

Multiple Comparison Procedures To A Control

For AN(C)OVA Models

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Basic Information

Automatic statistics for the file:

File
warpbreaks.csv

Your selection for the encoding: UTF-8

Your selection for the decimal character: .

Observations (rows with at least one non-missing value): 54

Variables (columns with at least one non-missing value): 3

Variables considered continuous: 1

Variables considered continuous
breaks

Variables considered categorical: 2

Variables considered categorical
wool
tension

Model Information

You defined the following linear model: `breaks~wool*tension`

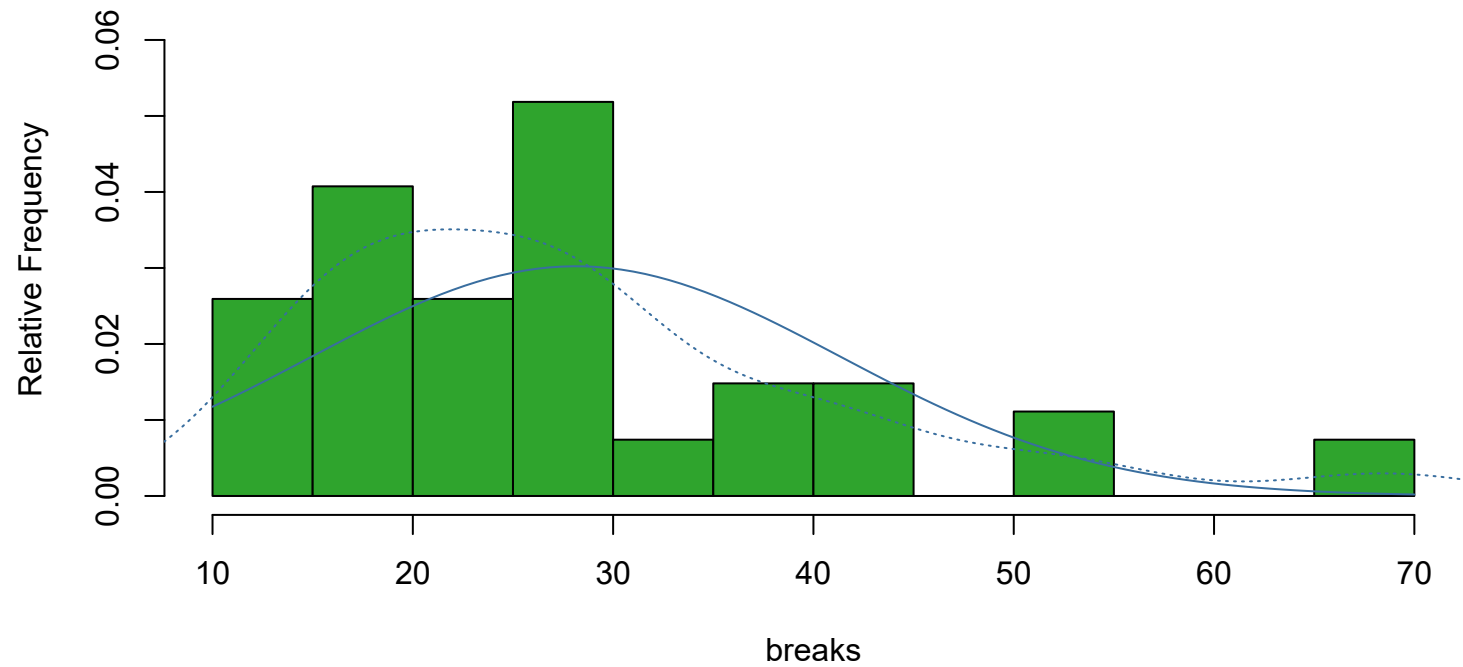
You are interested in the factor: `wool`

You are interested in pairwise comparisons to the reference level: `A`

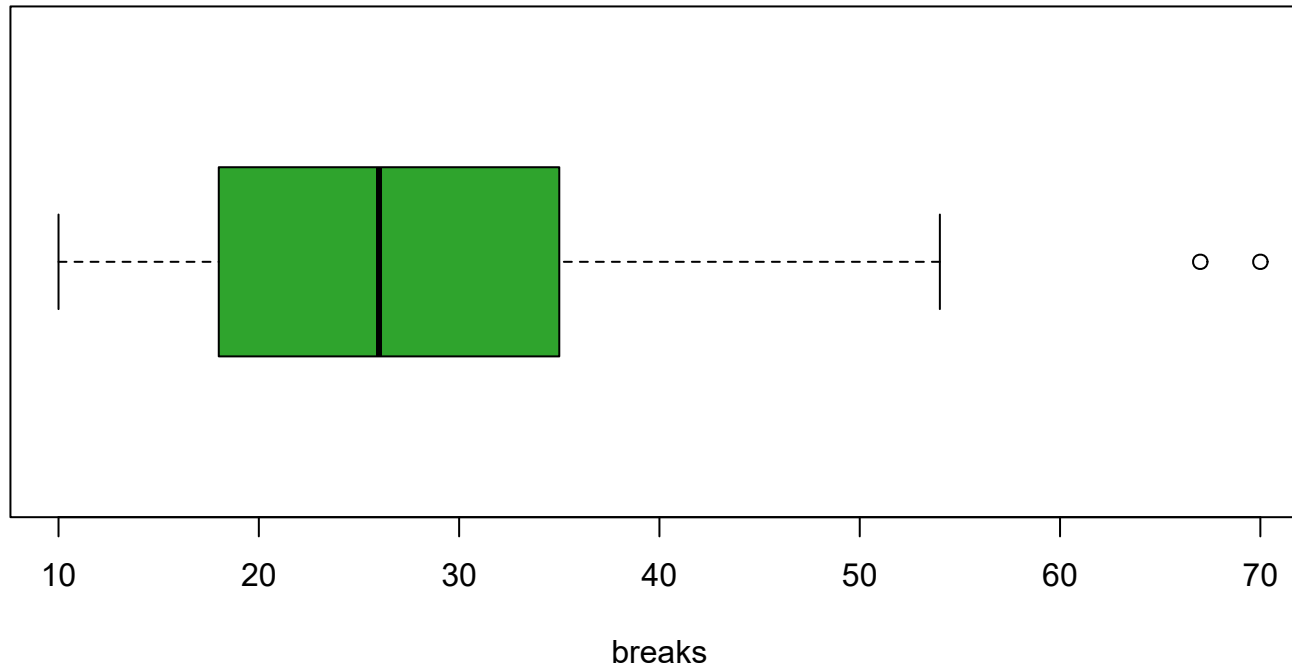
Descriptive Plots

Dependent Variable

Histogram of breaks

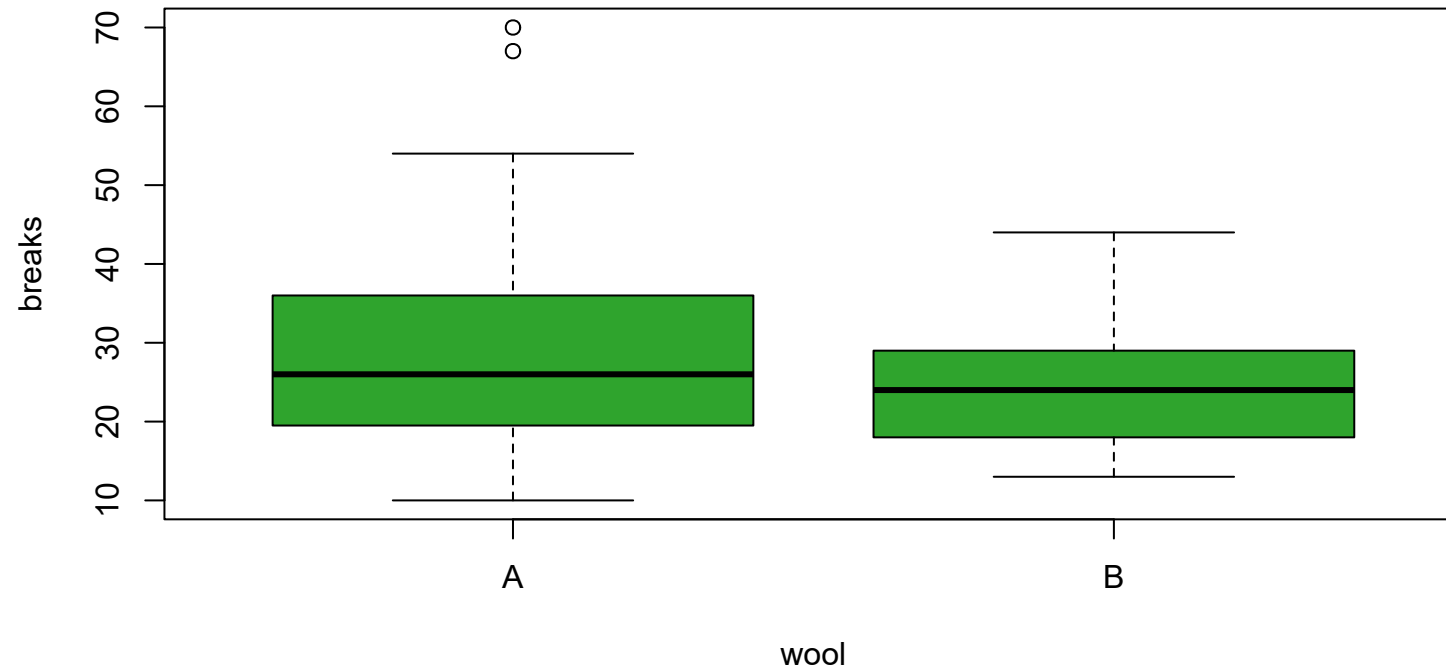


Boxplot of breaks

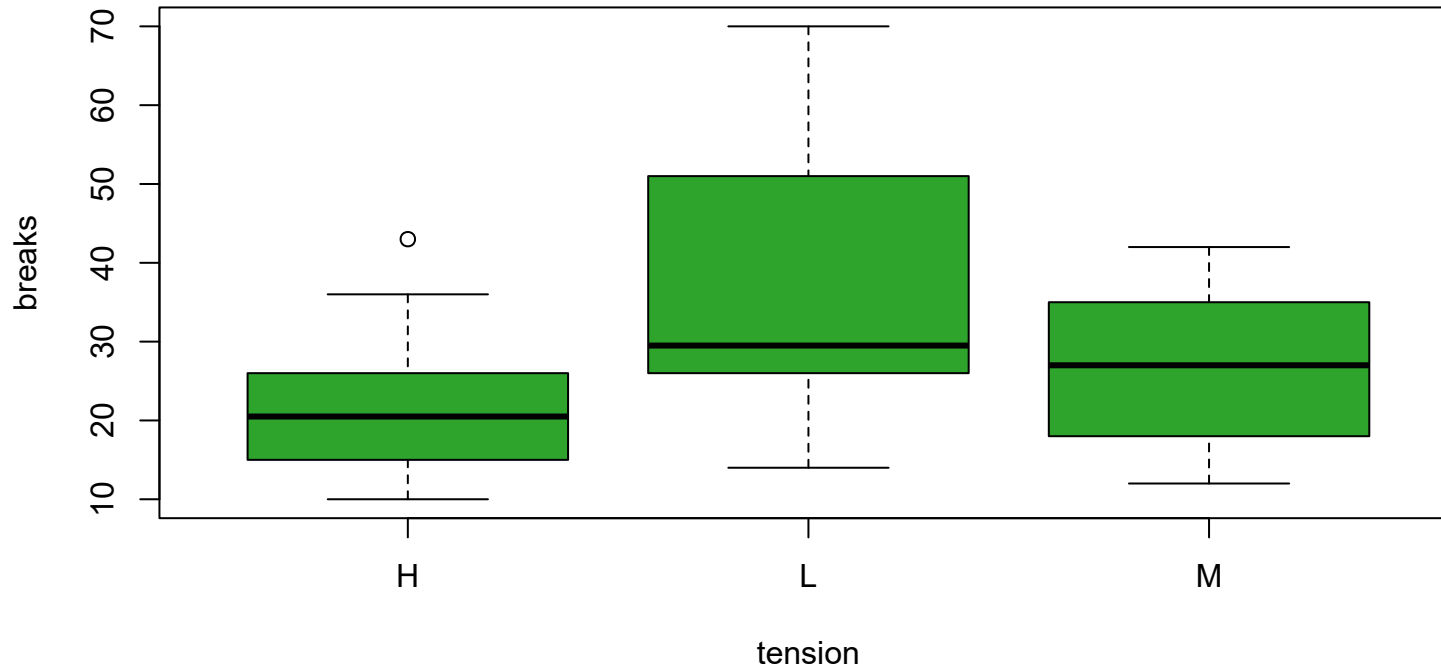


Dependent Against Categorical Factors

Boxplot of breaks ~ wool



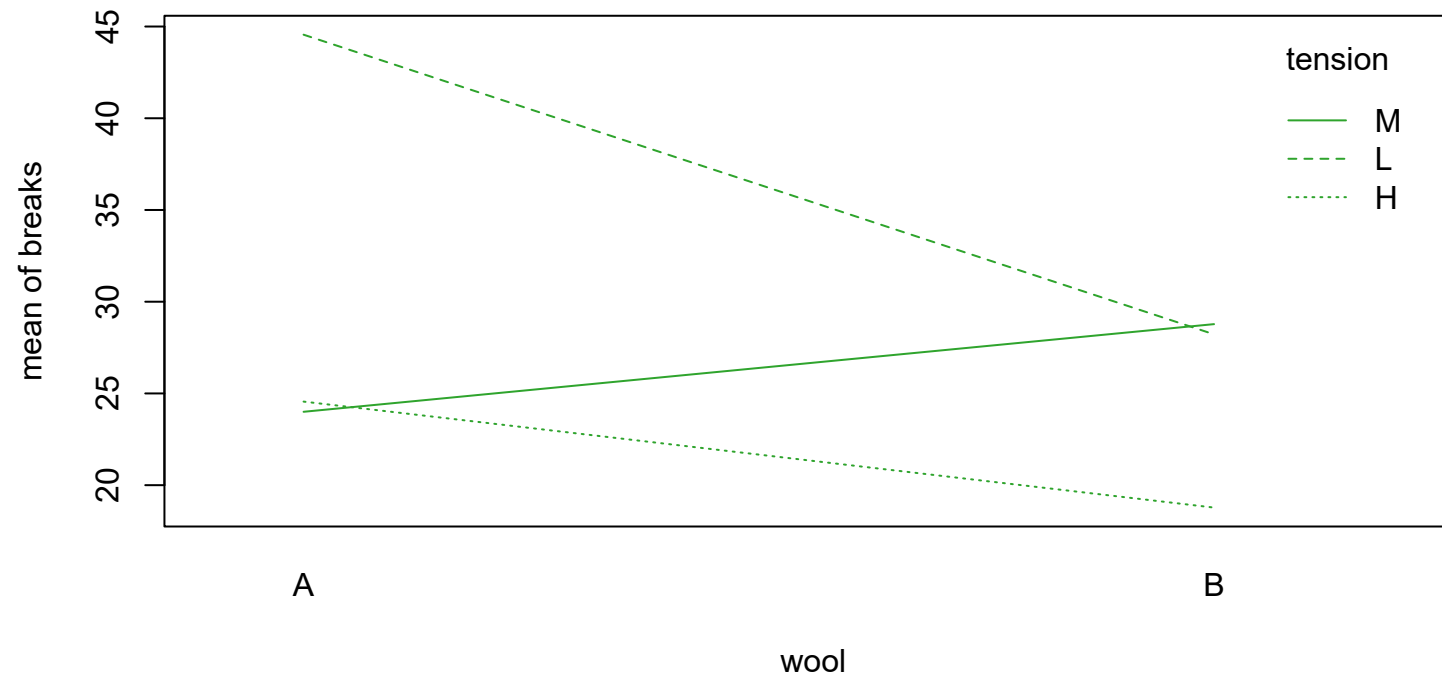
Boxplot of breaks ~ tension



Interaction Plot for Factors

Note: The more parallel the lines, the less likely is the significance of the interaction of the factors.

Interaction Plot of wool and tension



Analysis of Variance

Detailed Influence of Factors (Linear Model Parameter Estimates)

Variable	Value	Std.Error	T.value	P.value	sign. level ¹	Significance at 5 percent error
(Intercept)	28.15	1.49	18.91	<0.001	***	Intercept Significant.
wool1	2.89	1.49	1.94	0.058	.	Not Significant. No difference between the effect of wool1 and its reference.
tension1	-6.48	2.11	-3.08	0.003	**	Significant. A Difference between the effect of tension1 and its reference.
tension2	8.24	2.11	3.91	<0.001	***	Significant. A Difference between the effect of tension2 and its reference.
wool1:tension1	0.00	2.11	0.00	1		Interaction not Significant. Effect wool1 vs. reference don't depends on tension1.
wool1:tension2	5.28	2.11	2.51	0.016	*	Interaction Significant. Effect wool1 vs. reference depends on tension2.

¹ '***': sign. to 0.1% error. '**': sign. to 1% error. '*': sign. to 5% error. '.' : sign. to 10% error. ' ': not sign. ' - ': no statement.

Total Influence of Factors (ANOVA Type III)

Variable	Sum.Sq	Df	F.value	P.value	Interpretation (5% error)
(Intercept)	42785.19	1	357.47	<0.001	Intercept significantly different from zero.
wool	450.67	1	3.77	0.058	There exist significant differences between the levels of factor 2.
tension	2034.26	2	8.50	<0.001	There exist significant differences between the levels of factor 3.
wool:tension	1002.78	2	4.19	0.021	There exist significant differences between the levels of factor 4.
Residuals	5745.11	48			

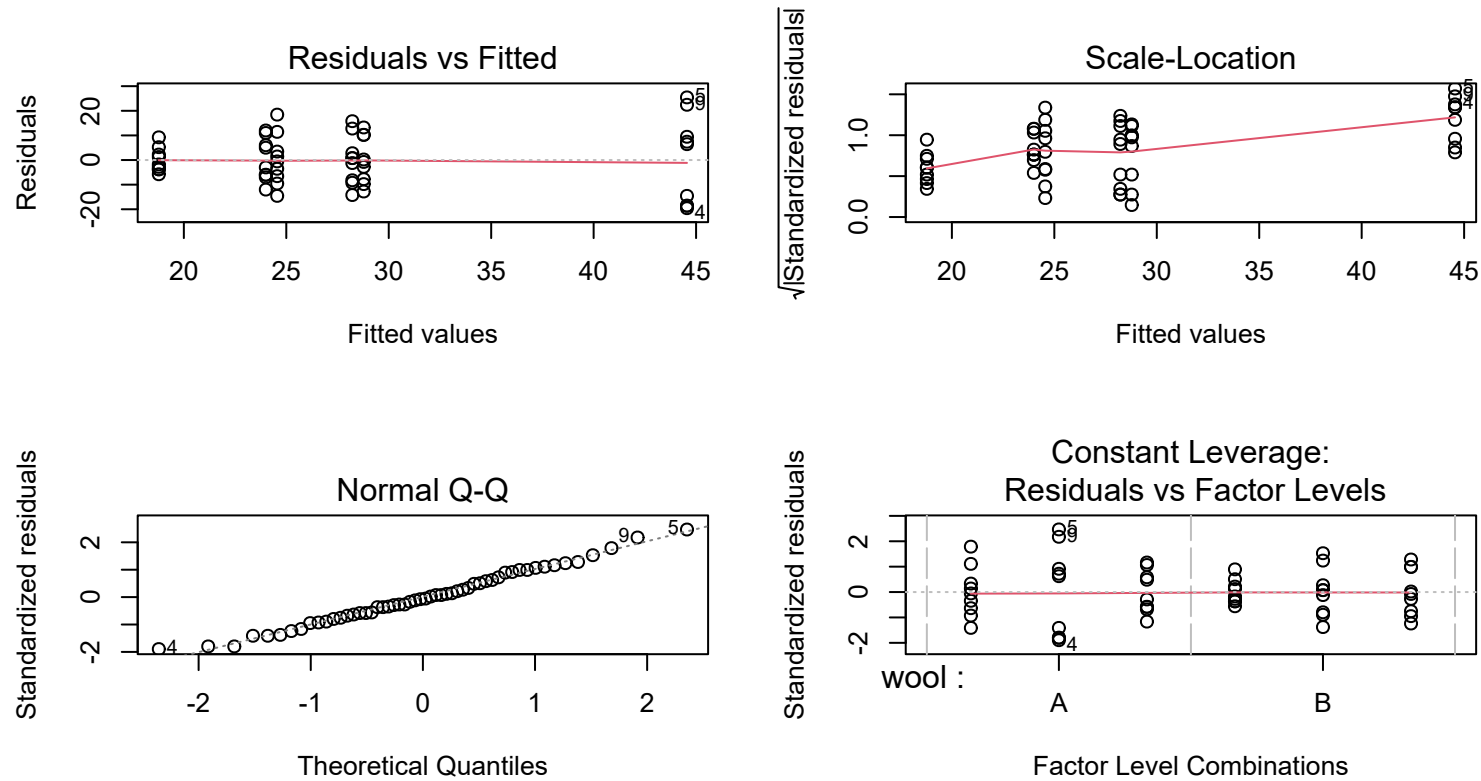
Goodness Of Fit Measures

To evaluate the model, some parameters are listed below.

	Values	Explanation	Interpretation
Multiple R-Squared	0.38	Fraction of variance explained by the model.	0: No fitting of data by the model. , 1: Perfect fit.
Adjusted R-Squared	0.31	Adjusted R-Squared by penalizing higher p.	A higher value means a better fit by the model.
F-statistic	3.77	Overall significance of the model.	Note the P-value to assess significance.
P-value	0.06	P-value of the F-statistic.	No significance to the 5% error. The model is better than the only-intercept model.

Diagnostics

Diagnostic Plots



Homogeneity of Variances

Levene Test of Homogeneity of Variances

	Df	F value	Pr(>F)
group	5	2.89	0.02
	48		

Warning: Group variances significantly heterogeneous at 1% error! The Analysis of Variance may not be valid.

Multiple Comparisons of Means to a Control

Theoretical background: Testing multiple hypotheses simultaneously and each at the same pre-specified significance level, increases the probability of false positive effects. The probability to commit at least one false positive decision increases with the number of hypotheses. A solution to overcome this problem is given by multiple comparisons procedures. Here, we do not control the per-hypothesis Type I error but the probability of committing at least one Type I error over all hypotheses. Using p-values adjusted for multiplicity, individual hypotheses can be finally compared with the pre-specified significance level.

Dunnet

Multiple Comparison: Dunnet Contrasts

Test whether the factor level A of the factor wool is less than the other levels. The Null Hypothesis is for example $B - A \geq 0$.

Null Hypothesis	Value	Std.Error	T.value	adjusted P.value	Sign. level ¹	Significance at 5 percent Type I error
$B - A \geq 0$	-5.78	2.98	-1.94	0.03	*	Significant. Level B of factor wool is significantly less than A ³

¹ '***': sign. to 0.1% error. '**': sign. to 1% error. '*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

² H1 does not hold significantly.

³ H1 holds significantly.

Simoultaneous Confidence Intervals: Dunnet Contrasts

Simultaneous Confidence Intervals which includes the true value of the difference between the reference level A and the other levels of wool

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
$B - A \geq 0$	-5.78	-Inf	-0.78	The interval $(-\text{Inf}, -0.78)$ traps the true difference B-A with probability 95 percent. ¹

¹ Remark: Zero is not in the confidence interval.

² Remark: Zero is in the confidence interval.

Dunnet Step-Down

Table 10: Multiple Comparison: Dunnet Contrasts

Null Hypothesis	Value	Std.Error	T.value	P.value	Sign. level ¹	Significance at 5 percent Type I error
$B - A \geq 0$	-5.78	2.98	-1.94	0.03	*	Significant. Level B of factor wool is significantly less than A ³

Table 10: Multiple Comparison: Dunnet Contrasts (continued)

Null Hypothesis	Value	Std.Error	T.value	P.value	Sign. level ¹	Significance at 5 percent Type I error
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¹ '***': sign. to 0.1% error. '**': sign. to 1% error. '*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

² H1 does not hold significantly.

³ H1 holds significantly.

Simultaneous Confidence Intervals which includes the true value of the difference between the reference level A and the other levels of wool

Table 11: Simoultaneous Confidence Intervals: Dunnet Contrasts

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
B - A >= 0	-5.78	-Inf	-0.78	The interval (-Inf, -0.78) traps the true difference B-A with probability 95 percent. ¹

¹ Remark: Zero is not in the confidence interval.

² Remark: Zero is in the confidence interval.

Dunnet Sandwich

The sandwich function provides a heteroskedasticity-consistent estimate of the covariance matrix. Thus, the sandwich estimate is resistant to the violation of the variance homogeneity assumption.

Multiple Comparison: Dunnett Contrasts Sandwich

Test whether the factor level A of the factor wool is less than the other levels. The Null Hypothesis is for example $B - A \geq 0$.

Null Hypothesis	Value	Std.Error	T.value	adjusted P.value	Sign. level ²	Significance at 5 percent Type I error
$B - A \geq 0$	-5.78	2.81	-2.06	0.02	*	Significant. Level B of factor wool is significantly less than A ⁴

¹ Note: Due to the applied sandwich estimator, the standard errors of the effects may be unequal.

² '***': sign. to 0.1% error. '**': sign. to 1% error. '*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

³ H1 does not hold significantly.

⁴ H1 holds significantly.

Simultaneous Confidence Intervals: Dunnett Contrasts Sandwich

Simultaneous Confidence Intervals which includes the true value of the difference between the reference level A and the other levels of wool.

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
$B - A \geq 0$	-5.78	-Inf	-1.07	The interval (-Inf, -1.07) traps the true difference B-A with probability 95 percent. ¹

¹ Remark: Zero is not in the confidence interval.

² Remark: Zero is in the confidence interval.

Dunnet Step-Down Sandwich

Multiple Comparison: Dunnet Contrasts step-down Sandwich

Test whether the factor level A of the factor wool is less than the other levels. The Null Hypothesis is for example $B - A \geq 0$.

Null Hypothesis	Value	Std.Error	T.value	P.value	Sign. level ¹	Significance at 5 percent Type I error
$B - A \geq 0$	-5.78	2.81	-2.06	0.02	*	Significant. Level B of factor wool is significantly less than A ³

¹ '***': sign. to 0.1% error. '**': sign. to 1% error. '*': sign. to 5% error. ' . ': sign. to 10% error. ' ': not sign. ' - ': no statement.

² H1 does not hold significantly.

³ H1 holds significantly.

Simultaneous Confidence Intervals: Dunnet Contrasts step-down Sandwich

Simultaneous Confidence Intervals which include the true value of the difference between the reference level A and the other levels of wool

Null Hypothesis	Value	Lower bound	Upper bound	Interpretation
$B - A \geq 0$	-5.78	-Inf	-1.07	The interval (-Inf, -1.07) traps the true difference B-A with probability 95 percent. ¹

¹ Remark: Zero is not in the confidence interval.

² Remark: Zero is in the confidence interval.

References

- Bretz, Frank, and Peter Westfall Torsten Hothorn. 2010. Multiple Comparisons Using R. 1st Edition. Chapman; Hall/CRC. <https://doi.org/10.1201/9781420010909>.
- Fox, John, and Sanford Weisberg. 2019. An R Companion to Applied Regression. Third. Thousand Oaks CA: Sage. <https://socialsciences.mcmaster.ca/jfox/Books/Companion/>.
- R Core Team. 2019. R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing. <https://www.R-project.org/>.
- Zeileis, Achim, and Torsten Hothorn. 2002. "Diagnostic Checking in Regression Relationships." R News 2 (3): 7–10. <https://CRAN.R-project.org/doc/Rnews/>.