Fault tolerant spanners

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1 Introduction

Let $\mathcal{F}$ be a family of regions in the plane, which we call the fault regions. For a fault region $F \in \mathcal{F}$ and a geometric graph G on a point set P , we define $G \ominus F$ to be the part of G that remains after the points from P inside F and all edges that intersect F have been removed from the graph. For simplicity, we assume that a region fault F does not contain its boundary, i.e., only vertices and edges intersecting the interior of F will be affected.

Let $\mathcal{L}$ be a family of regions in the plane, which we call the local regions. For a fault region $L \in \mathcal{L}$ and a geometric graph G on a point set P , we define $G \mid_F$ to be the part of G contained in the interior of F, meaning only the vertices and edges that are fully containd in the interior of F.

The problem: Given a set P of points in $\mathbb{R}^2$, and a family $\mathcal{F}$ of regions, compute a graph G that is a t-spanner of P under any fault $F \in \mathcal{F}$.

2 Complement of disk faults / disk local spanners

Let $\mathcal{L}$ be the set of disks, we can use the algorithm of Abam et al. [**?**], and replace the set of edges between every pair (A_i, B_i) in the SSPD with the edges of the Delaunay triangulation $\mathcal{DT}(P)$ with one end in A_i and the other in B_i. We only need to prove that for any disk $d \in \mathcal{L}$, and for any set P' of points we have that $\mathcal{DT}(P') \mid_d$ is connected, as that will imply that for every pair (A_i, B_i) of the SSPD, if $A_i \cap d$ and $B_i \cap d$ are not empty, there exists an edge between them in the constructed graph G.

Claim 1. *For a set of points $P \subseteq \mathbb{R}^2$ and for any disk d, $\mathcal{DT}(P) \mid_d$ is connected.*

Proof. We prove a different claim that immediately implies the desired one. Let d be a disk with two points $p, q \in P$ on its boundary. Then there is a path between p and q in $\mathcal{DT}(P) \mid_d$.

We prove by induction over the number points in the interior of d.

$|d \cap (P \setminus \partial d)| = 0$: Then by construction of the Delaunay triangulation the edge $\{p, q\}$ is in $\mathcal{DT}(P)$ and is contained in the interior of D.

$|d \cap (P \setminus \partial d)| > 0$: Let $x \in P$ be a point in the interior of d. We move the center of d in the direction of p, shrinking d in the process, until we get a disk $d' \subseteq d$ such that x is on the boundary of d'. By induction there is a path between p and x in $\mathcal{DT}\mid_{d'}$, and since $\mathcal{DT}\mid_{d'} \subseteq \mathcal{DT}\mid_{d}$ we have that the same path exists in $\mathcal{DT}\mid_{d}$. The same proof gives us a path between x and q and thus we are done.

$\square$

3 Complement of square faults / square local spanners

For the same reasons, if we can prove that for a square s we have that $\mathcal{DT}_\infty\mid_s$, where $\mathcal{DT}_\infty$ is the L_∞ norm Delaunay triangulation, is connected, we would be able to use the same algorithm almost verbatim. Fortunately, the same proof works just as well when replacing disks with squares.

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