

# Confirmatory Factor Analysis

Statsomat.com

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## Basic Information

Automatic statistics for the file:

File
case1.csv

Your selection for the encoding: Auto

Your selection for the decimal character: Auto

Observations (rows with at least one non-missing value): 301

Variables (columns with at least one non-missing value): 9

Variables considered continuous: 9

Variables considered continuous
x1
x2
x3
x4
x5
x6
x7
x8
x9

## Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
visual	=~	x1	1	0
visual	=~	x2	1	1
visual	=~	x3	1	2
textual	=~	x4	1	0
textual	=~	x5	1	3
textual	=~	x6	1	4
speed	=~	x7	1	0
speed	=~	x8	1	5
speed	=~	x9	1	6
x1	~~	x1	0	7
x2	~~	x2	0	8
x3	~~	x3	0	9
x4	~~	x4	0	10
x5	~~	x5	0	11
x6	~~	x6	0	12
x7	~~	x7	0	13
x8	~~	x8	0	14
x9	~~	x9	0	15
visual	~~	visual	0	16
textual	~~	textual	0	17
speed	~~	speed	0	18
visual	~~	textual	0	19
visual	~~	speed	0	20
textual	~~	speed	0	21

## Assumptions

Open issue

## Model Settings

## Outputs

## Model Fit Summary

lavaan 0.6-7 ended normally after 35 iterations

Estimator	ML
Optimization method	NLMINB
Number of free parameters	21
Number of observations	301

Model Test User Model:

Test statistic	85.535
Degrees of freedom	24
P-value (Chi-square)	0.000

Model Test Baseline Model:

Test statistic	918.242
Degrees of freedom	36
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.930
Tucker-Lewis Index (TLI)	0.895

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-3737.933
Loglikelihood unrestricted model (H1)	-3695.165
Akaike (AIC)	7517.865
Bayesian (BIC)	7595.715
Sample-size adjusted Bayesian (BIC)	7529.115

Root Mean Square Error of Approximation:

RMSEA	0.092
90 Percent confidence interval - lower	0.072
90 Percent confidence interval - upper	0.114
P-value RMSEA <= 0.05	0.001

Standardized Root Mean Square Residual:

SRMR	0.065
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Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
visual =~				
x1	1.000			
x2	0.554	0.100	5.552	0.000
x3	0.730	0.109	6.676	0.000
textual =~				
x4	1.000			
x5	1.113	0.065	17.015	0.000
x6	0.926	0.055	16.703	0.000
speed =~				
x7	1.000			
x8	1.174	0.164	7.153	0.000
x9	1.068	0.149	7.153	0.000

Covariances:

	Estimate	Std.Err	z-value	P(> z )
visual ~~				
textual	0.408	0.074	5.551	0.000
speed	0.262	0.056	4.648	0.000
textual ~~				
speed	0.175	0.050	3.528	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.x1	0.550	0.114	4.832	0.000
.x2	1.134	0.102	11.142	0.000
.x3	0.844	0.091	9.308	0.000
.x4	0.371	0.048	7.778	0.000
.x5	0.446	0.058	7.643	0.000
.x6	0.356	0.043	8.277	0.000
.x7	0.795	0.081	9.768	0.000
.x8	0.486	0.074	6.545	0.000
.x9	0.573	0.071	8.098	0.000
visual	0.809	0.146	5.557	0.000
textual	0.980	0.112	8.737	0.000
speed	0.388	0.087	4.470	0.000

**Completely Standardized Parameter Estimates**

Latent Variables:

	est.std	Std.Err	z-value	P(> z )	ci.lower	ci.upper
visual =~						
x1	0.772	0.055	14.016	0.000	0.664	0.880
x2	0.424	0.060	7.107	0.000	0.307	0.541
x3	0.581	0.055	10.533	0.000	0.473	0.689

```

textual =~
  x4          0.852    0.023   37.778    0.000    0.807    0.896
  x5          0.855    0.022   38.273    0.000    0.811    0.899
  x6          0.838    0.023   35.882    0.000    0.792    0.884
speed =~
  x7          0.572    0.053   10.773    0.000    0.468    0.677
  x8          0.724    0.051   14.289    0.000    0.624    0.823
  x9          0.660    0.051   12.881    0.000    0.560    0.761

```

Covariances:

	est.std	Std.Err	z-value	P(> z )	ci.lower	ci.upper
visual ~~						
textual	0.458	0.064	7.188	0.000	0.333	0.584
speed	0.468	0.073	6.418	0.000	0.325	0.611
textual ~~						
speed	0.284	0.069	4.130	0.000	0.149	0.419

Variances:

	est.std	Std.Err	z-value	P(> z )	ci.lower	ci.upper
.x1	0.405	0.085	4.762	0.000	0.238	0.571
.x2	0.820	0.051	16.228	0.000	0.721	0.919
.x3	0.662	0.064	10.321	0.000	0.536	0.788
.x4	0.275	0.038	7.157	0.000	0.200	0.350
.x5	0.269	0.038	7.038	0.000	0.194	0.344
.x6	0.298	0.039	7.607	0.000	0.221	0.374
.x7	0.672	0.061	11.049	0.000	0.553	0.792
.x8	0.476	0.073	6.501	0.000	0.333	0.620
.x9	0.564	0.068	8.333	0.000	0.431	0.697
visual	1.000				1.000	1.000
textual	1.000				1.000	1.000
speed	1.000				1.000	1.000

**Communalities**

Table 4: Communalities

Variable	Communality
x1	0.60
x2	0.18
x3	0.34
x4	0.73
x5	0.73
x6	0.70
x7	0.33
x8	0.52
x9	0.44

## Factor Discriminant Validity

Table 5: Factor Discriminant Validity Test at Cutoff 0.85

			Factor Correlation	Chisq diff	Df diff	P-Value
visual	~~	textual	0.458	55.954	1	<0.001
visual	~~	speed	0.468	28.504	1	<0.001
textual	~~	speed	0.284	106.112	1	<0.001

## Factor Reliability

Table 6: Factor Reliability

	visual	textual	speed	total
Omega (Bentler)	0.63	0.89	0.69	0.85
Omega (McDonald)	0.61	0.89	0.69	0.86
AVE	0.37	0.72	0.42	0.51

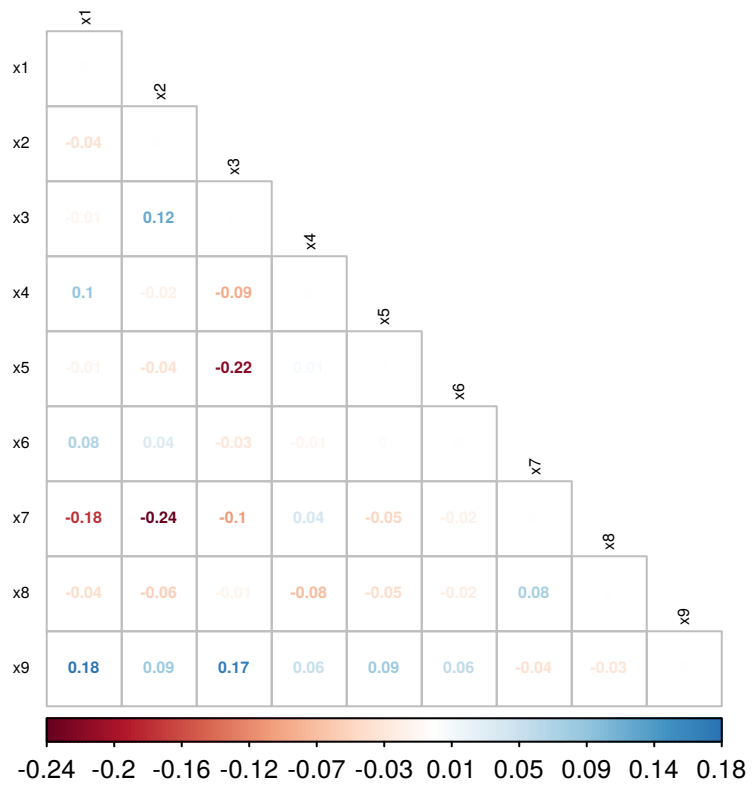
## Observed Covariance Matrix

x1	1.36								
x2	0.41	1.38							
x3	0.58	0.45	1.27						
x4	0.5	0.21	0.21	1.35					
x5	0.44	0.21	0.11	1.1	1.66				
x6	0.45	0.25	0.24	0.9	1.01	1.2			
x7	0.08	-0.1	0.09	0.22	0.14	0.14	1.18		
x8	0.26	0.11	0.21	0.13	0.18	0.17	0.54	1.02	
x9	0.46	0.24	0.37	0.24	0.3	0.24	0.37	0.45	1.02

## Model-Implied Covariance Matrix

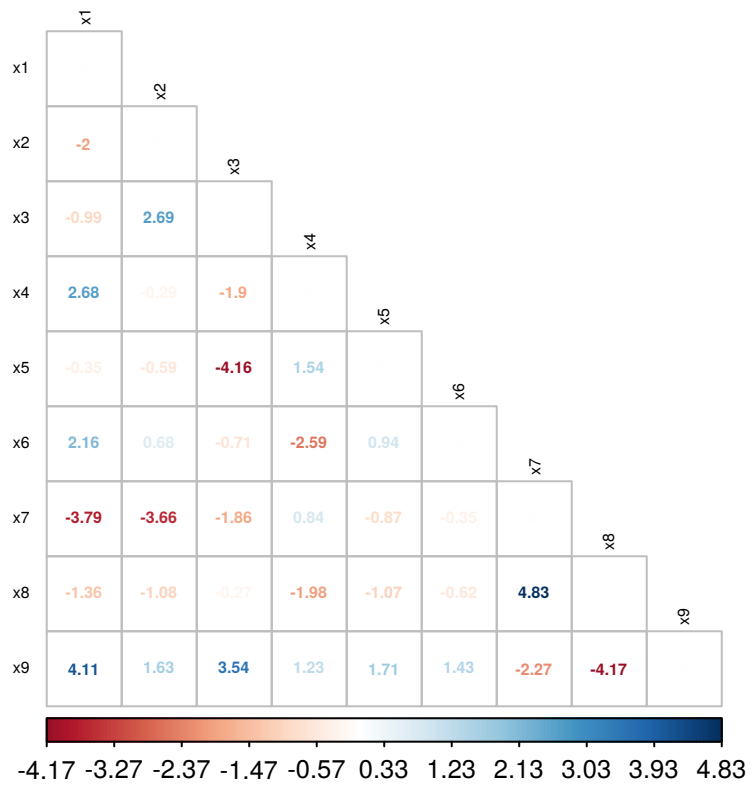
x1	1.36								
x2	0.45	1.38							
x3	0.59	0.33	1.27						
x4	0.41	0.23	0.3	1.35					
x5	0.45	0.25	0.33	1.09	1.66				
x6	0.38	0.21	0.28	0.91	1.01	1.2			
x7	0.26	0.15	0.19	0.17	0.19	0.16	1.18		
x8	0.31	0.17	0.22	0.21	0.23	0.19	0.46	1.02	
x9	0.28	0.16	0.2	0.19	0.21	0.17	0.41	0.49	1.02

## Residual Covariance Matrix

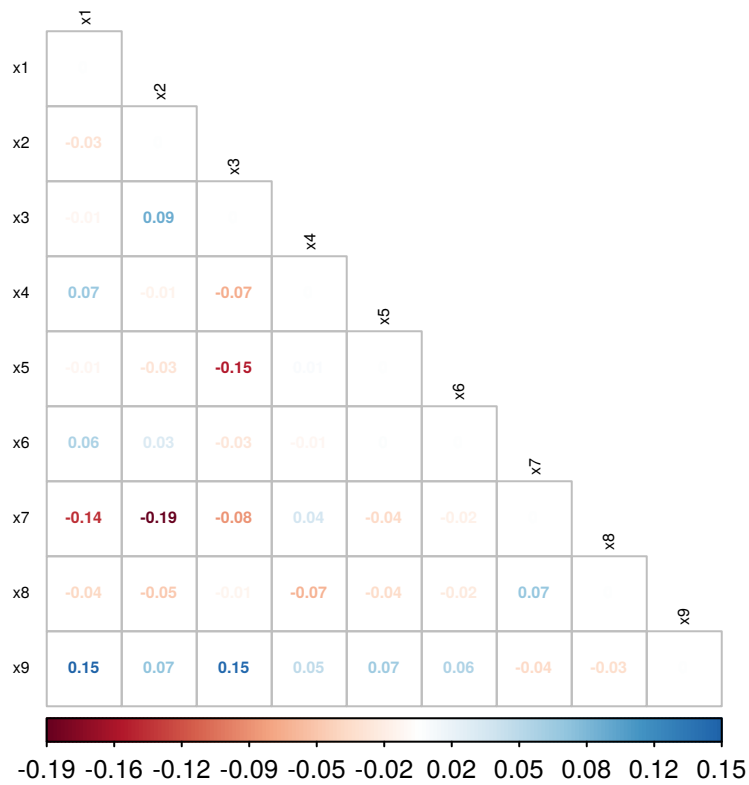




## Standardized Residual Matrix



## Residual Correlation Matrix



## Modification Indices

Table 7: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
x7	~~	x8	34.132	0.543	0.1	0.190	** (m) **
x8	~~	x9	15.123	-0.422	0.1	0.151	** (m) **
x2	~~	x7	8.974	-0.183	0.1	0.373	** (m) **
x2	~~	x3	8.528	0.219	0.1	0.267	** (m) **
x3	~~	x5	7.856	-0.130	0.1	0.577	** (m) **
x1	~~	x9	7.498	0.139	0.1	0.501	** (m) **

Note:

Maximum 10 modification indices in descending order of their magnitude are listed.

Table 8: Modification Indices With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
visual	=~	x9	36.538	0.575	0.4	0.988	*epc:m*
visual	=~	x7	18.922	-0.425	0.4	0.984	epc:nm
textual	=~	x3	9.180	-0.272	0.4	0.994	epc:nm
textual	=~	x1	8.954	0.352	0.4	0.926	epc:nm

Note:

Table 8: Modification Indices With Respect To Factor Loadings (continued)

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
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Maximum 10 modification indices in descending order of their magnitude are listed.

## Interpretation

### Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [@brown], [@kline].

#### Model Test User Model

The degrees of freedom are calculated as the number of known parameters minus the number of free parameters:  $45 - 21 = 24$ . The 24 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 85.535 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.01 then the test is not statistically significant at 1 % error, the hypothesis cannot be rejected, which would be in favour of the model.

In our case, the p-value is  $< 0.001$  suggesting that the model may not be acceptable for the data. The Chi-square model fit index is based on a very stringent statistical hypothesis which may have no practical relevance. We will consider it only in connection with other model fit indices.

#### Model Test Baseline Model

The test statistic with the value 918.242 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is  $< 0.001$  suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

#### Root Mean Square Error of Approximation:

The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.

In our case, the RMSEA is 0.092. The upper bound of the 90% confidence interval of the RMSEA is 0.114 and greater or equal than the threshold value 0.1, suggesting a poor model fit.

#### Standardized Root Mean Square Residual:

The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.

In our case, the SRMR is 0.07 which is smaller than the threshold value 0.08 suggesting an acceptable model fit.

#### User Model versus Baseline Model

The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of

departure from the baseline model).

In our case, the CFI is 0.93 which is greater or equal than the threshold value 0.90, suggesting an acceptable model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.

In our case, the TLI is 0.895 which is smaller or equal than the threshold value 0.90, suggesting a poor model fit.

## Summary of the Goodness of Fit Indices

The CFI and the SRMR tend to support the model but the Chi-square and the RSMSEA model fit indicate the opposite. The goodness-of-fit of the model is uncertain. We proceed by diagnosing the sources of possible misspecification.

## Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.

A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.

Following variable pairs have standardized residuals which are larger or equal than the considered threshold 2.58 [brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [kline]. In these cases, the covariance relationship between the involved variables is probably underestimated:

Table 9: Pair(s) with Underestimated Covariance

Pair 1	x1	x4
Pair 2	x1	x9
Pair 3	x3	x9
Pair 4	x7	x8

Following variable pairs have standardized residuals which are smaller or equal than the considered threshold -2.58 [brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [kline]. In these cases, the covariance relationship between the involved variables is probably overestimated:

Table 10: Pair(s) with Overestimated Covariance

Pair 1	x1	x7
Pair 2	x2	x7
Pair 3	x3	x5
Pair 4	x4	x6
Pair 5	x8	x9

Depending on the sample size, the misspecification detected by the analysis of the residual covariance resp. correlation matrices can be statistically significant but not relevant and in practice negligible. This is matter of subject in

the next section(s).

## Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [Brown] and [M]. We cite from [Brown]: “The modification index reflects an approximation of how much the overall model  $\chi^2$  will decrease if the fixed or constrained parameter is freely estimated.” In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.

We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label “epc:m”. For more information with regard to the labels of the decision column, please consult the Appendix.

We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.

Following modification indices with respect to factor loadings fulfill these requirements:

Table 11: Significant and Relevant MIs With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
visual	=~	x9	36.538	0.575	0.4	0.988	*epc:m*

We remark that there exist no modification indices with respect to factor covariances.

## Parameter Estimates

### Factor Loadings

We remark that the completely standardized factor loadings (section “Completely Standardized Parameter Estimates”) are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [Brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 12: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading <sup>1</sup>	P-Value	Significant? <sup>2</sup>	Relevance <sup>3</sup>	Sign <sup>4</sup>	Check
visual	x1	0.77	<0.001	Yes	**	—	Ok
visual	x2	0.42	<0.001	Yes	*	—	Ok
visual	x3	0.58	<0.001	Yes	*	—	Ok
textual	x4	0.85	<0.001	Yes	***	—	Ok
textual	x5	0.86	<0.001	Yes	***	—	Ok
textual	x6	0.84	<0.001	Yes	***	—	Ok
speed	x7	0.57	<0.001	Yes	*	—	Ok
speed	x8	0.72	<0.001	Yes	**	—	Ok

sometimes  
 $\chi^2$ -Square &  
sometimes  $\chi^2$   
→ not consistent

Table 12: Check Completely Standardized Factor Loadings (continued)

Latent Variable	Observed Variable	Loading <sup>1</sup>	P-Value	Significant? <sup>2</sup>	Relevance <sup>3</sup>	Sign <sup>4</sup>	Check
speed	x9	0.66	<0.001	Yes	**	—	Ok

<sup>1</sup> The completely standardized factor loading can be interpreted as the correlation with the factor.

<sup>2</sup> Completely standardized factor loading significance at 5% type I error.

<sup>3</sup> Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.

<sup>4</sup> No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.

? {  
5 \_\_\_\_\_  
6 \_\_\_\_\_  
7 \_\_\_\_\_

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section “Model Fit Summary”, for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the “Model Fit Summary” section:

Table 13: Interpretation of Unstandardized Factor Loadings

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in visual leads to a 1.00 -unit increase in the x1
A 1-unit increase in visual leads to a 0.55 -unit increase in the x2
A 1-unit increase in visual leads to a 0.73 -unit increase in the x3
A 1-unit increase in textual leads to a 1.00 -unit increase in the x4
A 1-unit increase in textual leads to a 1.11 -unit increase in the x5
A 1-unit increase in textual leads to a 0.93 -unit increase in the x6
A 1-unit increase in speed leads to a 1.00 -unit increase in the x7
A 1-unit increase in speed leads to a 1.17 -unit increase in the x8
A 1-unit increase in speed leads to a 1.07 -unit increase in the x9

### Factor Discriminant Validity

As noted by [Brown], “the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context.” Though, the largest (absolute) estimated factor intercorrelation within the section “Completely Standardized Parameter Estimates” is 0.47 which we regard as a proof of a reasonable discriminant validity. Moreover, the statistical test(s) for factor discriminant validity are statistically significant at 5% type I error.

### Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 14: Completely Standardized Error Variances and Communality

Observed Variable	Error Variance <sup>1</sup>	Communality <sup>23</sup>	P-Value	Significant Error Variance? <sup>4</sup>
x1	0.40	0.60	<0.001	Yes
x2	0.82	0.18	<0.001	Yes
x3	0.66	0.34	<0.001	Yes

Table 14: Completely Standardized Error Variances and Communality  
(continued)

Observed Variable	Error Variance <sup>1</sup>	Communality <sup>2,3</sup>	P-Value	Significant Error Variance? <sup>4</sup>
x4	0.27	0.73	<0.001	Yes
x5	0.27	0.73	<0.001	Yes
x6	0.30	0.70	<0.001	Yes
x7	0.67	0.33	<0.001	Yes
x8	0.48	0.52	<0.001	Yes
x9	0.56	0.44	<0.001	Yes

<sup>1</sup> Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).

<sup>2</sup> Corresponds to the squared factor loading.

<sup>3</sup> Can be interpreted as proportion of explained variance by the latent factor(s) (%).

<sup>4</sup> 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

### Factor Reliability

The table “Factor Reliability” contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretability of the reliability measures depend on the specific research context. Nevertheless, omega values below 0.6 or AVE values below 0.5 (at least one of these existent in your case) should be regarded with criticism. The factor reliability estimates are not further considered in the final summary.

### Final Summary

In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.

Considering the goodness-of-fit indices, the quality of the model is uncertain. Moreover, there exist localized areas of ill fit. Therefore, we assume that the model is non-acceptable. Please reconsider your data and the theory behind. Only if supported by theory, you could try to respecify the model and improve the goodness-of-fit by applying (one of the) following recommendations or call for actions.

### Addings to the Model

#### Observed Variables

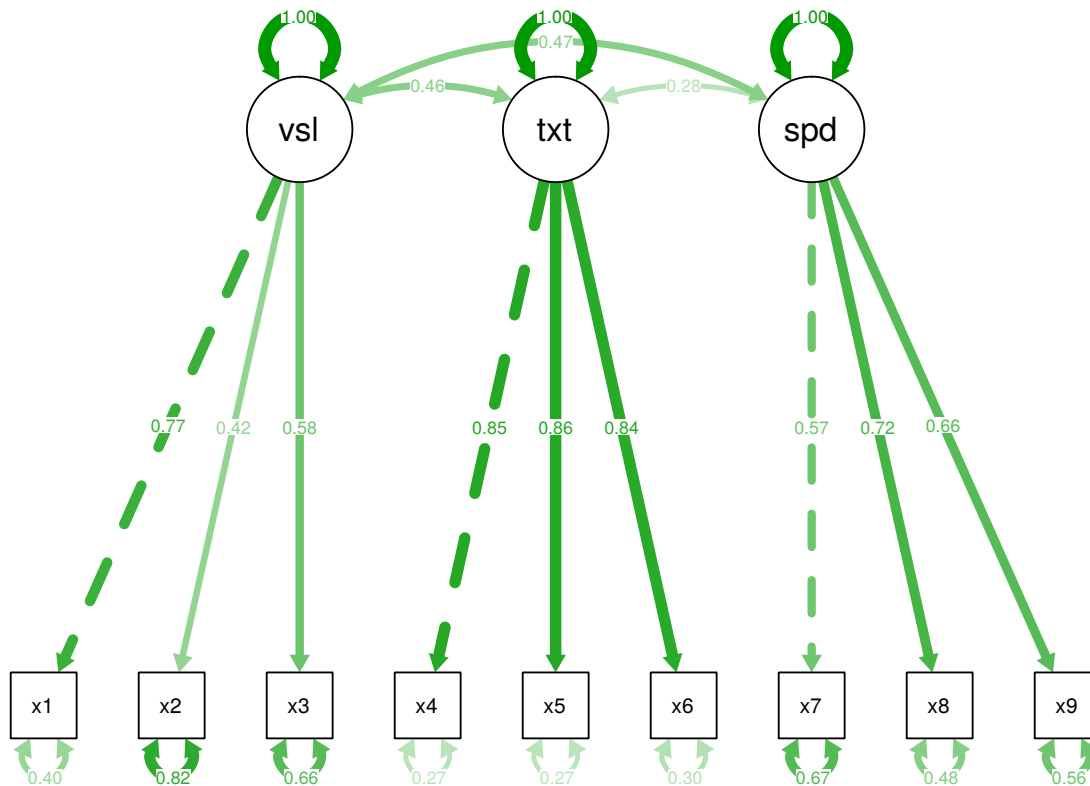
Update the model by adding to the model cross-loadings indicated within the table “Significant and Relevant MIs With Respect To Factor Loadings”.

Start updating the model by adding a parameter with the largest modification index (and EPC) and only if this parameter can be interpreted meaningfully. Other possible localized areas of misfit may be remedied by adding, i.e. freeing a single parameter.

## Final Comments

Please consider that this summary depends on hard-coded cutoff-values which may be too liberal or too conservative for your research area. If you have sound theory-based reasons and decide to respecify the model, we strongly recommend the replication of the CFA in an independent sample.

## Path Diagram



## APPENDIX

### Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

## References