

# Multiple Comparison Procedures To A Control

For AN(C)OVA Models

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## Basic Information

Automatic statistics for the file:

File
warpbreaks.csv

Your selection for the encoding: UTF-8

Your selection for the decimal character: .

Observations (rows with at least one non-missing value): 54

Variables (columns with at least one non-missing value): 4

Variables considered continuous: 2

Variables considered continuous
V1
breaks

Variables considered categorical: 2

Variables considered categorical
wool
tension

## Model Information

You defined the following linear model: `breaks~wool*tension`

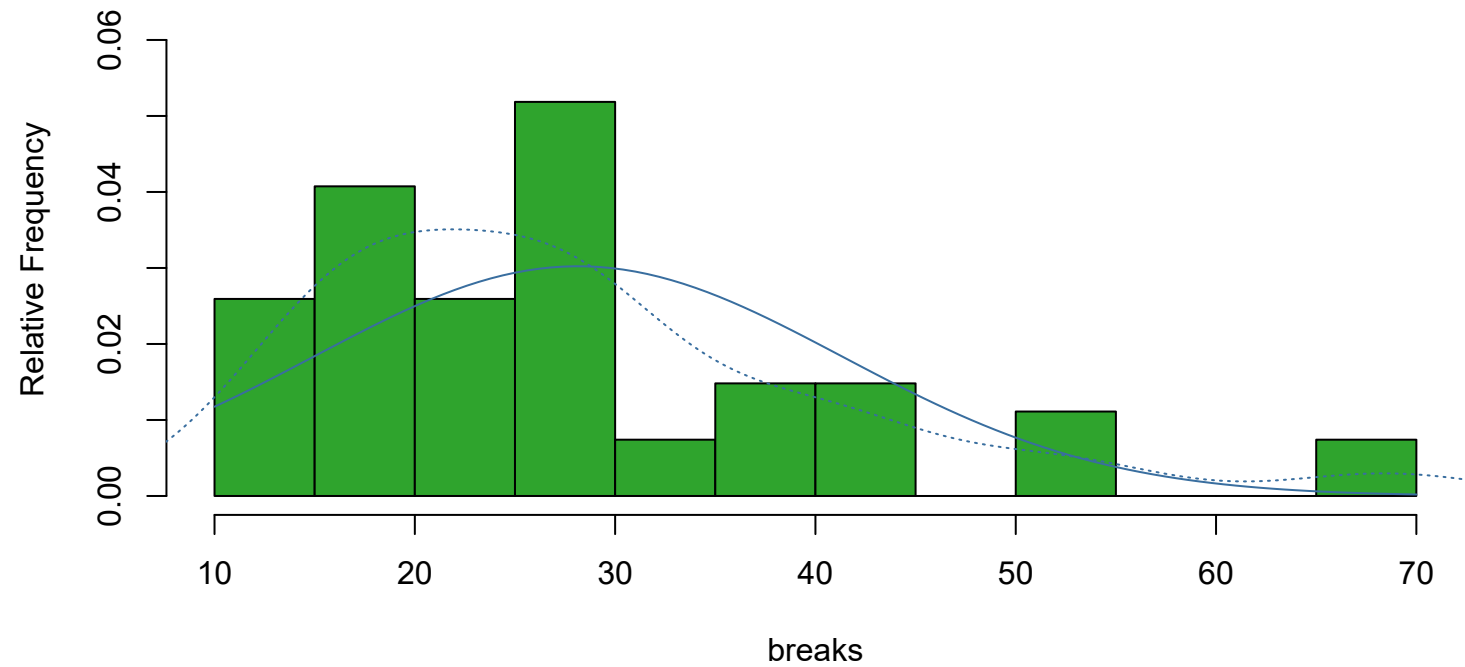
You are interested in the factor: `wool`

You are interested in pairwise comparisons to the reference level: `A`

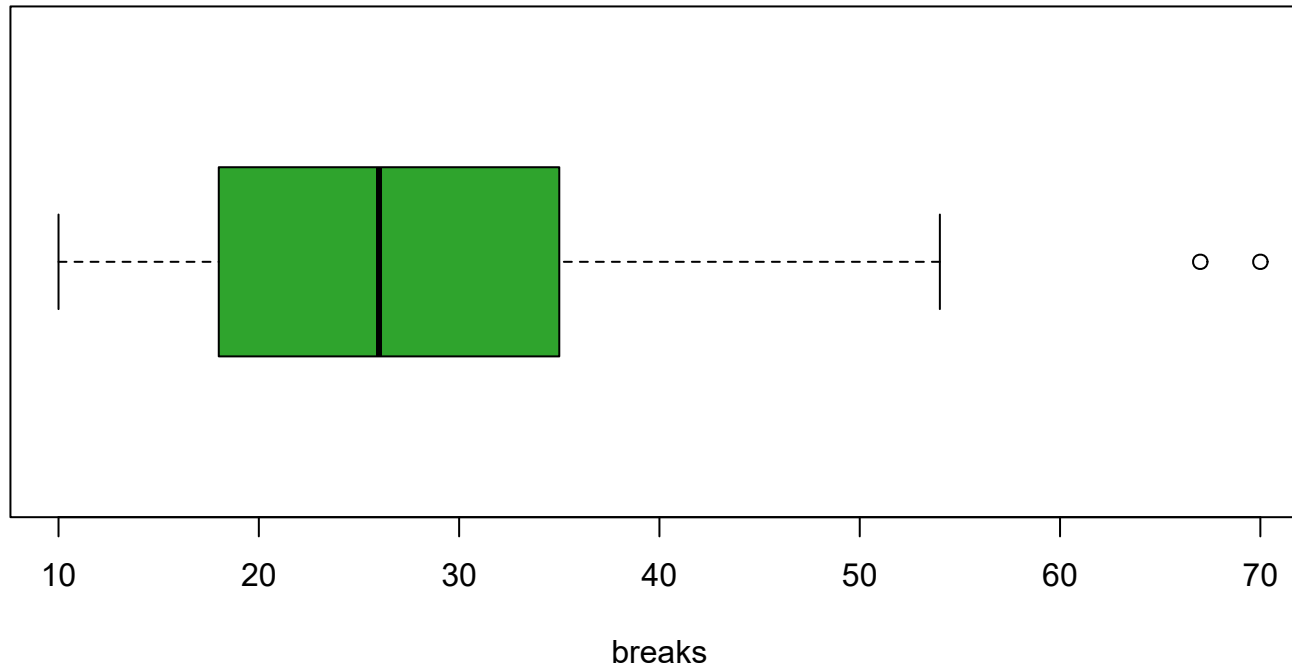
## Descriptive Plots

Dependent Variable

Histogram of breaks

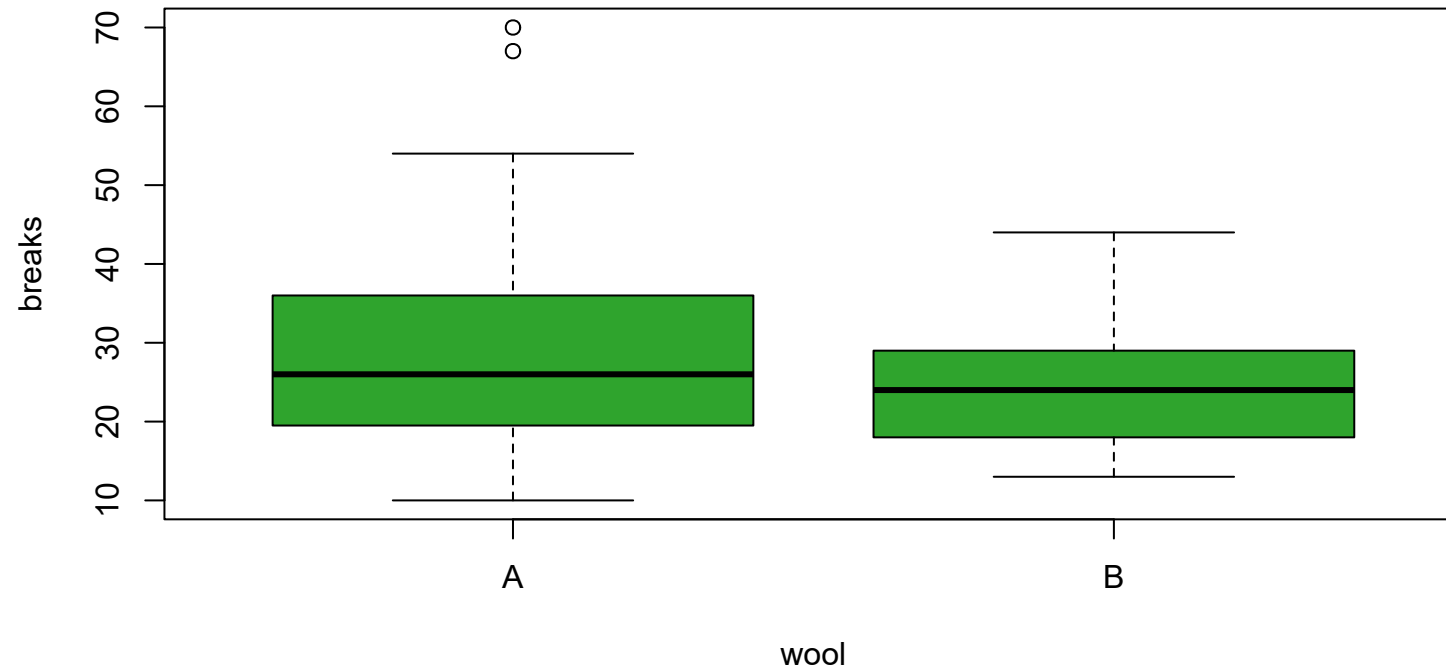


**Boxplot of breaks**

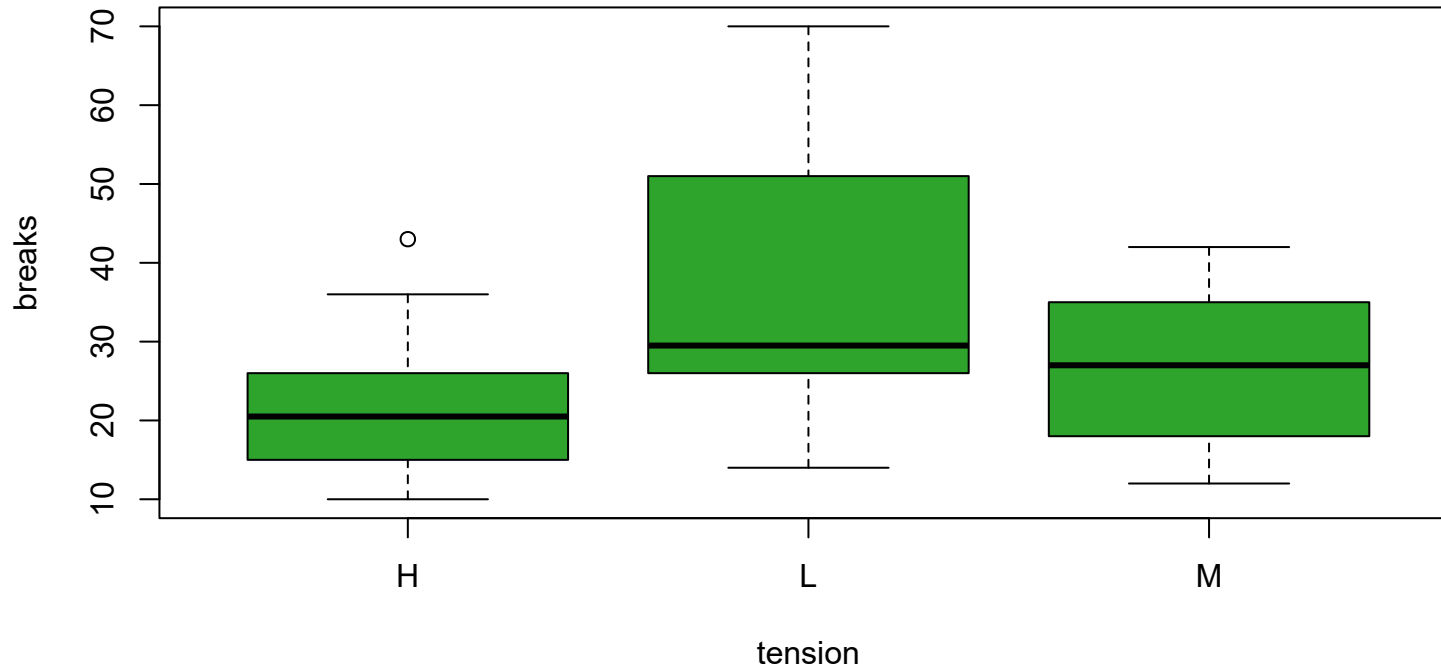


## Dependent Against Categorical Factors

Boxplot of breaks ~ wool



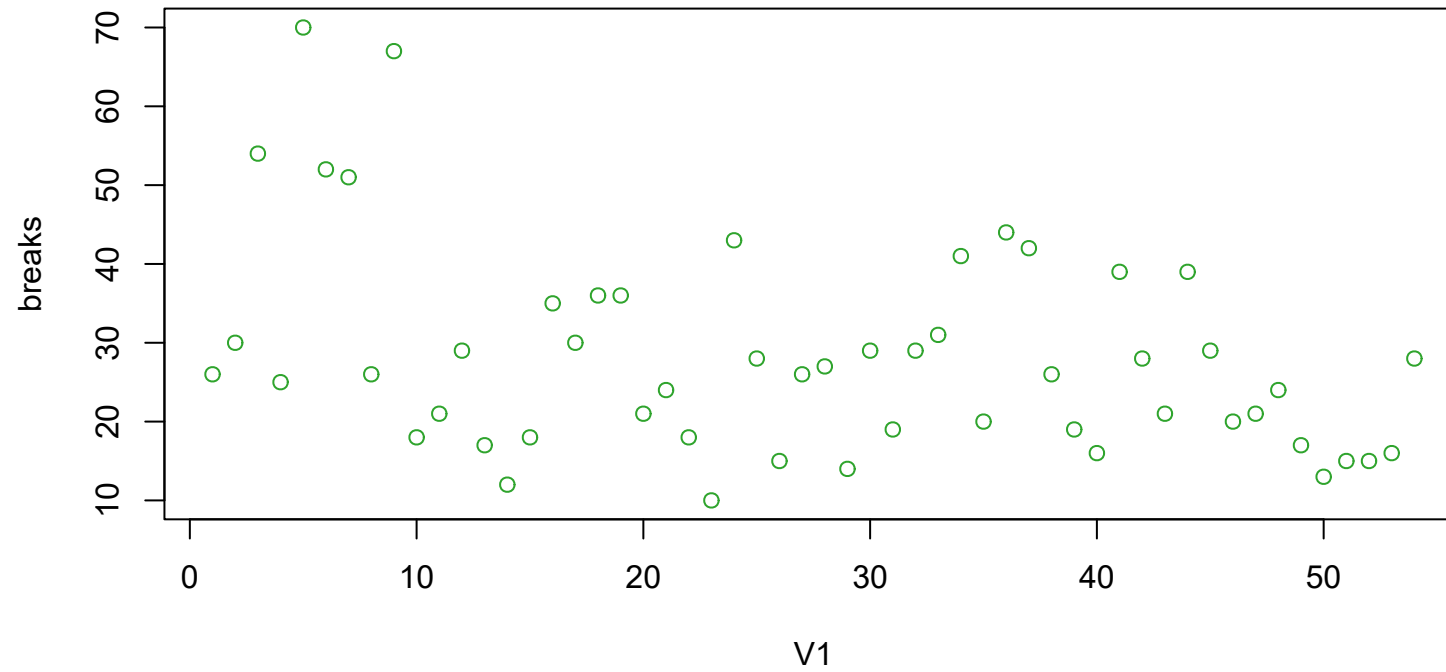
**Boxplot of breaks ~ tension**





## Dependent against Covariates

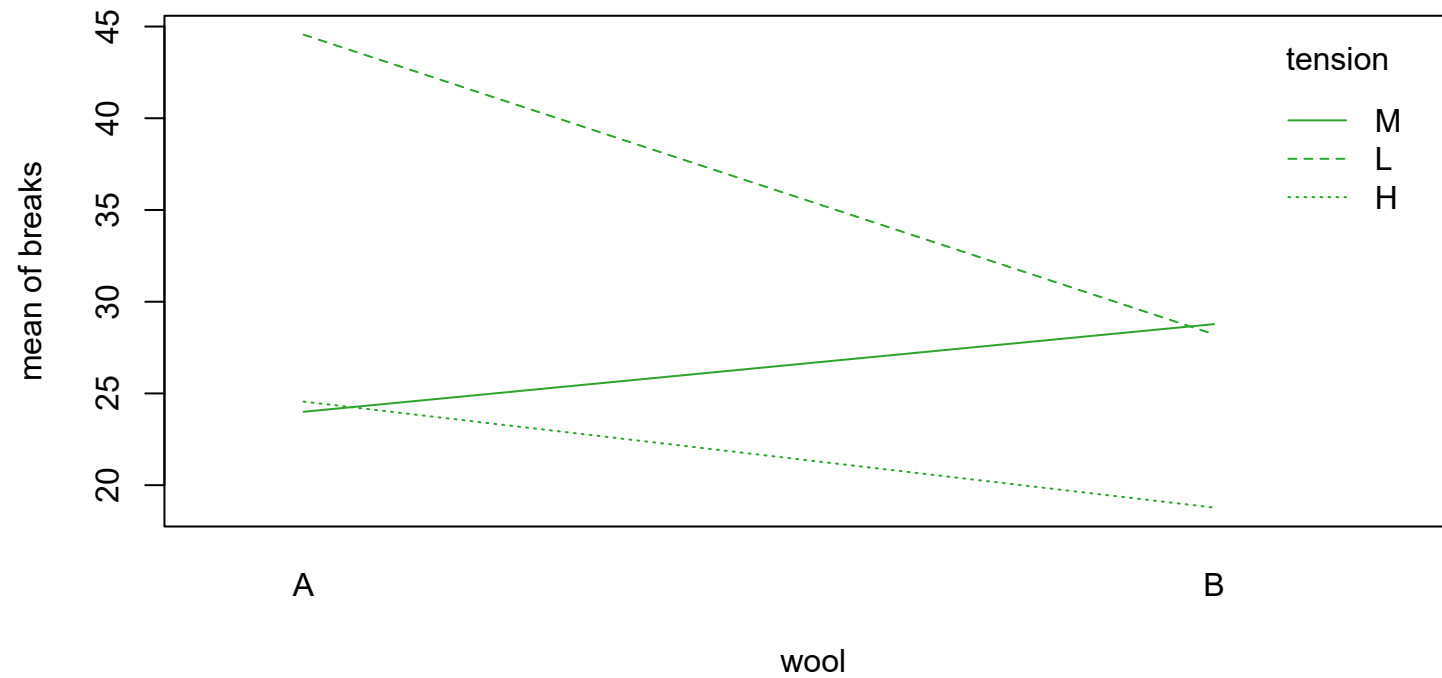
Scatterplot of V1



## Interaction Plot for Factors

Note: The more parallel the lines, the less likely is the significance of the interaction of the factors.

**Interaction Plot of wool and tension**



## Analysis of Variance

### Detailed Influence of Factors (Linear Model Parameter Estimates)

Variable	Value	Std.Error	T.value	P.value	sign. level <sup>1</sup>	Significance at 5 percent error
(Intercept)	28.15	1.49	18.91	<0.001	***	Intercept Significant.
wool1	2.89	1.49	1.94	0.058	.	Not Significant. No difference between the effect of wool1 and its reference.
tension1	-6.48	2.11	-3.08	0.003	**	Significant. A Difference between the effect of tension1 and its reference.
tension2	8.24	2.11	3.91	<0.001	***	Significant. A Difference between the effect of tension2 and its reference.
wool1:tension1	0.00	2.11	0.00	1		Interaction not Significant. Effect wool1 vs. reference don't depends on tension1.
wool1:tension2	5.28	2.11	2.51	0.016	*	Interaction Significant. Effect wool1 vs. reference depends on tension2.

<sup>1</sup> '\*\*\*': sign. to 0.1% error. '\*\*': sign. to 1% error. '\*': sign. to 5% error. '.' : sign. to 10% error. ' ': not sign. ' - ': no statement.

### Total Influence of Factors (ANOVA Type III)

Variable	Sum.Sq	Df	F.value	P.value	Interpretation (5% error)
(Intercept)	42785.19	1	357.47	<0.001	Intercept significantly different from zero.
wool	450.67	1	3.77	0.058	There exist significant differences between the levels of factor 2.
tension	2034.26	2	8.50	<0.001	There exist significant differences between the levels of factor 3.
wool:tension	1002.78	2	4.19	0.021	There exist significant differences between the levels of factor 4.
Residuals	5745.11	48			

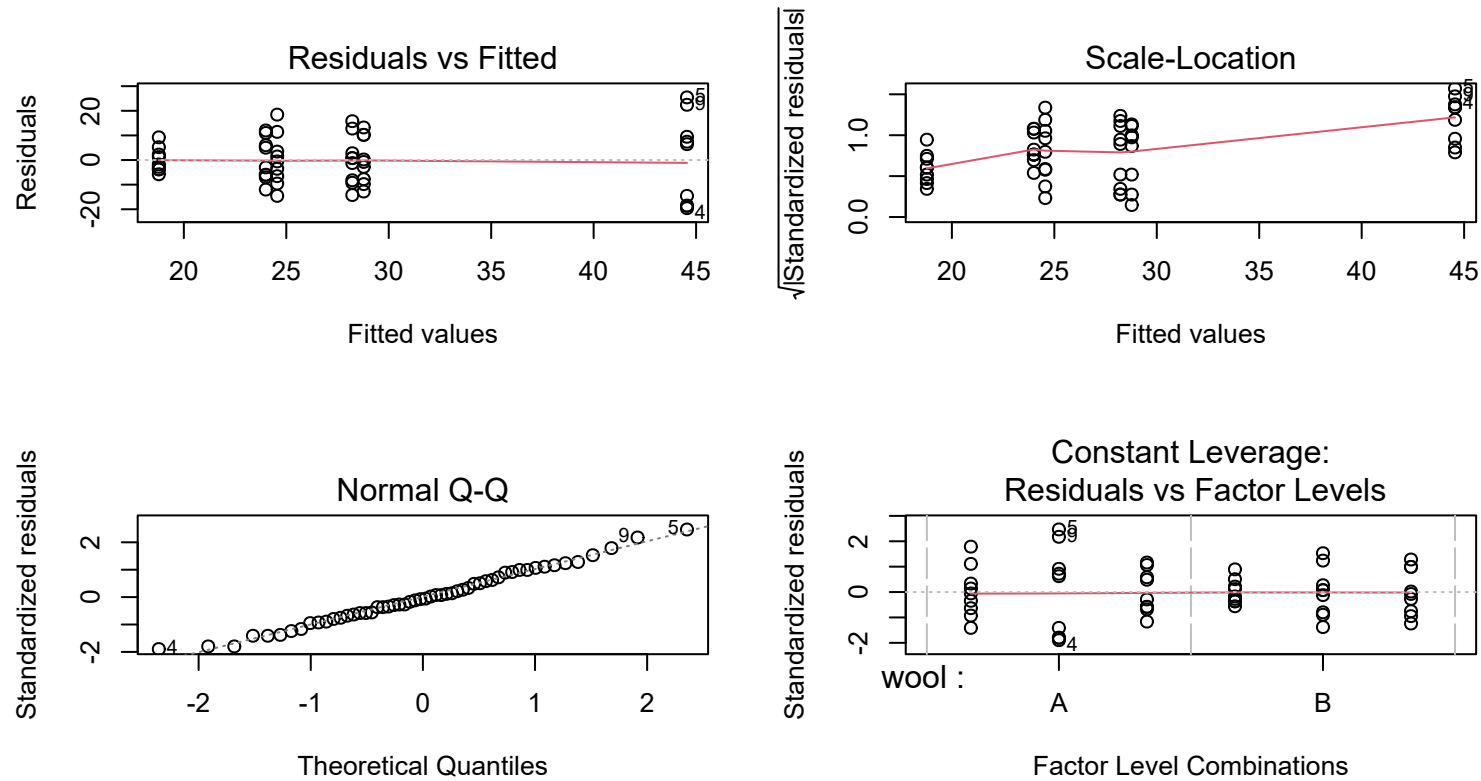
### Goodness Of Fit Measures

To evaluate the model, some parameters are listed below.

	Values	Explanation	Interpretation
Multiple R-Squared	0.38	Fraction of variance explained by the model.	0: No fitting of data by the model. , 1: Perfect fit.
Adjusted R-Squared	0.31	Adjusted R-Squared by penalizing higher p.	A higher value means a better fit by the model.
F-statistic	3.77	Overall significance of the model.	Note the P-value to assess significance.
P-value	0.06	P-value of the F-statistic.	No significance to the 5% error. The model is better than the only-intercept model.

## Diagnostics

### Diagnostic Plots



## Homogeneity of Variances

### Levene Test of Homogeneity of Variances

	Df	F value	Pr(>F)
group	5	2.89	0.02
	48		

Warning: Group variances significantly heterogeneous at 1% error! The Analysis of Variance may not be valid.

## Multiple Comparisons of Means to a Control

Theoretical background: Testing multiple hypotheses simultaneously and each at the same pre-specified significance level, increases the probability of false positive effects. The probability to commit at least one false positive decision increases with the number of hypotheses. A solution to overcome this problem is given by multiple comparisons procedures. Here, we do not control the per-hypothesis Type I error but the probability of committing at least one Type I error over all hypotheses. Using p-values adjusted for multiplicity, individual hypotheses can be finally compared with the pre-specified significance level.

### Dunnet

#### Multiple Comparison: Dunnet Contrasts

Test whether the factor level A of the factor wool is less than the other levels. The Null Hypothesis is for example  $B - A \geq 0$ .

Null Hypothesis	Value	Std.Error	T.value	adjusted P.value	Sign. level <sup>1</sup>	Significance at 5 percent Type I error
$B - A \geq 0$	-5.78	2.98	-1.94	0.03	*	Significant. Level B of factor wool is significantly less than A <sup>3</sup>

<sup>1</sup> '\*\*\*': sign. to 0.1% error. '\*\*': sign. to 1% error. '\*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

<sup>2</sup> H1 does not hold significantly.

<sup>3</sup> H1 holds significantly.

#### Simoultaneous Confidence Intervals: Dunnet Contrasts

Simultaneous Confidence Intervals which includes the true value of the difference between the reference level A and the other levels of wool

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
$B - A \geq 0$	-5.78	-Inf	-0.78	The interval $(-\text{Inf}, -0.78)$ traps the true difference B-A with probability 95 percent. <sup>1</sup>

<sup>1</sup> Remark: Zero is not in the confidence interval.

<sup>2</sup> Remark: Zero is in the confidence interval.

## Dunnet Sandwich

The sandwich function provides a heteroskedasticity-consistent estimate of the covariance matrix. Thus, the sandwich estimate is resistant to the violation of the variance homogeneity assumption.

### Multiple Comparison: Dunnett Contrasts Sandwich

Test whether the factor level A of the factor wool is less than the other levels. The Null Hypothesis is for example  $B - A \geq 0$ .

Null Hypothesis	Value	Std.Error	T.value	adjusted P.value	Sign. level <sup>2</sup>	Significance at 5 percent Type I error
$B - A \geq 0$	-5.78	2.81	-2.06	0.02	*	Significant. Level B of factor wool is significantly less than A <sup>4</sup>

<sup>1</sup> Note: Due to the applied sandwich estimator, the standard errors of the effects may be unequal.

<sup>2</sup> '\*\*\*': sign. to 0.1% error. '\*\*': sign. to 1% error. '\*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

<sup>3</sup> H1 does not hold significantly.

<sup>4</sup> H1 holds significantly.

### Simultaneous Confidence Intervals: Dunnett Contrasts Sandwich

Simultaneous Confidence Intervals which includes the true value of the difference between the reference level A and the other levels of wool.

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
$B - A \geq 0$	-5.78	-Inf	-1.07	The interval (-Inf, -1.07) traps the true difference B-A with probability 95 percent. <sup>1</sup>

<sup>1</sup> Remark: Zero is not in the confidence interval.

<sup>2</sup> Remark: Zero is in the confidence interval.

## Dunnet Step Down

Table 12: Multiple Comparison: Dunnet Contrasts

Null Hypothesis	Value	Std.Error	T.value	P.value	Sign. level <sup>1</sup>	Significance at 5 percent Type I error
B - A >= 0	-5.78	2.98	-1.94	0.03	*	Significant. Level B of factor wool is significantly less than A <sup>3</sup>

<sup>1</sup> '\*\*\*': sign. to 0.1% error. '\*\*': sign. to 1% error. '\*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

<sup>2</sup> H1 does not hold significantly.

<sup>3</sup> H1 holds significantly.

Simultaneous Confidence Intervals which includes the true value of the difference between the reference level A and the other levels of wool

Table 13: Simoultaneous Confidence Intervals: Dunnet Contrasts

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
B - A >= 0	-5.78	-Inf	-0.78	The interval (-Inf, -0.78) traps the true difference B-A with probability 95 percent. <sup>1</sup>

<sup>1</sup> Remark: Zero is not in the confidence interval.

<sup>2</sup> Remark: Zero is in the confidence interval.



## Dunnet Step-Down Sandwich

### Multiple Comparison: Dunnet Contrasts step-down Sandwich

Test whether the factor level A of the factor wool is less than the other levels. The Null Hypothesis is for example  $B - A \geq 0$ .

Null Hypothesis	Value	Std.Error	T.value	P.value	Sign. level <sup>1</sup>	Significance at 5 percent Type I error
$B - A \geq 0$	-5.78	2.81	-2.06	0.02	*	Significant. Level B of factor wool is significantly less than A <sup>3</sup>

<sup>1</sup> '\*\*\*': sign. to 0.1% error. '\*\*': sign. to 1% error. '\*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

<sup>2</sup> H1 does not hold significantly.

<sup>3</sup> H1 holds significantly.

### Simultaneous Confidence Intervals: Dunnet Contrasts step-down Sandwich

Simultaneous Confidence Intervals which include the true value of the difference between the reference level A and the other levels of wool

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
$B - A \geq 0$	-5.78	-Inf	-1.07	The interval (-Inf, -1.07) traps the true difference B-A with probability 95 percent. <sup>1</sup>

<sup>1</sup> Remark: Zero is not in the confidence interval.

<sup>2</sup> Remark: Zero is in the confidence interval.

## References

- Bretz, Frank, and Peter Westfall Torsten Hothorn. 2010. Multiple Comparisons Using R. 1st Edition. Chapman; Hall/CRC. <https://doi.org/10.1201/9781420010909>.
- Fox, John, and Sanford Weisberg. 2019. An R Companion to Applied Regression. Third. Thousand Oaks CA: Sage. <https://socialsciences.mcmaster.ca/jfox/Books/Companion/>.
- R Core Team. 2019. R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing. <https://www.R-project.org/>.
- Zeileis, Achim, and Torsten Hothorn. 2002. "Diagnostic Checking in Regression Relationships." R News 2 (3): 7–10. <https://CRAN.R-project.org/doc/Rnews/>.