Confirmatory Factor Analysis

Statsomat.com

07 January 2021

Warning: The automatic computation and interpretation delivered by the Statsomat should not completely replace the classical, made by humans graphical exploratory data analysis and statistical analysis. There may be data cases for which the Statsomat does not deliver the most optimal solution or output interpretation.

Basic Information

Automatic statistics for the file:

File
case4.csv

Your selection for the encoding: Auto
Your selection for the decimal character: Auto
Observations (rows with at least one non-missing value): 250
Variables (columns with at least one non-missing value): 12
Variables considered continuous: 12

Variables considered continuous
x1
x2
x3
x4
x5
x6
x7
x8
x9
x10
x11
x12

Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
Coping	=~	x1	1	0
Coping	=~	x2	1	1
Coping	=~	x3	1	2
Coping	=~	x4	1	3
Social	=~	x5	1	0
Social	=~	x6	1	4
Social	=~	x7	1	5
Social	=~	x8	1	6
Enhance	=~	x9	1	0
Enhance	=~	x10	1	7
Enhance	=~	x11	1	8
Enhance	=~	x12	1	9
x1	~~	x1	0	10
x2	~~	x2	0	11
x3	~~	x3	0	12
x4	~~	x4	0	13
x5	~~	x5	0	14
x6	~~	x6	0	15
x7	~~	x7	0	16
x8	~~	x8	0	17
x9	~~	x9	0	18
x10	~~	x10	0	19
x11	~~	x11	0	20
x12	~~	x12	0	21
Coping	~~	Coping	0	22
Social	~~	Social	0	23
Enhance	~~	Enhance	0	24
Coping	~~	Social	0	25
Coping	~~	Enhance	0	26
Social	~~	Enhance	0	27

Assumptions

Open issue

Model Settings

Outputs

Model Fit Summary

```
lavaan 0.6-7 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of free parameters                         27

  Number of observations                           250

Model Test User Model:

  Test statistic                                43.389
  Degrees of freedom                                51
  P-value (Chi-square)                           0.767

Model Test Baseline Model:

  Test statistic                               832.013
  Degrees of freedom                                66
  P-value                                        0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.013

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -5970.532
  Loglikelihood unrestricted model (H1)      -5948.837

  Akaike (AIC)                               11995.063
  Bayesian (BIC)                             12090.143
  Sample-size adjusted Bayesian (BIC)        12004.551

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent confidence interval - lower         0.000
  90 Percent confidence interval - upper         0.029
  P-value RMSEA <= 0.05                          0.999
```

```
Standardized Root Mean Square Residual:

  SRMR                                           0.037

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Coping =~
    x1                1.000
    x2                0.745    0.144    5.158    0.000
    x3                0.974    0.185    5.267    0.000
    x4                1.512    0.231    6.554    0.000
  Social =~
    x5                1.000
    x6                1.208    0.131    9.246    0.000
    x7                1.568    0.179    8.741    0.000
    x8                1.510    0.166    9.087    0.000
  Enhance =~
    x9                1.000
    x10               0.648    0.098    6.630    0.000
    x11               1.053    0.151    6.981    0.000
    x12               1.097    0.157    7.000    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  Coping ~~
    Social            0.775    0.156    4.952    0.000
    Enhance           0.460    0.140    3.281    0.001
  Social ~~
    Enhance           0.471    0.160    2.947    0.003

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .x1                3.439    0.316   10.868    0.000
   .x2                1.864    0.172   10.858    0.000
   .x3                2.924    0.270   10.819    0.000
   .x4                0.181    0.130    1.394    0.163
   .x5                1.786    0.183    9.764    0.000
   .x6                1.375    0.163    8.456    0.000
   .x7                3.236    0.350    9.244    0.000
   .x8                2.407    0.275    8.754    0.000
   .x9                4.561    0.517    8.823    0.000
   .x10               1.962    0.221    8.881    0.000
```

```
   .x11               3.707    0.467    7.939    0.000
   .x12               3.929    0.500    7.860    0.000
    Coping            0.787    0.232    3.395    0.001
    Social            1.195    0.231    5.167    0.000
    Enhance           2.592    0.586    4.421    0.000
```

Completely Standardized Parameter Estimates

```
Latent Variables:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  Coping =~
    x1               0.432    0.055    7.799    0.000    0.323    0.540
    x2               0.436    0.055    7.901    0.000    0.328    0.544
    x3               0.451    0.054    8.294    0.000    0.345    0.558
    x4               0.953    0.035   27.554    0.000    0.885    1.021
  Social =~
    x5               0.633    0.045   14.187    0.000    0.546    0.721
    x6               0.748    0.036   20.763    0.000    0.677    0.818
    x7               0.690    0.040   17.079    0.000    0.611    0.769
    x8               0.729    0.037   19.459    0.000    0.655    0.802
  Enhance =~
    x9               0.602    0.055   11.018    0.000    0.495    0.709
    x10              0.597    0.055   10.887    0.000    0.490    0.705
    x11              0.661    0.052   12.715    0.000    0.559    0.763
    x12              0.665    0.052   12.846    0.000    0.564    0.767

Covariances:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
  Coping ~~
    Social           0.799    0.043   18.490    0.000    0.714    0.883
    Enhance          0.322    0.072    4.480    0.000    0.181    0.463
  Social ~~
    Enhance          0.268    0.079    3.406    0.001    0.114    0.422

Variances:
                   est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
   .x1               0.814    0.048   17.031    0.000    0.720    0.907
   .x2               0.810    0.048   16.855    0.000    0.716    0.904
   .x3               0.796    0.049   16.220    0.000    0.700    0.893
   .x4               0.091    0.066    1.385    0.166   -0.038    0.221
   .x5               0.599    0.057   10.596    0.000    0.488    0.710
   .x6               0.441    0.054    8.183    0.000    0.335    0.546
   .x7               0.524    0.056    9.400    0.000    0.415    0.633
   .x8               0.469    0.055    8.592    0.000    0.362    0.576
   .x9               0.638    0.066    9.692    0.000    0.509    0.767
   .x10              0.643    0.066    9.811    0.000    0.515    0.772
   .x11              0.563    0.069    8.206    0.000    0.429    0.698
```

```
.x12                0.558    0.069    8.094    0.000    0.423    0.693
 Coping             1.000                               1.000    1.000
 Social             1.000                               1.000    1.000
 Enhance            1.000                               1.000    1.000
```

Communality

Table 4: Communality

Variable	Communality
x1	0.19
x10	0.36
x11	0.44
x12	0.44
x2	0.19
x3	0.20
x4	0.91
x5	0.40
x6	0.56
x7	0.48
x8	0.53
x9	0.36

Factor Discriminant Validity

Table 5: Factor Discriminant Validity Test at Cutoff 0.85

			Factor Correlation	Chisq diff	Df diff	P-Value
Coping	~~	Social	0.799	1.819	1	0.177
Coping	~~	Enhance	0.322	85.970	1	<0.001
Social	~~	Enhance	0.268	104.571	1	<0.001

Factor Reliability

Table 6: Factor Reliability

	Coping	Social	Enhance	total
Omega (Bentler)	0.63	0.79	0.73	0.83
Omega (McDonald)	0.59	0.79	0.73	0.83
AVE	0.31	0.49	0.41	0.41

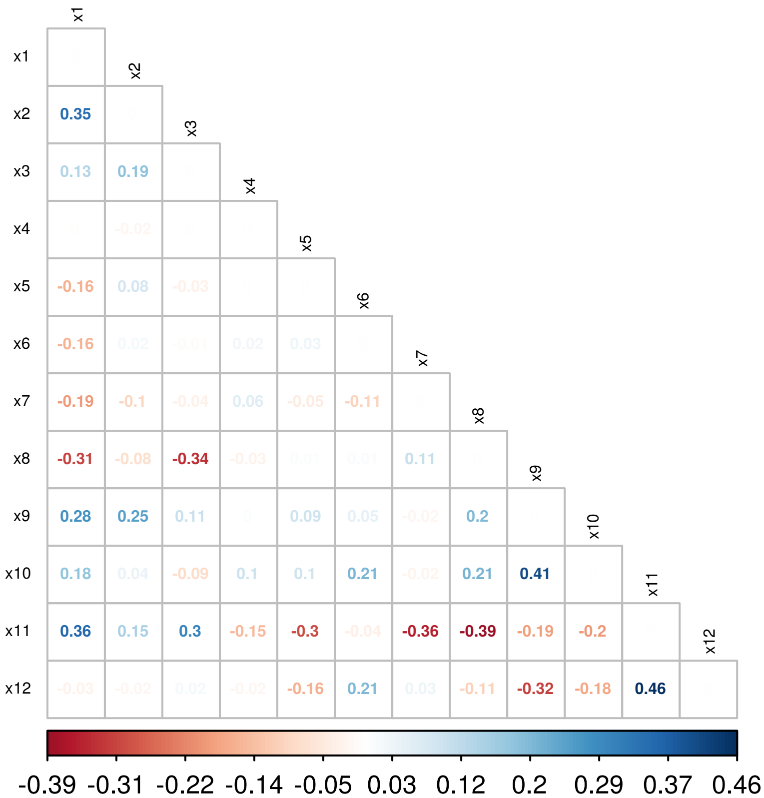
Observed Covariance Matrix

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12
x1	4.23											
x2	0.94	2.3										
x3	0.9	0.76	3.67									
x4	1.19	0.87	1.16	1.98								
x5	0.61	0.66	0.72	1.17	2.98							
x6	0.78	0.72	0.9	1.44	1.48	3.12						
x7	1.02	0.81	1.15	1.9	1.83	2.16	6.18					
x8	0.86	0.79	0.8	1.74	1.81	2.19	2.94	5.13				
x9	0.74	0.59	0.55	0.7	0.56	0.62	0.72	0.91	7.15			
x10	0.48	0.26	0.2	0.55	0.4	0.58	0.46	0.67	2.09	3.05		
x11	0.84	0.51	0.78	0.58	0.19	0.56	0.42	0.35	2.54	1.57	6.58	
x12	0.47	0.35	0.51	0.74	0.35	0.83	0.84	0.67	2.53	1.66	3.45	7.05

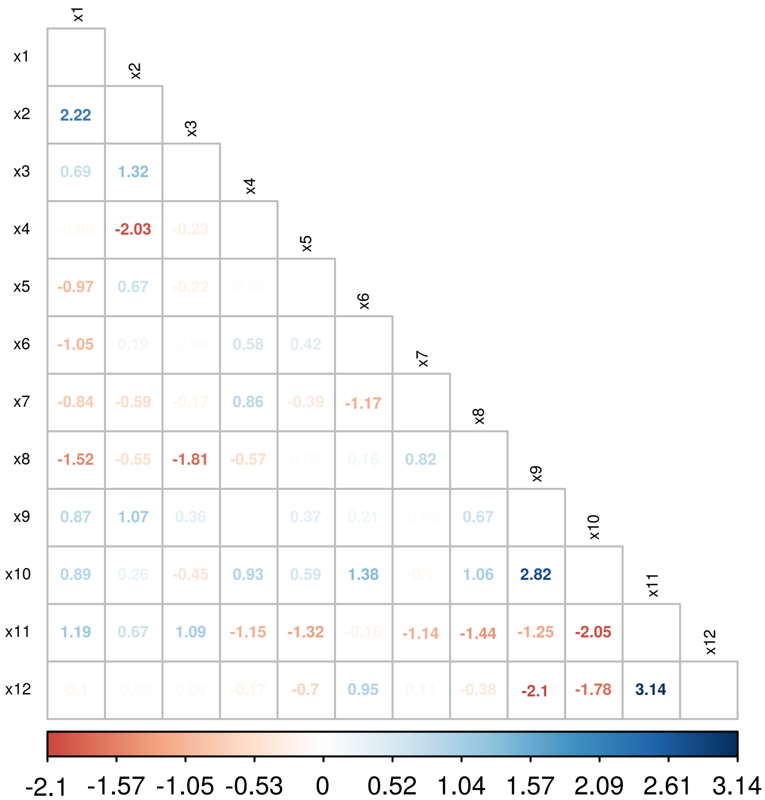
Model-Implied Covariance Matrix

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12
x1	4.23											
x2	0.59	2.3										
x3	0.77	0.57	3.67									
x4	1.19	0.89	1.16	1.98								
x5	0.77	0.58	0.76	1.17	2.98							
x6	0.94	0.7	0.91	1.42	1.44	3.12						
x7	1.22	0.91	1.18	1.84	1.87	2.26	6.18					
x8	1.17	0.87	1.14	1.77	1.8	2.18	2.83	5.13				
x9	0.46	0.34	0.45	0.7	0.47	0.57	0.74	0.71	7.15			
x10	0.3	0.22	0.29	0.45	0.31	0.37	0.48	0.46	1.68	3.05		
x11	0.48	0.36	0.47	0.73	0.5	0.6	0.78	0.75	2.73	1.77	6.58	
x12	0.5	0.38	0.49	0.76	0.52	0.62	0.81	0.78	2.84	1.84	2.99	7.05

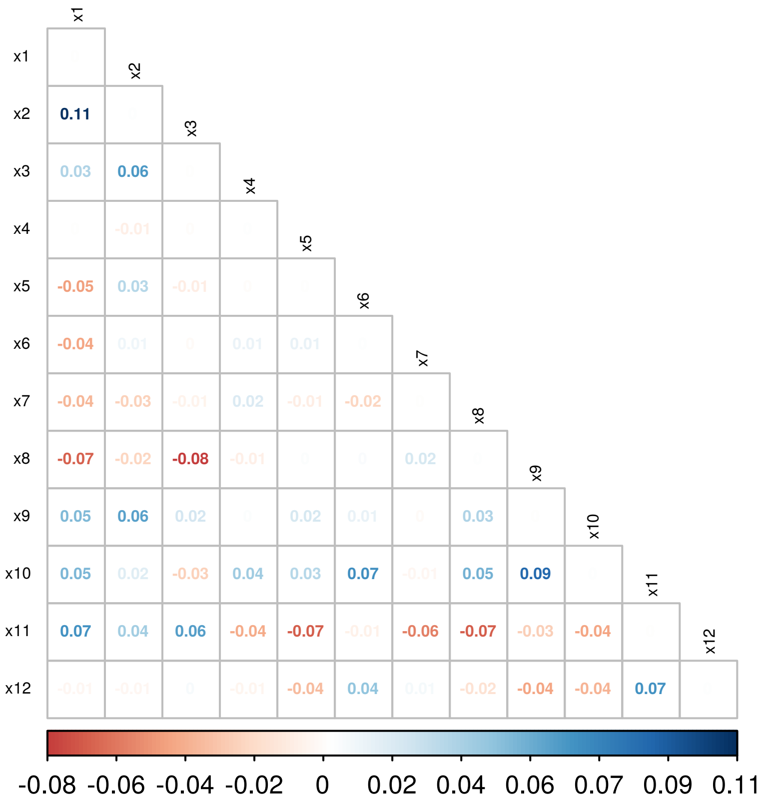
Residual Covariance Matrix



Standardized Residual Matrix



Residual Correlation Matrix



Modification Indices

Table 7: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
x11	~~	x12	13.474	1.708	0.1	0.055	**(m)**
x9	~~	x10	9.923	0.860	0.1	0.065	**(m)**
x1	~~	x2	5.149	0.378	0.1	0.092	**(m)**
x9	~~	x12	3.848	-0.874	0.1	0.056	**(m)**
x10	~~	x11	3.666	-0.532	0.1	0.065	(i)
x2	~~	x4	3.390	-0.228	0.1	0.127	(i)
x3	~~	x11	2.983	0.408	0.1	0.071	(i)
x10	~~	x12	2.786	-0.483	0.1	0.064	(i)

Note:
Maximum 10 modification indices in descending order of their magnitude are listed.

Table 8: Modification Indices With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
Social	=~	x4	9.458	1.30	0.4	0.157	**(m)**
Social	=~	x1	3.463	-0.52	0.4	0.299	(i)

Table 8: Modification Indices With Respect To Factor Loadings (continued)

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision

Note:
Maximum 10 modification indices in descending order of their magnitude are listed.

Interpretation

Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [@brown], [@kline].

Model Test User Model
The degrees of freedom are calculated as the number of known parameters minus the number of free parameters: 78 - 27 = 51 . The 51 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 43.389 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.05 then the test is not statistically significant at 5 % error, the hypothesis cannot be rejected, which would be in favour of the model.
In our case, the p-value is 0.767 suggesting an acceptable model fit.

Model Test Baseline Model
The test statistic with the value 832.013 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is <0.001 suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

Root Mean Square Error of Approximation:
The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.
In our case, the RMSEA is 0. The upper bound of the 90% confidence interval of the RMSEA is 0.029 and smaller than the threshold value 0.05, suggesting an excellent model fit.

Standardized Root Mean Square Residual:
The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.
In our case, the SRMR is 0.04 which is smaller than the threshold value 0.1 suggesting an acceptable model fit.

User Model versus Baseline Model
The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of departure from the baseline model).
In our case, the CFI is 1 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.
In our case, the TLI is 1.013 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Summary of the Goodness of Fit Indices

The TLI model fit index suggests an acceptable model fit. Moreover the Chi-square model fit index and the RMSEA suggest an acceptable model fit. We tentatively assume an acceptable model fit and verify this assertion by considering further metrics.

Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.
A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.
Following variable pairs have standardized residuals which are larger or equal than the considered threshold 2.58 [@brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably underestimated:

Table 9: Pair(s) with Underestimated Covariance

Pair 1	x1	x2
Pair 2	x11	x12
Pair 3	x12	x11

There are no variable pairs with standardized residuals which are smaller or equal than the considered threshold -2.58 [@brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [@kline]. Therefore, no relationships among the variables are substantially overestimated by the model. Depending on the sample size, the misspecification detected by the analysis of the residual covariance resp. correlation matrices can be statistically significant but not relevant and in practice negligible. This is matter of subject in the next section(s).

Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [@brown] and [@mi]. We cite from [@brown]: “The modification index reflects an approximation of how much the overall model Chi² will decrease if the fixed or constrained parameter is freely estimated.” In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.
We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized

in the decision column by the label “epc:m”. For more information with regard to the labels of the decision column, please consult the Appendix.
We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.
We remark that these conditions are not fulfilled for modification indices with respect to factor loadings. Therefore, there exist no significant and relevant modification indices with respect to factor loadings.
We remark that there exist no modification indices with respect to factor covariances.

Parameter Estimates

Factor Loadings

We remark that the completely standardized factor loadings (section “Completely Standardized Parameter Estimates”) are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [@brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 10: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading[1]	P-Value	Significant?[2]	Relevance[3]	Sign[4]	Check
Coping	x1	0.43	<0.001	Yes	*	—	Ok
Coping	x2	0.44	<0.001	Yes	*	—	Ok
Coping	x3	0.45	<0.001	Yes	*	—	Ok
Coping	x4	0.95	<0.001	Yes	***	—	Ok
Social	x5	0.63	<0.001	Yes	**	—	Ok
Social	x6	0.75	<0.001	Yes	**	—	Ok
Social	x7	0.69	<0.001	Yes	**	—	Ok
Social	x8	0.73	<0.001	Yes	**	—	Ok
Enhance	x9	0.60	<0.001	Yes	**	—	Ok
Enhance	x10	0.60	<0.001	Yes	*	—	Ok
Enhance	x11	0.66	<0.001	Yes	**	—	Ok
Enhance	x12	0.67	<0.001	Yes	**	—	Ok

[1] The completely standardized factor loading can be interpreted as the correlation with the factor.
[2] Completely standardized factor loading significance at 5% type I error.
[3] Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.
[4] No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.
[5] _____
[6] _____
[7] _____

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section “Model Fit Summary”, for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the “Model Fit Summary” section:

Table 11: Interpretation of Unstandardized Factor Loadings

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in Coping leads to a 1.00 -unit increase in the x1
A 1-unit increase in Coping leads to a 0.74 -unit increase in the x2
A 1-unit increase in Coping leads to a 0.97 -unit increase in the x3
A 1-unit increase in Coping leads to a 1.51 -unit increase in the x4
A 1-unit increase in Social leads to a 1.00 -unit increase in the x5
A 1-unit increase in Social leads to a 1.21 -unit increase in the x6
A 1-unit increase in Social leads to a 1.57 -unit increase in the x7
A 1-unit increase in Social leads to a 1.51 -unit increase in the x8
A 1-unit increase in Enhance leads to a 1.00 -unit increase in the x9
A 1-unit increase in Enhance leads to a 0.65 -unit increase in the x10
A 1-unit increase in Enhance leads to a 1.05 -unit increase in the x11
A 1-unit increase in Enhance leads to a 1.10 -unit increase in the x12

Factor Discriminant Validity

As noted by [@brown], "the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context." Still, large or statistically significant factor covariances resp. correlations are questionable and provide evidence of poor discriminant validity. There is evidence to question the distinctness of the following factor pair(s), since their correlation approaches in absolute value 1.0 and their test of discriminant validity is non-statistically significant at 5% type I error:

Table 12: Factor with Poor Discriminant Validity

				Factor Correlation	Chisq diff	Df diff	P-Value
Pair 1	Coping	~~	Social	0.799	1.819	1	0.177

Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 13: Completely Standardized Error Variances and Communality

Observed Variable	Error Variance[1]	Communality[23]	P-Value	Significant Error Variance?[4]
x1	0.81	0.19	<0.001	Yes
x10	0.64	0.36	<0.001	Yes
x11	0.56	0.44	<0.001	Yes
x12	0.56	0.44	<0.001	Yes
x2	0.81	0.19	<0.001	Yes
x3	0.80	0.20	<0.001	Yes
x4	0.09	0.91	0.166	No
x5	0.60	0.40	<0.001	Yes
x6	0.44	0.56	<0.001	Yes
x7	0.52	0.48	<0.001	Yes

Table 13: Completely Standardized Error Variances and Communality (continued)

Observed Variable	Error Variance[1]	Communality[23]	P-Value	Significant Error Variance?[4]
x8	0.47	0.53	<0.001	Yes
x9	0.64	0.36	<0.001	Yes

[1] Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).
[2] Corresponds to the squared factor loading.
[3] Can be interpreted as proportion of explained variance by the latent factor(s) (%).
[4] 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

Factor Reliability

The table “Factor Reliability” contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretatibility of the reliability measures depend on the specific research context. Nevertheless, omega values below 0.6 or AVE values below 0.5 (at least one of these existent in your case) should be regarded with criticism. The factor reliability estimates are not further considered in the final summary.

Final Summary

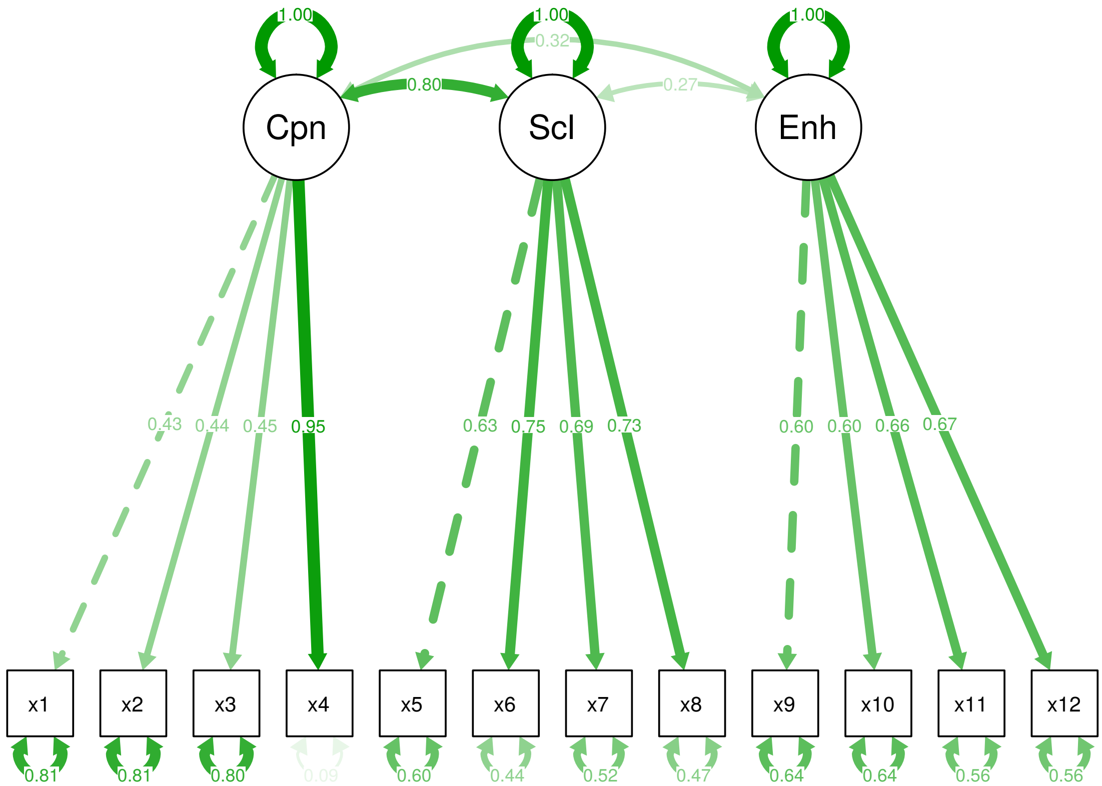
In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.
Considering the goodness-of-fit indices, the model is acceptable. Nevertheless, there exist localized areas of ill fit. Therefore, the quality of the model is uncertain. Only if supported by theory, you could try to respecify the model and improve the goodness-of-fit by applying (one of the) following recommendations or call for actions.

Eliminations from the Model

Factors*
Only if supported by theory, you could try to collapse the factors which were identified as probably non-distinguishable.

Final comments: Please consider that this summary depends on hard-coded cutoff-values which may be too liberal or too conservative for your research area. If you have sound theory-based reasons and decide to respicify the model, we strongly recommend the replication of the CFA in an independent sample.

Path Diagram



APPENDIX

Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

References