1) Introduction

Witten 1989: Jones Polynomial $\longleftrightarrow$ Holonomy in Chern Simons Theory on S^3
$\mathbb{Z}[t^{-1}, t^{1}]$

$$J(K) = \int \mathcal{D}A \; \underbrace{\mathrm{Tr}\left(P\exp\left(\oint_K A \, ds\right)\right)}_{\text{Holonomy / Wilson loop}} \; \exp\Big(\underbrace{\frac{ik}{4\pi}\int_{S^3} \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A\wedge A \wedge A\right)}_{S[A] - \text{Independent of } g}\Big)$$

$\underbrace{\text{Metric Independence}}_{\text{Topological}}$ $\rightarrow$ Diffeomorphism.

$$Z(M) = \int \mathcal{D}A \; \exp(iS[A]) \quad , M \text{ is a 3-manifold}$$

If $S[A]$ is topological $\rightarrow$ <u>$Z(M)$ is an invariant of M</u>

Recipe for Invariants (M)

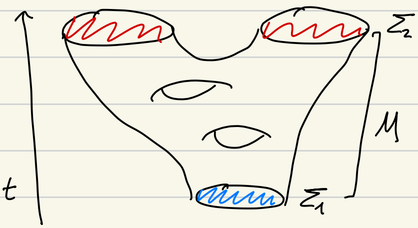
1) Endow M with a Connection

2) Compute $S[A]$

3) Weighted over all As.

Cobordisms

3-Manifold M , $\partial M = \Sigma = \Sigma_1 \sqcup \Sigma_2$



M is a cobordism from Σ_1 to Σ_2.

Cobordisms & QFT:

To each Σ (2d closed) we associate a Hilbert Space $\mathcal{H}_\Sigma = \mathcal{A}(\Sigma)$: Space of fields living on Σ.

Physical state $\psi \rightarrow$ Functional of $\mathcal{A}(\Sigma)$. $A \in \mathcal{A}(\Sigma)$, $\psi(A) \rightarrow$ Amplitude : ψ in A.

$\mathcal{A}(\Sigma)$ basis: Delta Functionals $\langle \hat{A} | \hat{A}' \rangle = \delta(A - A') \rightarrow \psi$ over $\hat{A}$

$$\langle \hat{A}_2 | U | \hat{A}_1 \rangle = \int_{A|_{\Sigma} = \hat{A}_1}^{A|_{\Sigma_2} = \hat{A}_2} DA \, \exp(i S[A])$$

$U(M)$: Time Evolution Operator.

To every spacelike manifold Σ we associate a Hilbert space $A(\Sigma) = \mathcal{H}_\Sigma$

To each cobordism M from Σ_1 to Σ_2, we associate a time evolution operator $U(M): \Sigma_1 \to \Sigma_2$

Definition QFT

A functor from nCob to Hilb $\rightarrow$ Z: nCob $\rightarrow$ Hilb

φ^4 Theory: $\Sigma_t = \mathbb{R}^3 \times t$, $M_{[t_1, t_2]} = \mathbb{R}^3 \times [t_1, t_2]$

TQFT: Correlation function that are topological.

$\mathcal{O}_1, \mathcal{O}_2 \dots \mathcal{O}_n : A(\Sigma) \to \mathbb{C}$

metric independent

$$\langle \mathcal{O}_1 \mathcal{O}_n \rangle = \frac{1}{Z} \int DA \; \mathcal{O}_1 \mathcal{O}_n \exp(i S[A])$$

Metric - Independence $\rightarrow$ Diffeomorphism Invariance