5	X6
186	2.228949	6.473265	-0.9784323	4.034067	3.687871	4.32723

Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
AUDITORY	=~	X1	1	0
AUDITORY	=~	X2	1	1
AUDITORY	=~	X3	1	2
VISUAL	=~	X4	1	0
VISUAL	=~	X5	1	3
VISUAL	=~	X6	1	4
X1	~~	X1	0	5
X2	~~	X2	0	6
X3	~~	X3	0	7
X4	~~	X4	0	8
X5	~~	X5	0	9
X6	~~	X6	0	10
AUDITORY	~~	AUDITORY	0	11
VISUAL	~~	VISUAL	0	12
AUDITORY	~~	VISUAL	0	13

Assumptions

Open issue

Model Settings

Outputs

Model Fit Summary

```
lavaan 0.6-7 ended normally after 39 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         13

  Number of observations                           250

Model Test User Model:

  Test statistic                                 6.097
```

```
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.636

Model Test Baseline Model:

  Test statistic                               899.393
  Degrees of freedom                                15
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.004

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -2923.228
  Loglikelihood unrestricted model (H1)      -2920.180

  Akaike (AIC)                                5872.456
  Bayesian (BIC)                              5918.235
  Sample-size adjusted Bayesian (BIC)         5877.024

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent confidence interval - lower         0.000
  90 Percent confidence interval - upper         0.062
  P-value RMSEA <= 0.05                          0.898

Standardized Root Mean Square Residual:

  SRMR                                           0.012

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  AUDITORY =~
    X1                1.000
    X2                1.039    0.083   12.461    0.000
    X3                0.959    0.079   12.097    0.000
  VISUAL =~
    X4                1.000
```

```
    X5                1.023    0.051   19.995    0.000
    X6                1.023    0.052   19.686    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  AUDITORY ~~
    VISUAL            1.410    0.289    4.879    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .X1                2.368    0.333    7.117    0.000
   .X2                2.279    0.343    6.641    0.000
   .X3                2.623    0.334    7.856    0.000
   .X4                0.663    0.105    6.338    0.000
   .X5                0.877    0.120    7.328    0.000
   .X6                0.957    0.125    7.677    0.000
    AUDITORY          4.417    0.624    7.079    0.000
    VISUAL            3.085    0.341    9.046    0.000
```

Completely Standardized Parameter Estimates

```
Latent Variables:
                    est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  AUDITORY =~
    X1                0.807    0.032   25.056    0.000    0.744    0.870
    X2                0.823    0.031   26.197    0.000    0.761    0.884
    X3                0.779    0.034   23.114    0.000    0.713    0.845
  VISUAL =~
    X4                0.907    0.017   53.665    0.000    0.874    0.940
    X5                0.887    0.018   48.285    0.000    0.851    0.923
    X6                0.878    0.019   46.220    0.000    0.841    0.916

Covariances:
                    est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  AUDITORY ~~
    VISUAL            0.382    0.063    6.108    0.000    0.259    0.505

Variances:
                    est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
   .X1                0.349    0.052    6.718    0.000    0.247    0.451
   .X2                0.323    0.052    6.262    0.000    0.222    0.425
   .X3                0.393    0.053    7.470    0.000    0.290    0.496
   .X4                0.177    0.031    5.767    0.000    0.117    0.237
   .X5                0.214    0.033    6.565    0.000    0.150    0.278
   .X6                0.229    0.033    6.850    0.000    0.163    0.294
    AUDITORY          1.000                               1.000    1.000
    VISUAL            1.000                               1.000    1.000
```

Communality

Table 5: Communality

Variable	Communality
X1	0.65
X2	0.68
X3	0.61
X4	0.82
X5	0.79
X6	0.77

Factor Discriminant Validity

Table 6: Factor Discriminant Validity Test at Cutoff 0.85

			Factor Correlation	Chisq diff	Df diff	P-Value
AUDITORY	~~	VISUAL	0.382	123.616	1	<0.001

Factor Reliability

Table 7: Factor Reliability

	AUDITORY	VISUAL	total
Omega (Bentler)	0.85	0.92	0.91
Omega (McDonald)	0.85	0.92	0.91
AVE	0.65	0.79	0.70

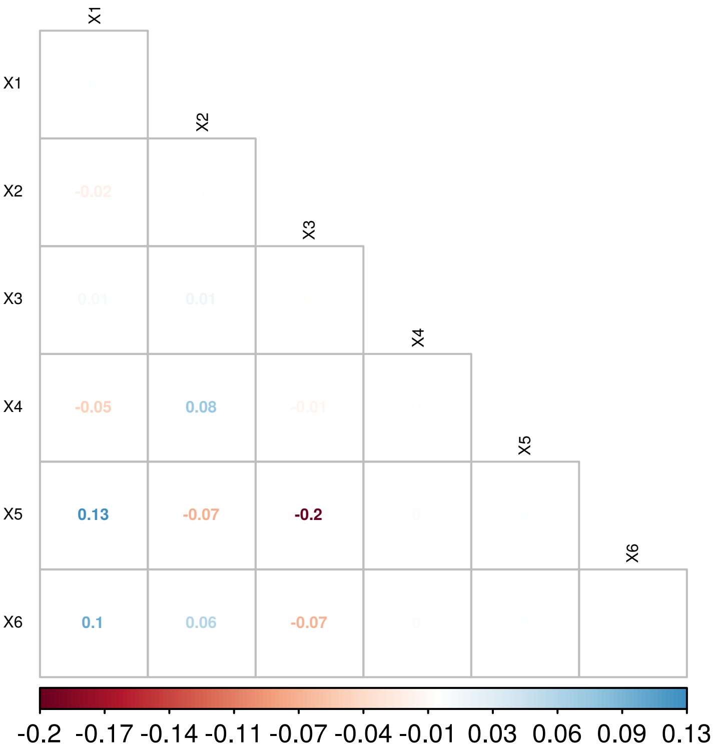
Observed Covariance Matrix

	X1	X2	X3	X4	X5	X6
X1	6.78					
X2	4.57	7.05				
X3	4.24	4.41	6.68			
X4	1.36	1.54	1.34	3.75		
X5	1.57	1.43	1.18	3.16	4.1	
X6	1.55	1.56	1.31	3.15	3.23	4.19

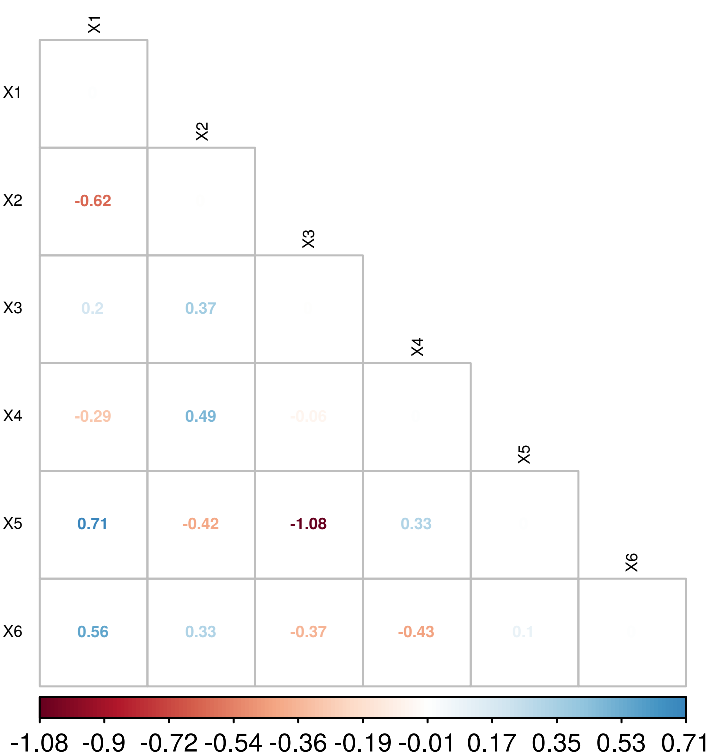
Model-Implied Covariance Matrix

	X1	X2	X3	X4	X5	X6
X1	6.78					
X2	4.59	7.05				
X3	4.23	4.4	6.68			
X4	1.41	1.47	1.35	3.75		
X5	1.44	1.5	1.38	3.16	4.1	
X6	1.44	1.5	1.38	3.16	3.23	4.19

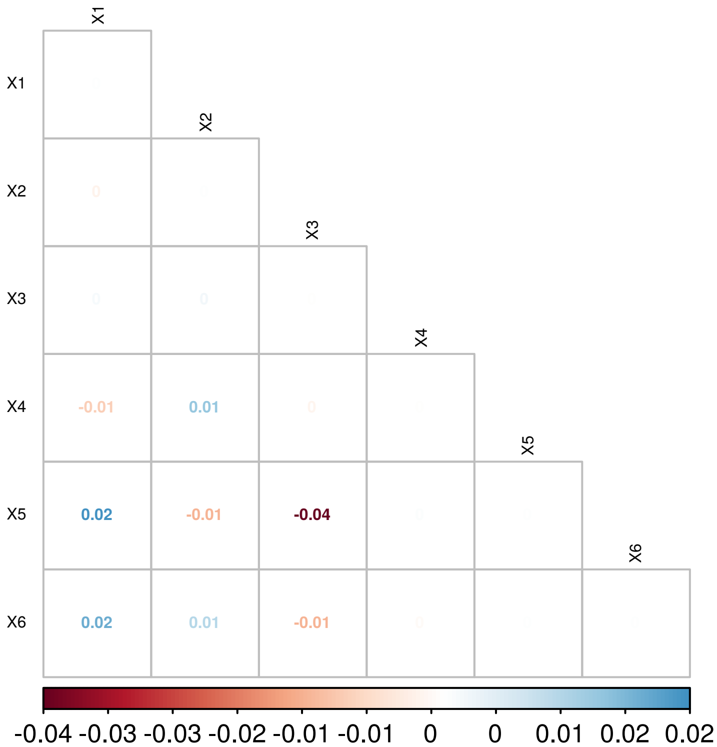
Residual Covariance Matrix



Standardized Residual Matrix



Residual Correlation Matrix



Modification Indices

Table 8: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
X1	~~	X4	3.460	-0.219	0.1	0.136	(i)
X1	~~	X5	3.217	0.229	0.1	0.123	(i)
X3	~~	X5	1.557	-0.162	0.1	0.120	(i)
X3	~~	X4	1.128	0.127	0.1	0.133	(i)
X2	~~	X5	0.882	-0.121	0.1	0.122	(i)
X2	~~	X4	0.835	0.108	0.1	0.135	(i)
X1	~~	X2	0.371	-0.440	0.1	0.052	(i)
X4	~~	X6	0.183	-0.146	0.1	0.060	(i)

Note:
Maximum 10 modification indices in descending order of their magnitude are listed.

Table 9: Modification Indices With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
VISUAL	=~	X3	0.371	-0.049	0.4	0.999	(nm)
AUDITORY	=~	X5	0.183	-0.018	0.4	1.000	(nm)

Note:
Maximum 10 modification indices in descending order of their magnitude are listed.

Interpretation

Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [@brown], [@kline].

Model Test User Model
The degrees of freedom are calculated as the number of known parameters minus the number of free parameters: 21 - 13 = 8 . The 8 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 6.097 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.05 then the test is not statistically significant at 5 % error, the hypothesis cannot be rejected, which would be in favour of the model.
In our case, the p-value is 0.636 suggesting an acceptable model fit.

Model Test Baseline Model
The test statistic with the value 899.393 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is <0.001 suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

Root Mean Square Error of Approximation:
The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.
In our case, the RMSEA is 0. The upper bound of the 90% confidence interval of the RMSEA is 0.062 and smaller than the threshold value 0.08, suggesting a good model fit.

Standardized Root Mean Square Residual:
The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.
In our case, the SRMR is 0.01 which is smaller than the threshold value 0.1 suggesting an acceptable model fit.

User Model versus Baseline Model
The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of departure from the baseline model).
In our case, the CFI is 1 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.
In our case, the TLI is 1.004 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Summary of the Goodness of Fit Indices

The TLI model fit index suggests an acceptable model fit. Moreover the Chi-square model fit index and the RMSEA suggest an acceptable model fit. We tentatively assume an acceptable model fit and verify this assertion by considering further metrics.

Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.
A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.
There are no variable pairs with standardized residuals which are larger or equal than the considered threshold 2.58 [@brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [@kline]. Therefore, no relationships among the variables are substantially underestimated by the model. There are no variable pairs with standardized residuals which are smaller or equal than the considered threshold -2.58 [@brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [@kline]. Therefore, no relationships among the variables are substantially overestimated by the model.

Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [@brown] and [@mi]. We cite from [@brown]: “The modification index reflects an approximation of how much the overall model Chi² will decrease if the fixed or constrained parameter is freely estimated.” In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.
We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label “epc:m”. For more information with regard to the labels of the decision column, please consult the Appendix.
We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.
We remark that these conditions are not fulfilled for modification indices with respect to factor loadings. Therefore, there exist no significant and relevant modification indices with respect to factor loadings.
We remark that there exist no modification indices with respect to factor covariances.

Parameter Estimates

Factor Loadings

We remark that the completely standardized factor loadings (section “Completely Standardized Parameter Estimates”) are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [@brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 10: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading[1]	P-Value	Significant?[2]	Relevance[3]	Sign[4]	Check
AUDITORY	X1	0.81	<0.001	Yes	***	—	Ok
AUDITORY	X2	0.82	<0.001	Yes	***	—	Ok
AUDITORY	X3	0.78	<0.001	Yes	**	—	Ok
VISUAL	X4	0.91	<0.001	Yes	***	—	Ok
VISUAL	X5	0.89	<0.001	Yes	***	—	Ok
VISUAL	X6	0.88	<0.001	Yes	***	—	Ok

[1] The completely standardized factor loading can be interpreted as the correlation with the factor.

[2] Completely standardized factor loading significance at 5% type I error.

[3] Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.

[4] No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.

[5] _____

[6] _____

[7] _____

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section "Model Fit Summary", for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the "Model Fit Summary" section:

Table 11: Interpretation of Unstandardized Factor Loadings

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in AUDITORY leads to a 1.00 -unit increase in the X1
A 1-unit increase in AUDITORY leads to a 1.04 -unit increase in the X2
A 1-unit increase in AUDITORY leads to a 0.96 -unit increase in the X3
A 1-unit increase in VISUAL leads to a 1.00 -unit increase in the X4
A 1-unit increase in VISUAL leads to a 1.02 -unit increase in the X5
A 1-unit increase in VISUAL leads to a 1.02 -unit increase in the X6

Factor Discriminant Validity

As noted by [@brown], "the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context." Though, the largest estimated factor intercorrelation within the section "Completely Standardized Parameter Estimates" is 0.38 which we regard as a proof of a reasonable discriminant validity. Moreover, the statistical test(s) for factor discriminant validity are statistically significant at 5% type I error.

Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 12: Completely Standardized Error Variances and Communality

Observed Variable	Error Variance[1]	Communality[23]	P-Value	Significant Error Variance?[4]
X1	0.35	0.65	<0.001	Yes
X2	0.32	0.68	<0.001	Yes
X3	0.39	0.61	<0.001	Yes
X4	0.18	0.82	<0.001	Yes
X5	0.21	0.79	<0.001	Yes
X6	0.23	0.77	<0.001	Yes

[1] Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).
[2] Corresponds to the squared factor loading.
[3] Can be interpreted as proportion of explained variance by the latent factor(s) (%).
[4] 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

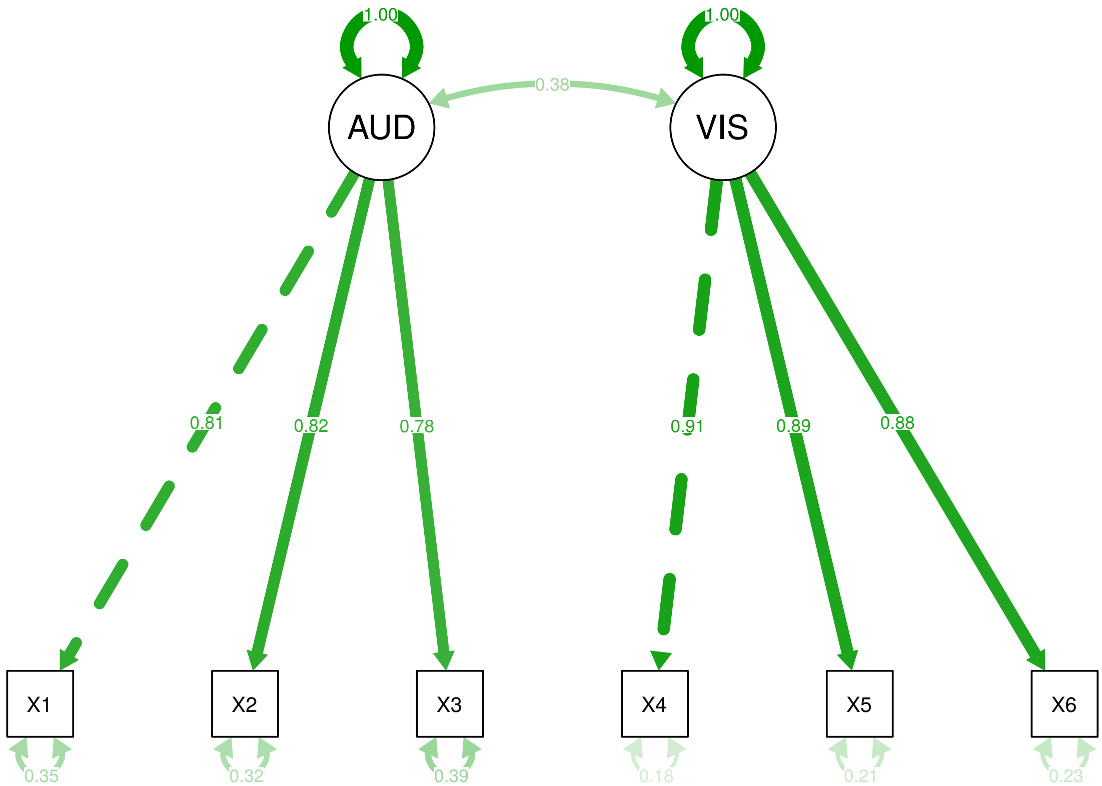
Factor Reliability

The table "Factor Reliability" contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretatibility of the reliability measures depend on the specific research context. In some fields of research, omega values greater or equal than 0.6 and AVE values greater or equal than 0.5 (fulfilled by and large in your case) could be sufficient.

Final Summary

In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.
Considering the goodness-of-fit indices, the model is acceptable. Moreover, we cannot identify localized areas of ill fit. We conclude that the model is acceptable.

Path Diagram



APPENDIX

Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

References