

# Confirmatory Factor Analysis

Statsomat.com

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## Basic Information

Automatic statistics for the file:

File
neuroticism.csv

Your selection for the encoding: Auto

Your selection for the decimal character: Auto

Observations (rows with at least one non-missing value): 250

Variables (columns with at least one non-missing value): 8

Variables considered continuous: 8

Variables considered continuous
N1
N2
N3
N4
E1
E2
E3
E4

## Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

Left hand side	Operator	Right hand side	User	Free parameter
NEUROT	=~	N1	1	0
NEUROT	=~	N2	1	1
NEUROT	=~	N3	1	2
NEUROT	=~	N4	1	3
EXTRAV	=~	E1	1	0
EXTRAV	=~	E2	1	4
EXTRAV	=~	E3	1	5
EXTRAV	=~	E4	1	6
N1	~~	N1	0	7
N2	~~	N2	0	8
N3	~~	N3	0	9
N4	~~	N4	0	10
E1	~~	E1	0	11
E2	~~	E2	0	12
E3	~~	E3	0	13
E4	~~	E4	0	14
NEUROT	~~	NEUROT	0	15
EXTRAV	~~	EXTRAV	0	16
NEUROT	~~	EXTRAV	0	17

## Assumptions

Open issue

## Model Settings

## Outputs

### Model Fit Summary

lavaan 0.6-7 ended normally after 59 iterations

Estimator	ML
Optimization method	NLMINB
Number of free parameters	17
Number of observations	250

Model Test User Model:

Test statistic	13.285
Degrees of freedom	19
P-value (Chi-square)	0.824

Model Test Baseline Model:

Test statistic	1253.791
Degrees of freedom	28
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.007

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-5748.501
Loglikelihood unrestricted model (H1)	-5741.858
Akaike (AIC)	11531.002
Bayesian (BIC)	11590.866
Sample-size adjusted Bayesian (BIC)	11536.975

Root Mean Square Error of Approximation:

RMSEA	0.000
90 Percent confidence interval - lower	0.000
90 Percent confidence interval - upper	0.034
P-value RMSEA <= 0.05	0.990

Standardized Root Mean Square Residual:

SRMR	0.019
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Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
NEUROT =~				
N1	1.000			

N2	0.942	0.052	17.981	0.000
N3	1.071	0.060	17.791	0.000
N4	0.997	0.052	19.312	0.000
EXTRAV =~				
E1	1.000			
E2	1.074	0.079	13.636	0.000
E3	0.935	0.072	12.927	0.000
E4	0.814	0.072	11.247	0.000

Covariances:

	Estimate	Std.Err	z-value	P(> z )
NEUROT ~~				
EXTRAV	-10.512	1.920	-5.476	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.N1	7.025	0.907	7.746	0.000
.N2	8.746	0.999	8.755	0.000
.N3	11.760	1.328	8.855	0.000
.N4	7.188	0.916	7.846	0.000
.E1	12.802	1.581	8.100	0.000
.E2	11.671	1.602	7.285	0.000
.E3	12.192	1.458	8.361	0.000
.E4	15.972	1.666	9.587	0.000
NEUROT	25.335	2.900	8.736	0.000
EXTRAV	23.054	3.188	7.232	0.000

**Completely Standardized Parameter Estimates**

Latent Variables:

	est.std	Std.Err	z-value	P(> z )	ci.lower	ci.upper
NEUROT =~						
N1	0.885	0.018	49.547	0.000	0.850	0.920
N2	0.849	0.021	39.953	0.000	0.807	0.890
N3	0.844	0.022	38.868	0.000	0.801	0.886
N4	0.882	0.018	48.691	0.000	0.846	0.917
EXTRAV =~						
E1	0.802	0.029	27.203	0.000	0.744	0.860
E2	0.834	0.027	30.772	0.000	0.781	0.887
E3	0.789	0.030	25.931	0.000	0.730	0.849
E4	0.699	0.038	18.478	0.000	0.625	0.773

Covariances:

	est.std	Std.Err	z-value	P(> z )	ci.lower	ci.upper
NEUROT ~~						
EXTRAV	-0.435	0.059	-7.410	0.000	-0.550	-0.320

### Variances:

	est.std	Std.Err	z-value	P(> z )	ci.lower	ci.upper
.N1	0.217	0.032	6.869	0.000	0.155	0.279
.N2	0.280	0.036	7.770	0.000	0.209	0.351
.N3	0.288	0.037	7.871	0.000	0.216	0.360
.N4	0.222	0.032	6.952	0.000	0.159	0.285
.E1	0.357	0.047	7.553	0.000	0.264	0.450
.E2	0.305	0.045	6.747	0.000	0.216	0.393
.E3	0.377	0.048	7.838	0.000	0.283	0.471
.E4	0.511	0.053	9.668	0.000	0.408	0.615
NEUROT	1.000				1.000	1.000
EXTRAV	1.000				1.000	1.000

### Communality

Table 4: Communality

Variable	Communality
E1	0.64
E2	0.70
E3	0.62
E4	0.49
N1	0.78
N2	0.72
N3	0.71
N4	0.78

### Factor Discriminant Validity

Table 5: Factor Discriminant Validity Test at Cutoff 0.85

			Factor Correlation	Chisq diff	Df diff	P-Value
NEUROT	~~	EXTRAV	-0.435	109.647	1	<0.001

### Factor Reliability

Table 6: Factor Reliability

	NEUROT	EXTRAV	total
Omega (Bentler)	0.92	0.86	0.83
Omega (McDonald)	0.92	0.87	0.83
AVE	0.75	0.62	0.68

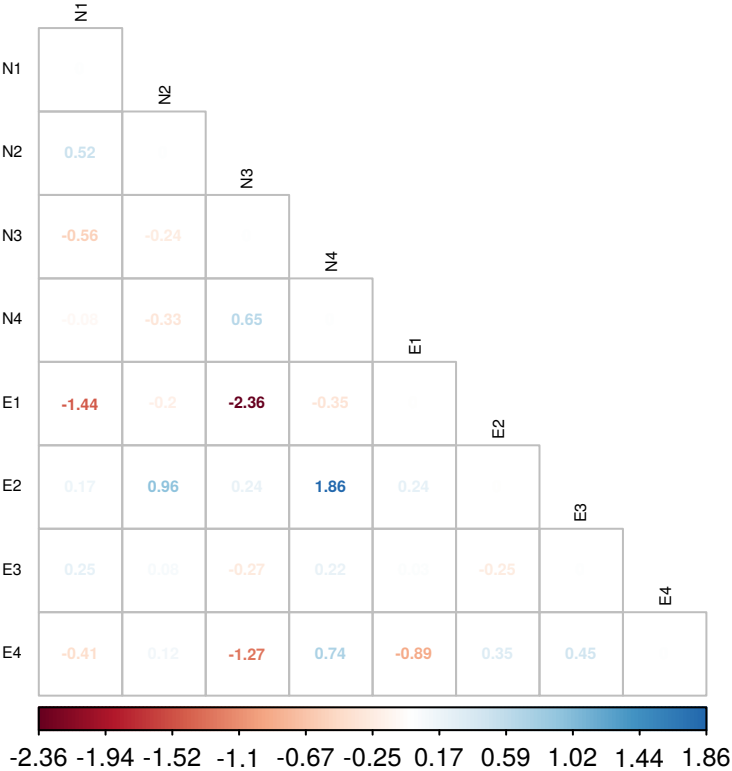
## Observed Covariance Matrix

N1	N1							
	32.36							
N2		N2						
	24.38	31.23						
N3			N3					
	26.56	25.31	40.8					
N4				N4				
	25.18	23.46	27.69	32.36				
E1					E1			
	-11.96	-10.11	-13.62	-10.83	35.86			
E2						E2		
	-11.12	-9.68	-11.86	-9.4	25.01	38.29		
E3							E3	
	-9.58	-9.19	-10.79	-9.58	21.6	22.91	32.36	
E4								E4
	-8.97	-7.93	-10.42	-7.79	17.87	20.51	17.99	31.23

## Model-Implied Covariance Matrix

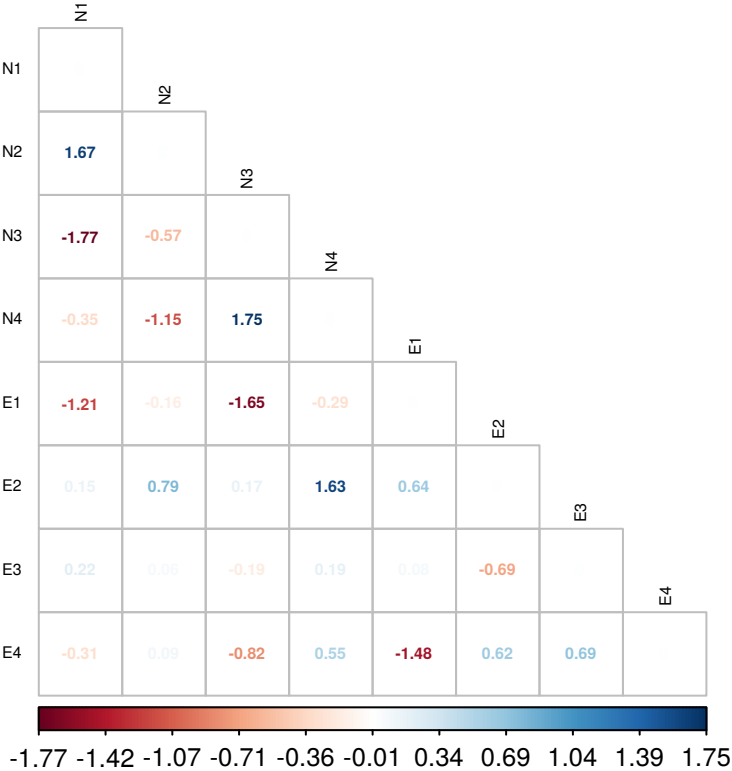
N1	32.36						
N2	23.87	31.23					
N3	27.12	25.55	40.8				
N4	25.25	23.79	27.04	32.36			
E1	-10.51	-9.9	-11.25	-10.48	35.86		
E2	-11.29	-10.64	-12.09	-11.26	24.77	38.29	
E3	-9.83	-9.26	-10.53	-9.8	21.56	23.17	32.36
E4	-8.55	-8.06	-9.16	-8.53	18.76	20.15	17.54
							31.23

Residual Covariance Matrix

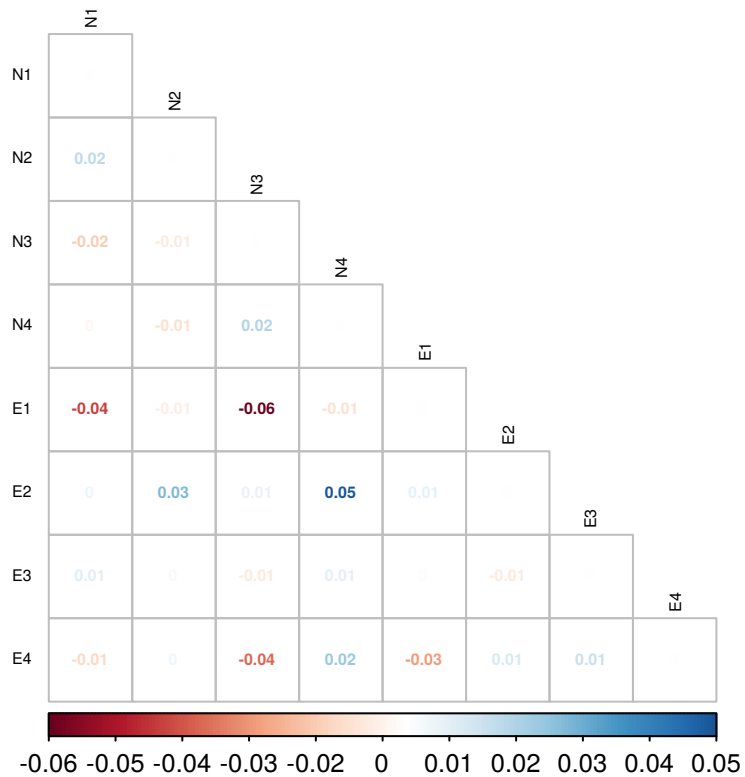




Standardized Residual Matrix



## Residual Correlation Matrix



## Modification Indices

Table 7: Modification Indices With Respect To Error Covariances

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
N3	~~	N4	3.507	1.820	0.1	0.051	(i)
N1	~~	N2	3.227	1.538	0.1	0.052	(i)
N1	~~	N3	2.747	-1.615	0.1	0.051	(i)
E1	~~	E4	1.983	-1.827	0.1	0.051	(i)
N4	~~	E2	1.276	0.906	0.1	0.052	(i)
N2	~~	N4	1.253	-0.956	0.1	0.052	(i)
N3	~~	E1	1.101	-1.028	0.1	0.051	(i)

Note:

Maximum 10 modification indices in descending order of their magnitude are listed.

Table 8: Modification Indices With Respect To Factor Loadings

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
NEUROT	=~	E1	1.328	-0.073	0.4	1	(nm)
NEUROT	=~	E2	1.121	0.068	0.4	1	(nm)
EXTRAV	=~	N4	1.067	0.053	0.4	1	(nm)

Note:

Table 8: Modification Indices With Respect To Factor Loadings (continued)

Left	Operator	Right	Modification Index	Expected Parameter Change	Delta	Power	Decision
Maximum 10 modification indices in descending order of their magnitude are listed.							

## Interpretation

### Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [Brown], [Kline].

#### Model Test User Model

The degrees of freedom are calculated as the number of known parameters minus the number of free parameters:  $36 - 17 = 19$ . The 19 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 13.285 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.05 then the test is not statistically significant at 5 % error, the hypothesis cannot be rejected, which would be in favour of the model.

In our case, the p-value is 0.824 suggesting an acceptable model fit.

#### Model Test Baseline Model

The test statistic with the value 1253.791 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is  $<0.001$  suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

#### Root Mean Square Error of Approximation:

The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.

In our case, the RMSEA is 0. The upper bound of the 90% confidence interval of the RMSEA is 0.034 and smaller than the threshold value 0.05, suggesting an excellent model fit.

#### Standardized Root Mean Square Residual:

The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.

In our case, the SRMR is 0.02 which is smaller than the threshold value 0.1 suggesting an acceptable model fit.

#### User Model versus Baseline Model

The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of departure from the baseline model).

In our case, the CFI is 1 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.

In our case, the TLI is 1.007 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

## Summary of the Goodness of Fit Indices

The TLI model fit index suggests an acceptable model fit. Moreover the Chi-square model fit index and the RMSEA suggest an acceptable model fit. We tentatively assume an acceptable model fit and verify this assertion by considering further metrics.

## Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.

A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.

Following variable pairs have standardized residuals which are larger or equal than the considered threshold 2.58 [brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [kline]. In these cases, the covariance relationship between the involved variables is probably underestimated:

Table 9: Pair(s) with Underestimated Covariance

Pair 1	E4	E4
--------	----	----

There are no variable pairs with standardized residuals which are smaller or equal than the considered threshold -2.58 [brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [kline]. Therefore, no relationships among the variables are substantially overestimated by the model. Depending on the sample size, the misspecification detected by the analysis of the residual covariance resp. correlation matrices can be statistically significant but not relevant and in practice negligible. This is matter of subject in the next section(s).

## Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [brown] and [mi]. We cite from [brown]: “The modification index reflects an approximation of how much the overall model  $\chi^2$  will decrease if the fixed or constrained parameter is freely estimated.” In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.

We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label “epc:m”. For more information with regard to the labels of the decision column, please consult the Appendix.

We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.

We remark that these conditions are not fulfilled for modification indices with respect to factor loadings. Therefore, there exist no significant and relevant modification indices with respect to factor loadings.

We remark that there exist no modification indices with respect to factor covariances.

## Parameter Estimates

### Factor Loadings

We remark that the completely standardized factor loadings (section “Completely Standardized Parameter Estimates”) are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [Brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 10: Check Completely Standardized Factor Loadings

Latent Variable	Observed Variable	Loading <sup>1</sup>	P-Value	Significant? <sup>2</sup>	Relevance <sup>3</sup>	Sign <sup>4</sup>	Check
NEUROT	N1	0.88	<0.001	Yes	***	—	Ok
NEUROT	N2	0.85	<0.001	Yes	***	—	Ok
NEUROT	N3	0.84	<0.001	Yes	***	—	Ok
NEUROT	N4	0.88	<0.001	Yes	***	—	Ok
EXTRAV	E1	0.80	<0.001	Yes	***	—	Ok
EXTRAV	E2	0.83	<0.001	Yes	***	—	Ok
EXTRAV	E3	0.79	<0.001	Yes	**	—	Ok
EXTRAV	E4	0.70	<0.001	Yes	**	—	Ok

<sup>1</sup> The completely standardized factor loading can be interpreted as the correlation with the factor.

<sup>2</sup> Completely standardized factor loading significance at 5% type I error.

<sup>3</sup> Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.

<sup>4</sup> No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.

<sup>5</sup> \_\_\_\_\_

<sup>6</sup> \_\_\_\_\_

<sup>7</sup> \_\_\_\_\_

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section “Model Fit Summary”, for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the “Model Fit Summary” section:

Table 11: Interpretation of Unstandardized Factor Loadings

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in NEUROT leads to a 1.00 -unit increase in the N1
A 1-unit increase in NEUROT leads to a 0.94 -unit increase in the N2
A 1-unit increase in NEUROT leads to a 1.07 -unit increase in the N3
A 1-unit increase in NEUROT leads to a 1.00 -unit increase in the N4
A 1-unit increase in EXTRAV leads to a 1.00 -unit increase in the E1

Table 11: Interpretation of Unstandardized Factor Loadings (continued)

Interpretation of Unstandardized Factor Loadings
A 1-unit increase in EXTRAV leads to a 1.07 -unit increase in the E2
A 1-unit increase in EXTRAV leads to a 0.94 -unit increase in the E3
A 1-unit increase in EXTRAV leads to a 0.81 -unit increase in the E4

### Factor Discriminant Validity

As noted by [Brown], “the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context.” Though, the largest estimated factor intercorrelation within the section “Completely Standardized Parameter Estimates” is -0.43 which we regard as a proof of a reasonable discriminant validity. Moreover, the statistical test(s) for factor discriminant validity are statistically significant at 5% type I error.

### Error Variances

We summarize the interpretation of the error variances and communalities in the next table:

Table 12: Completely Standardized Error Variances and Communality

Observed Variable	Error Variance <sup>1</sup>	Communality <sup>23</sup>	P-Value	Significant Error Variance? <sup>4</sup>
E1	0.36	0.64	<0.001	Yes
E2	0.30	0.70	<0.001	Yes
E3	0.38	0.62	<0.001	Yes
E4	0.51	0.49	<0.001	Yes
N1	0.22	0.78	<0.001	Yes
N2	0.28	0.72	<0.001	Yes
N3	0.29	0.71	<0.001	Yes
N4	0.22	0.78	<0.001	Yes

<sup>1</sup> Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).

<sup>2</sup> Corresponds to the squared factor loading.

<sup>3</sup> Can be interpreted as proportion of explained variance by the latent factor(s) (%).

<sup>4</sup> 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

### Factor Reliability

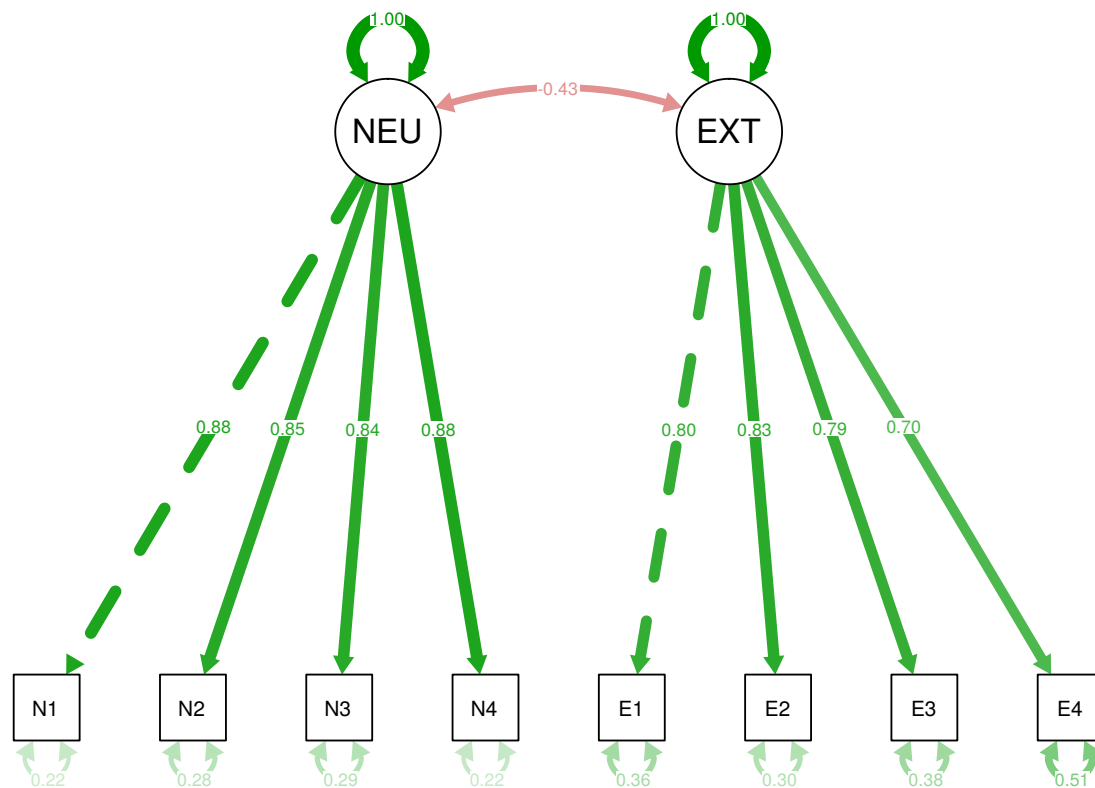
The table “Factor Reliability” contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretability of the reliability measures depend on the specific research context. In some fields of research, omega values greater or equal than 0.6 and AVE values greater or equal than 0.5 (fulfilled by and large in your case) could be sufficient.

## Final Summary

In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.

Considering the goodness-of-fit indices, the model is acceptable. Moreover, we cannot identify localized areas of ill fit. We conclude that the model is acceptable.

## Path Diagram



## APPENDIX

### Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

## References