# Confirmatory Factor Analysis

Statsomat.com

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**Warning: The automatic computation and interpretation delivered by the Statsomat should not completely replace the classical, made by humans graphical exploratory data analysis and statistical analysis. There may be data cases for which the Statsomat does not deliver the most optimal solution or output interpretation.**

## Basic Information

Automatic statistics for the file:

| File |
| --- |
| neuroticism.csv |

Your selection for the encoding: Auto
Your selection for the decimal character: Auto
Observations (rows with at least one non-missing value): 250
Variables (columns with at least one non-missing value): 8
Variables considered continuous: 8

| Variables considered continuous |
| --- |
| N1 |
| N2 |
| N3 |
| N4 |
| E1 |
| E2 |
| E3 |
| E4 |

## Model Syntax

The following table describes the applied model equations in lavaan model syntax, either as entered by you in the text area (denoted by User=1) or established internally (User=0). The last column numbers the free parameters which are estimated.

| Left hand side | Operator | Right hand side | User | Free parameter |
|---|---|---|---|---|
| NEUROT | =~ | N1 | 1 | 0 |
| NEUROT | =~ | N2 | 1 | 1 |
| NEUROT | =~ | N3 | 1 | 2 |
| NEUROT | =~ | N4 | 1 | 3 |
| EXTRAV | =~ | E1 | 1 | 0 |
| EXTRAV | =~ | E2 | 1 | 4 |
| EXTRAV | =~ | E3 | 1 | 5 |
| EXTRAV | =~ | E4 | 1 | 6 |
| N1 | ~~ | N1 | 0 | 7 |
| N2 | ~~ | N2 | 0 | 8 |
| N3 | ~~ | N3 | 0 | 9 |
| N4 | ~~ | N4 | 0 | 10 |
| E1 | ~~ | E1 | 0 | 11 |
| E2 | ~~ | E2 | 0 | 12 |
| E3 | ~~ | E3 | 0 | 13 |
| E4 | ~~ | E4 | 0 | 14 |
| NEUROT | ~~ | NEUROT | 0 | 15 |
| EXTRAV | ~~ | EXTRAV | 0 | 16 |
| NEUROT | ~~ | EXTRAV | 0 | 17 |

## Assumptions

Open issue

## Model Settings

## Outputs

### Model Fit Summary

```
lavaan 0.6-7 ended normally after 59 iterations

  Estimator                                   ML
  Optimization method                     NLMINB
  Number of free parameters                   17

  Number of observations                     250
```

```
Model Test User Model:

  Test statistic                                        13.285
  Degrees of freedom                                        19
  P-value (Chi-square)                                   0.824

Model Test Baseline Model:

  Test statistic                                      1253.791
  Degrees of freedom                                        28
  P-value                                                0.000

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                            1.000
  Tucker-Lewis Index (TLI)                               1.007

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)                      -5748.501
  Loglikelihood unrestricted model (H1)              -5741.858

  Akaike (AIC)                                       11531.002
  Bayesian (BIC)                                     11590.866
  Sample-size adjusted Bayesian (BIC)                11536.975

Root Mean Square Error of Approximation:

  RMSEA                                                  0.000
  90 Percent confidence interval - lower                 0.000
  90 Percent confidence interval - upper                 0.034
  P-value RMSEA <= 0.05                                  0.990

Standardized Root Mean Square Residual:

  SRMR                                                   0.019

Parameter Estimates:

  Standard errors                                     Standard
  Information                                         Expected
  Information saturated (h1) model                  Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  NEUROT =~
    N1                1.000
```

```
       N2                0.942    0.052   17.981    0.000
       N3                1.071    0.060   17.791    0.000
       N4                0.997    0.052   19.312    0.000
     EXTRAV =~
       E1                1.000
       E2                1.074    0.079   13.636    0.000
       E3                0.935    0.072   12.927    0.000
       E4                0.814    0.072   11.247    0.000


Covariances:
                     Estimate  Std.Err  z-value  P(>|z|)
     NEUROT ~~
       EXTRAV         -10.512    1.920   -5.476    0.000


Variances:
                     Estimate  Std.Err  z-value  P(>|z|)
      .N1               7.025    0.907    7.746    0.000
      .N2               8.746    0.999    8.755    0.000
      .N3              11.760    1.328    8.855    0.000
      .N4               7.188    0.916    7.846    0.000
      .E1              12.802    1.581    8.100    0.000
      .E2              11.671    1.602    7.285    0.000
      .E3              12.192    1.458    8.361    0.000
      .E4              15.972    1.666    9.587    0.000
       NEUROT          25.335    2.900    8.736    0.000
       EXTRAV          23.054    3.188    7.232    0.000
```

**Completely Standardized Parameter Estimates**

```
Latent Variables:
                      est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
     NEUROT =~
       N1                0.885    0.018   49.547    0.000    0.850    0.920
       N2                0.849    0.021   39.953    0.000    0.807    0.890
       N3                0.844    0.022   38.868    0.000    0.801    0.886
       N4                0.882    0.018   48.691    0.000    0.846    0.917
     EXTRAV =~
       E1                0.802    0.029   27.203    0.000    0.744    0.860
       E2                0.834    0.027   30.772    0.000    0.781    0.887
       E3                0.789    0.030   25.931    0.000    0.730    0.849
       E4                0.699    0.038   18.478    0.000    0.625    0.773


Covariances:
                      est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
     NEUROT ~~
       EXTRAV         -0.435    0.059   -7.410    0.000   -0.550   -0.320
```

```
Variances:
                est.std  Std.Err  z-value  P(>|z|) ci.lower ci.upper
    .N1           0.217    0.032    6.869    0.000    0.155    0.279
    .N2           0.280    0.036    7.770    0.000    0.209    0.351
    .N3           0.288    0.037    7.871    0.000    0.216    0.360
    .N4           0.222    0.032    6.952    0.000    0.159    0.285
    .E1           0.357    0.047    7.553    0.000    0.264    0.450
    .E2           0.305    0.045    6.747    0.000    0.216    0.393
    .E3           0.377    0.048    7.838    0.000    0.283    0.471
    .E4           0.511    0.053    9.668    0.000    0.408    0.615
    NEUROT        1.000                               1.000    1.000
    EXTRAV        1.000                               1.000    1.000
```

## Communality

Table 4: Communality

| Variable | Communality |
|----------|-------------|
| E1 | 0.64 |
| E2 | 0.70 |
| E3 | 0.62 |
| E4 | 0.49 |
| N1 | 0.78 |
| N2 | 0.72 |
| N3 | 0.71 |
| N4 | 0.78 |

## Factor Discriminant Validity

Table 5: Factor Discriminant Validity Test at Cutoff 0.85

| | | | Factor Correlation | Chisq diff | Df diff | P-Value |
|--------|----|--------|--------------------|------------|---------|---------|
| NEUROT | ~~ | EXTRAV | -0.435 | 109.647 | 1 | <0.001 |

## Factor Reliability

Table 6: Factor Reliability

| | NEUROT | EXTRAV | total |
|-----------------|--------|--------|-------|
| Omega (Bentler) | 0.92 | 0.86 | 0.83 |
| Omega (McDonald) | 0.92 | 0.87 | 0.83 |
| AVE | 0.75 | 0.62 | 0.68 |

## Observed Covariance Matrix

|       | N1     | N2     | N3     | N4     | E1    | E2    | E3    | E4    |
|-------|--------|--------|--------|--------|-------|-------|-------|-------|
| N1    | 32.36  |        |        |        |       |       |       |       |
| N2    | 24.38  | 31.23  |        |        |       |       |       |       |
| N3    | 26.56  | 25.31  | 40.8   |        |       |       |       |       |
| N4    | 25.18  | 23.46  | 27.69  | 32.36  |       |       |       |       |
| E1    | -11.96 | -10.11 | -13.62 | -10.83 | 35.86 |       |       |       |
| E2    | -11.12 | -9.68  | -11.86 | -9.4   | 25.01 | 38.29 |       |       |
| E3    | -9.58  | -9.19  | -10.79 | -9.58  | 21.6  | 22.91 | 32.36 |       |
| E4    | -8.97  | -7.93  | -10.42 | -7.79  | 17.87 | 20.51 | 17.99 | 31.23 |

## Model-Implied Covariance Matrix

|      | N1 | N2 | N3 | N4 | E1 | E2 | E3 | E4 |
|------|-------|-------|--------|-------|-------|-------|-------|-------|
| **N1** | 32.36 | | | | | | | |
| **N2** | 23.87 | 31.23 | | | | | | |
| **N3** | 27.12 | 25.55 | 40.8 | | | | | |
| **N4** | 25.25 | 23.79 | 27.04 | 32.36 | | | | |
| **E1** | -10.51 | -9.9 | -11.25 | -10.48 | 35.86 | | | |
| **E2** | -11.29 | -10.64 | -12.09 | -11.26 | 24.77 | 38.29 | | |
| **E3** | -9.83 | -9.26 | -10.53 | -9.8 | 21.56 | 23.17 | 32.36 | |
| **E4** | -8.55 | -8.06 | -9.16 | -8.53 | 18.76 | 20.15 | 17.54 | 31.23 |

## Residual Covariance Matrix

|      | N1    | N2    | N3    | N4    | E1    | E2    | E3    | E4 |
|------|-------|-------|-------|-------|-------|-------|-------|----|
| N1   |       |       |       |       |       |       |       |    |
| N2   | 0.52  |       |       |       |       |       |       |    |
| N3   | -0.56 | -0.24 |       |       |       |       |       |    |
| N4   | -0.08 | -0.33 | 0.65  |       |       |       |       |    |
| E1   | -1.44 | -0.2  | -2.36 | -0.35 |       |       |       |    |
| E2   | 0.17  | 0.96  | 0.24  | 1.86  | 0.24  |       |       |    |
| E3   | 0.25  | 0.08  | -0.27 | 0.22  | 0.10  | -0.25 |       |    |
| E4   | -0.41 | 0.12  | -1.27 | 0.74  | -0.89 | 0.35  | 0.45  |    |

-2.36  -1.94  -1.52  -1.1  -0.67  -0.25  0.17  0.59  1.02  1.44  1.86

**Standardized Residual Matrix**

|     | N1    | N2    | N3    | N4    | E1    | E2    | E3    | E4  |
|-----|-------|-------|-------|-------|-------|-------|-------|-----|
| N1  |       |       |       |       |       |       |       |     |
| N2  | 1.67  |       |       |       |       |       |       |     |
| N3  | -1.77 | -0.57 |       |       |       |       |       |     |
| N4  | -0.35 | -1.15 | 1.75  |       |       |       |       |     |
| E1  | -1.21 | -0.16 | -1.65 | -0.29 |       |       |       |     |
| E2  | 0.15  | 0.79  | 0.17  | 1.63  | 0.64  |       |       |     |
| E3  | 0.22  | 0.06  | -0.19 | 0.19  | 0.03  | -0.69 |       |     |
| E4  | -0.31 | 0.09  | -0.82 | 0.55  | -1.48 | 0.62  | 0.69  |     |

-1.77 -1.42 -1.07 -0.71 -0.36 -0.01 0.34 0.69 1.04 1.39 1.75
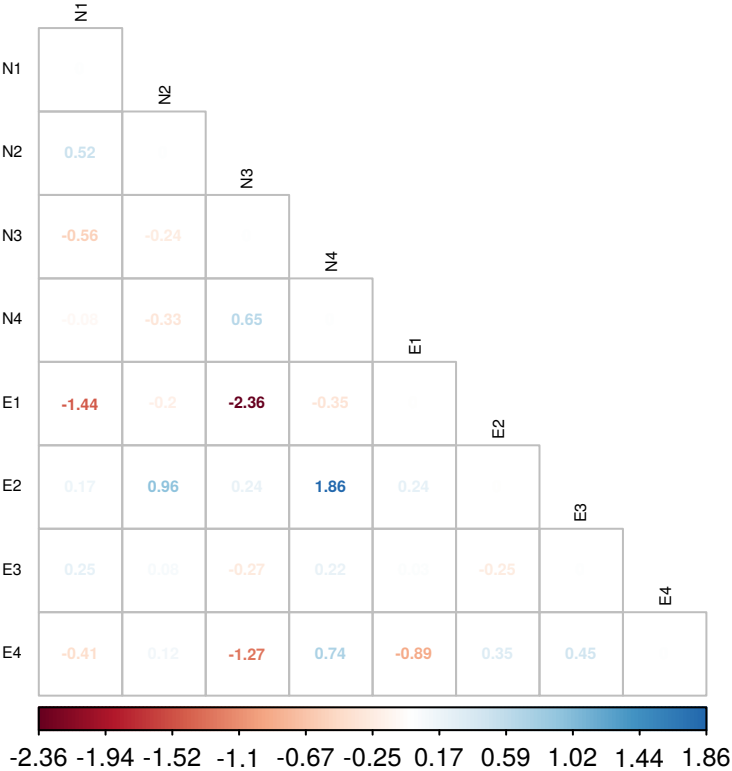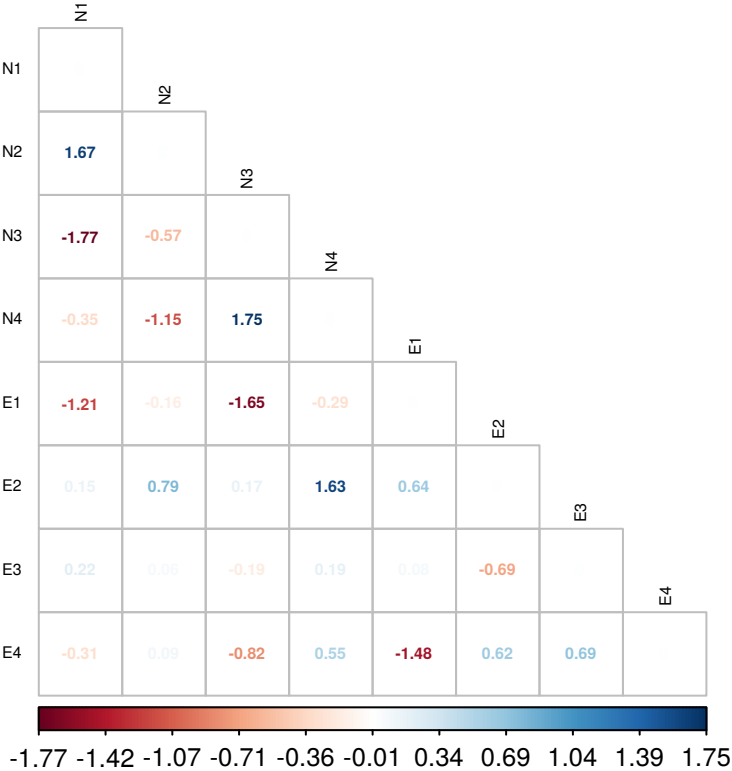
**Residual Correlation Matrix**
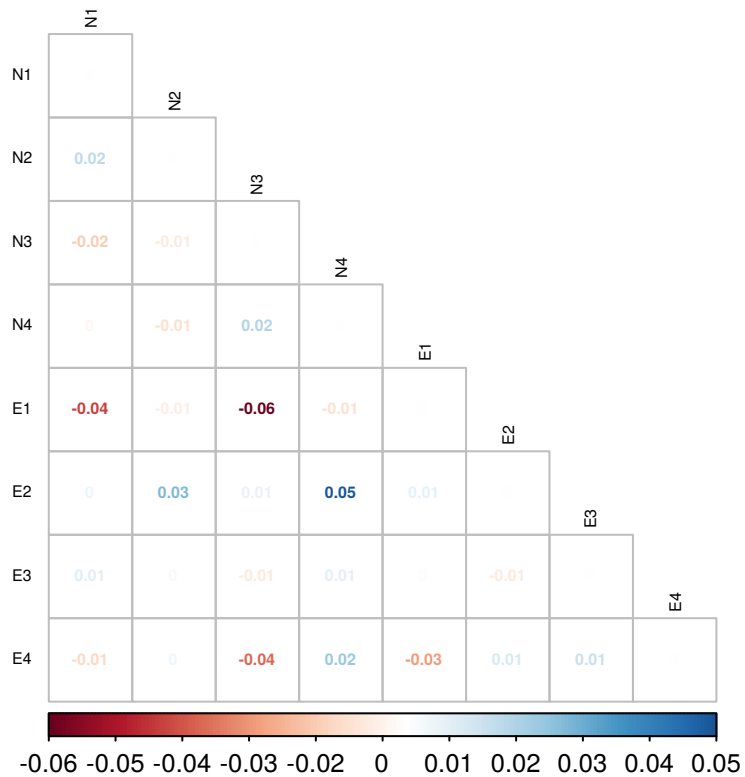


**Modification Indices**

Table 7: Modification Indices With Respect To Error Covariances

| Left | Operator | Right | Modification Index | Expected Parameter Change | Delta | Power | Decision |
|------|----------|-------|-------------------:|--------------------------:|-------|-------|----------|
| N3 | ~~ | N4 | 3.507 | 1.820 | 0.1 | 0.051 | (i) |
| N1 | ~~ | N2 | 3.227 | 1.538 | 0.1 | 0.052 | (i) |
| N1 | ~~ | N3 | 2.747 | -1.615 | 0.1 | 0.051 | (i) |
| E1 | ~~ | E4 | 1.983 | -1.827 | 0.1 | 0.051 | (i) |
| N4 | ~~ | E2 | 1.276 | 0.906 | 0.1 | 0.052 | (i) |
| N2 | ~~ | N4 | 1.253 | -0.956 | 0.1 | 0.052 | (i) |
| N3 | ~~ | E1 | 1.101 | -1.028 | 0.1 | 0.051 | (i) |

Note:

Maximum 10 modification indices in descending order of their magnitude are listed.

Table 8: Modification Indices With Respect To Factor Loadings

| Left | Operator | Right | Modification Index | Expected Parameter Change | Delta | Power | Decision |
|--------|----------|-------|-------------------:|--------------------------:|-------|-------|----------|
| NEUROT | =~ | E1 | 1.328 | -0.073 | 0.4 | 1 | (nm) |
| NEUROT | =~ | E2 | 1.121 | 0.068 | 0.4 | 1 | (nm) |
| EXTRAV | =~ | N4 | 1.067 | 0.053 | 0.4 | 1 | (nm) |

Note:

| Left | Operator | Right | Modification Index | Expected Parameter Change | Delta | Power | Decision |
|------|----------|-------|--------------------|-----------------------------|-------|-------|----------|

Maximum 10 modification indices in descending order of their magnitude are listed.

## Interpretation

### Goodness of Fit Indices

We consider some of the model fit indices from the Model Fit Summary section to check the goodness-of-fit of the model. To decide for an acceptable or non-acceptable model, we apply thresholds considered in the References: [@brown], [@kline].

**Model Test User Model**
The degrees of freedom are calculated as the number of known parameters minus the number of free parameters: 36 - 17 = 19 . The 19 degrees of freedom indicate an over-identified model, fact which basically enables further analysis and interpretation.

The test statistic with the value 13.285 is called the Chi-square model fit index and represents the difference between summaries of the model-implied covariance matrix and the observed covariance matrix which is hypothesized and desirable to be zero. In general, if the p-value is larger than 0.05 then the test is not statistically significant at 5 % error, the hypothesis cannot be rejected, which would be in favour of the model.
In our case, the p-value is 0.824 suggesting an acceptable model fit.

**Model Test Baseline Model**
The test statistic with the value 1253.791 represents the difference between summaries of the baseline model (an alternative model-implied covariance matrix having zero covariances, i.e. a worst fitting model assuming independent variables) and the observed covariance matrix. The p-value of the test of a zero difference is <0.001 suggesting that the baseline model does not fit good to the data. This result is used indirectly in the construction of other model fit indices.

**Root Mean Square Error of Approximation:**
The Root Mean Square Error of Approximation (RMSEA) is a fit index based on the chi-square test statistic, which corrects for parsimony, i.e. overly complex models are penalized. RMSEA can be greater or equal than zero, with values close to zero suggesting an acceptable model fit.
In our case, the RMSEA is 0. The upper bound of the 90% confidence interval of the RMSEA is 0.034 and smaller than the threshold value 0.05, suggesting an excellent model fit.

**Standardized Root Mean Square Residual:**
The Standardized Root Mean Square Residual (SRMR) is a fit index derived from the residual correlation matrix with a range between zero and one with values close to zero suggesting an acceptable model fit.
In our case, the SRMR is 0.02 which is smaller than the threshold value 0.1 suggesting an acceptable model fit.

**User Model versus Baseline Model**
The Comparative Fit Index (CFI), evaluates the fit of the the model in relation to the worst-fitting baseline model described above. It ranges between zero and one, with values close to one suggesting good models (in the sense of departure from the baseline model).
In our case, the CFI is 1 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

Similarly to the CFI, the Tucker-Lewis Index (TLI) evaluates the fit of the model in relation to the worst-fitting baseline model described above. Moreover, overly complex models are penalized. Values can range outside zero and one but the index is interpreted similarly to the CFI.

In our case, the TLI is 1.007 which is greater or equal than the threshold value 0.95, suggesting a good model fit.

## Summary of the Goodness of Fit Indices

The TLI model fit index suggests an acceptable model fit. Moreover the Chi-square model fit index and the RMSEA suggest an acceptable model fit. We tentatively assume an acceptable model fit and verify this assertion by considering further metrics.

## Residuals

We analyze the residual matrices from the Outputs chapter. The residual covariance matrix represents the difference between the observed covariance matrix and the fitted model-implied covariance matrix. Large absolute values indicate local areas of misfit. However, the residuals are affected by the raw metric and are difficult to interpret more precisely.

A better interpretation allows the standardized residual matrix (residuals divided by their estimated asymptotic standard error) and the residual correlation matrix.

Following variable pairs have standardized residuals which are larger or equal than the considered threshold 2.58 [@brown] or correlation residuals which are larger or equal than the considered threshold 0.1 [@kline]. In these cases, the covariance relationship between the involved variables is probably underestimated:

Table 9: Pair(s) with Underestimated Covariance

| Pair 1 | E4 | E4 |
|--------|----|----|

There are no variable pairs with standardized residuals which are smaller or equal than the considered threshold -2.58 [@brown] or correlation residuals which are smaller or equal than the considered threshold -0.1 [@kline]. Therefore, no relationships among the variables are substantially overestimated by the model. Depending on the sample size, the misspecification detected by the analysis of the residual covariance resp. correlation matrices can be statistically significant but not relevant and in practice negligible. This is matter of subject in the next section(s).

## Modification Indices

In the interpretation of the modification indices table(s) we rely mostly on [@brown] and [@mi]. We cite from [@brown]: "The modification index reflects an approximation of how much the overall model Chi² will decrease if the fixed or constrained parameter is freely estimated." In other words, if adding a line with a high modification index to the model, i.e. if adding a parameter, the overall goodness-of-fit may be improved. Nevertheless, this should be done only under certain conditions, described in the sequel.

We consider only modification indices greater or equal than 3.84 (which are statistically significant at 5% type I error). Next, we search only for modification indices which achieve a power of minimum 75% in detecting a (relevant) misspecification of at least 0.1 for error or factor correlations, respectively 0.4 for factor loadings. These are characterized in the decision column by the label "epc:m". For more information with regard to the labels of the decision column, please consult the Appendix.

We remark that these conditions are not fulfilled for modification indices with respect to error covariances. Therefore, there exist no significant and relevant modification indices with respect to error covariances.

We remark that these conditions are not fulfilled for modification indices with respect to factor loadings. Therefore, there exist no significant and relevant modification indices with respect to factor loadings.

We remark that there exist no modification indices with respect to factor covariances.

## Parameter Estimates

### Factor Loadings

We remark that the completely standardized factor loadings (section "Completely Standardized Parameter Estimates") are all statistically significant at 5% type I error. Moreover, in absolute value they are all greater than 0.4. This cutoff-value is considered in some CFA research areas a magnitude that is substantively meaningful [@brown]. Please consider also cutoff-values from your particular research area when interpreting the factor loadings. We summarize the interpretation of the completely standardized factor loadings in the next table:

Table 10: Check Completely Standardized Factor Loadings

| Latent Variable | Observed Variable | Loading[1] | P-Value | Significant?[2] | Relevance[3] | Sign[4] | Check |
|---|---|---|---|---|---|---|---|
| NEUROT | N1 | 0.88 | <0.001 | Yes | *** | — | Ok |
| NEUROT | N2 | 0.85 | <0.001 | Yes | *** | — | Ok |
| NEUROT | N3 | 0.84 | <0.001 | Yes | *** | — | Ok |
| NEUROT | N4 | 0.88 | <0.001 | Yes | *** | — | Ok |
| EXTRAV | E1 | 0.80 | <0.001 | Yes | *** | — | Ok |
| EXTRAV | E2 | 0.83 | <0.001 | Yes | *** | — | Ok |
| EXTRAV | E3 | 0.79 | <0.001 | Yes | ** | — | Ok |
| EXTRAV | E4 | 0.70 | <0.001 | Yes | ** | — | Ok |

[1] The completely standardized factor loading can be interpreted as the correlation with the factor.

[2] Completely standardized factor loading significance at 5% type I error.

[3] Stars correspond to factor loadings cutoff-values: 0.4, 0.6, 0.8.

[4] No (correct) information available. We assume the signs of the factor loadings correspond to your expectation.

[5] _____

[6] _____

[7] _____

Moreover, we remark that the significance test results for the completely standardized factor loadings from above coincide to those of the unstandardized factor loadings (within section "Model Fit Summary", for non-marker variables).

We proceed by interpreting the (unstandardized) factor loadings from the "Model Fit Summary" section:

Table 11: Interpretation of Unstandardized Factor Loadings

| Interpretation of Unstandardized Factor Loadings |
|---|
| A 1-unit increase in NEUROT leads to a 1.00 -unit increase in the N1 |
| A 1-unit increase in NEUROT leads to a 0.94 -unit increase in the N2 |
| A 1-unit increase in NEUROT leads to a 1.07 -unit increase in the N3 |
| A 1-unit increase in NEUROT leads to a 1.00 -unit increase in the N4 |
| A 1-unit increase in EXTRAV leads to a 1.00 -unit increase in the E1 |

Table 11: Interpretation of Unstandardized Factor Loadings (continued)

| Interpretation of Unstandardized Factor Loadings |
| --- |
| A 1-unit increase in EXTRAV leads to a 1.07 -unit increase in the E2 |
| A 1-unit increase in EXTRAV leads to a 0.94 -unit increase in the E3 |
| A 1-unit increase in EXTRAV leads to a 0.81 -unit increase in the E4 |

**Factor Discriminant Validity**

As noted by [@brown], "the interpretability of the size and statistical significance of factor intercorrelations depends on the specific research context." Though, the largest (absolute) estimated factor intercorrelation within the section "Completely Standardized Parameter Estimates" is -0.43 which we regard as a proof of a reasonable discriminant validity. Moreover, the statistical test(s) for factor discriminant validity are statistically significant at 5% type I error.

**Error Variances**

We summarize the interpretation of the error variances and communalities in the next table:

Table 12: Completely Standardized Error Variances and Communality

| Observed Variable | Error Variance[1] | Communality[23] | P-Value | Significant Error Variance?[4] |
| --- | --- | --- | --- | --- |
| E1 | 0.36 | 0.64 | <0.001 | Yes |
| E2 | 0.30 | 0.70 | <0.001 | Yes |
| E3 | 0.38 | 0.62 | <0.001 | Yes |
| E4 | 0.51 | 0.49 | <0.001 | Yes |
| N1 | 0.22 | 0.78 | <0.001 | Yes |
| N2 | 0.28 | 0.72 | <0.001 | Yes |
| N3 | 0.29 | 0.71 | <0.001 | Yes |
| N4 | 0.22 | 0.78 | <0.001 | Yes |

[1] Can be interpreted as proportion of unexplained variance by the latent factor(s) (%).
[2] Corresponds to the squared factor loading.
[3] Can be interpreted as proportion of explained variance by the latent factor(s) (%).
[4] 5% type I error. Typically significant since a large portion of variance is not explained by the latent variable.

**Factor Reliability**

The table "Factor Reliability" contains the omega measures of factor reliability given by Bentler (Bentler, 1972, 2009) and McDonald (McDonald, 1999) and the average variance extracted (AVE). The interpretatibility of the reliability measures depend on the specific research context. In some fields of research, omega values greater or equal than 0.6 and AVE values greater or equal than 0.5 (fulfilled by and large in your case) could be sufficient.

## Final Summary

In our final evaluation, we distinguish between following model quality categories: acceptable, non-acceptable or uncertain.
Considering the goodness-of-fit indices, the model is acceptable. Moreover, we cannot identify localized areas of ill fit. We conclude that the model is acceptable.

## Path Diagram
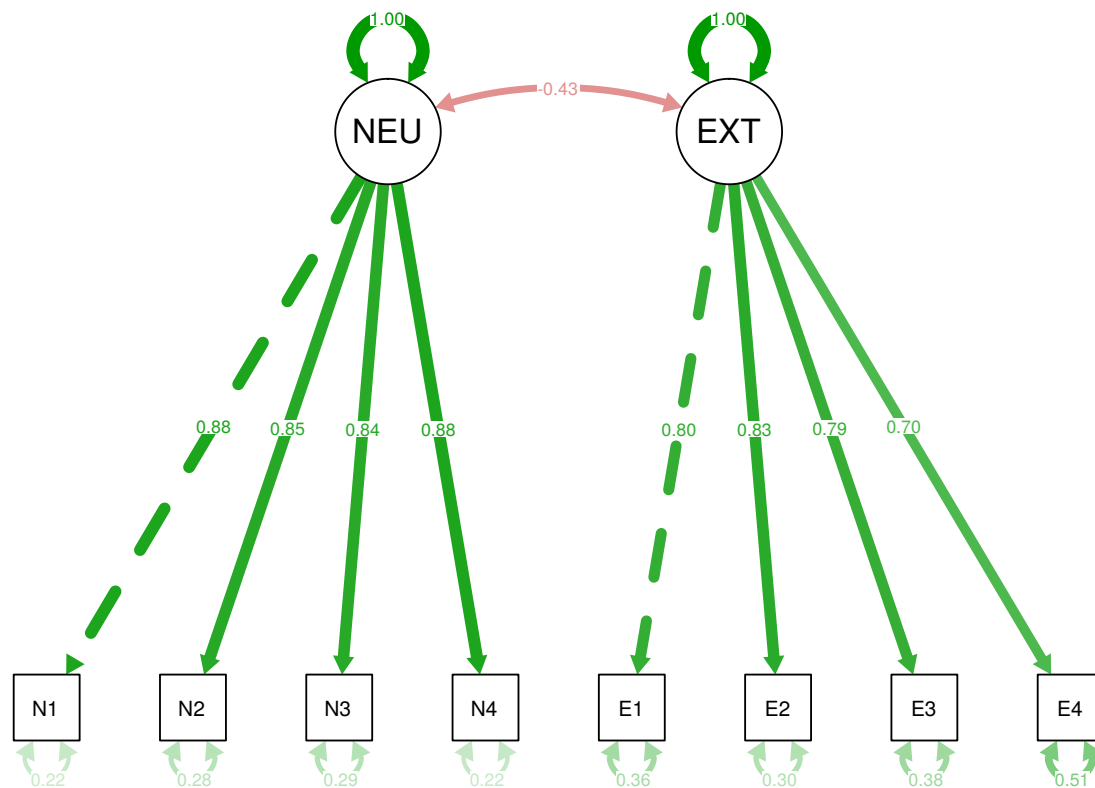


## APPENDIX

### Decision Column of the Modification Indices Table

```
not mi.significant & not high.power := "(i)"
mi.significant & not high.power := "**(m)**"
not mi.significant & high.power := "(nm)"
mi.significant & high.power & not epc.high := "epc:nm"
mi.significant & high.power & epc.high := "*epc:m*"
```

## References